M-N interaction in beam-to-column joints – Development of a design model

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ABSTRACT: The component method is a nowadays widely recognized procedure for the evaluation of the design properties of structural joints. It is used as a reference method in the Eurocodes and the proposed rules in these codes are mainly devoted to the characterization of joints subjected to bending moments and shear forces. However, in some situations, these joints can be subjected to combined axial loads and bending moments, for instance in frames subjected to an exceptional event leading to the loss of a column, situation where significant tying forces can developed in the structural beams above the lost column. In this paper, a design model, founded on the component method, aiming at predicting the behaviour of joints subjected to combined axial loads and bending moments is presented. In particular, it is illustrated how this model was validated through comparisons to recent experimental tests performed on steel-concrete composite beam-to-column joints.

1 INTRODUCTION

The component method is a nowadays well-known and widely recognized procedure for the evaluation of the design properties of structural joints. It is used as a reference method in Eurocode 3 and Eurocode 4, respectively for joints in steel and composite constructions, but it may apply to many other joint configuration and connection types.

In the component method, any joint is seen as a set of elements (called components). The mechanical properties of these components, in terms of elastic deformation, design resistance and deformation capacity are evaluated through appropriate design models; then the components properties are "assembled" to finally derive the mechanical properties of the full joint, i.e. its rotational stiffness, its moment and shear design resistances, its collapse mode and its level of rotational capacity.

In the Eurocodes, the proposed rules are mainly devoted to the characterization of joints subjected to bending moments and shear forces. It is the reason why, in Part 1.8 of Eurocode 3 dedicated to the design of steel joints, the proposed field of application is limited to joints in which the force (N_{Ed}, also noted N in the paper for sake of simplicity – and the same applies to M_{Ed}, noted M -) acting in the joint remains lower than 5% of the axial design resistance $N_{pl,Rd}$ of the connected beam (and not of the joint what is quite surprising as far as the influence of the applied axial load on the joint response is of concern):

$$\left|\frac{N_{Ed}}{N_{pl,Rd}}\right| \leq 0.05 \tag{1}$$

Under this limit it is considered that the rotational response of the joints is not significantly influenced by the axial forces. It has however to be stated that this value is a fully arbitrary one and is not at all scientifically justified.

However, in some situations, these joints can be subjected to combined axial loads and bending moments, for instance at the extremities of inclined roof beams or in frames subjected to an exceptional event leading to the loss of a column, situation where significant tying forces can developed in the structural beams above the lost column.

If the criterion of Equation 1 is not satisfied, the Eurocodes recommend to check the resistance by referring to "*M-N*" interaction diagram defined by the polygon linking the four points corresponding respectively to the hogging and sagging bending resistances in absence of axial forces and to the tension and compression axial resistances in absence of bending (see Figure 1).

In a previous study (Cerfontaine, 2003), it has been shown that the proposed method is quite questionable. So, an improved design analytical procedure, based on the component method concept, has been (i) developed by Cerfontaine to predict the response of ductile and non-ductile steel joints subjected to combined axial loads and bending moments and (ii) extended to composite joints in (Demonceau, 2008). This method is briefly introduced in the present article; in particular, it is illustrated how this model was validated through comparisons to recent experimental tests performed on steel-concrete composite beam-to-column joints.

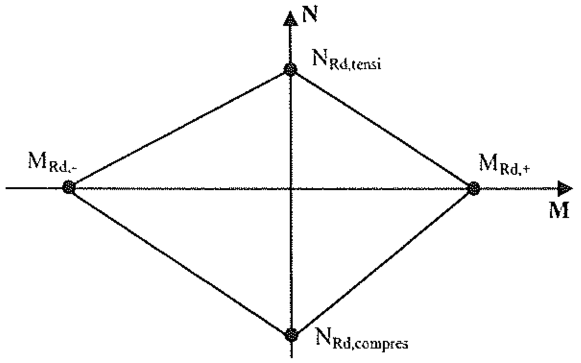

Figure 1. M-N resistant curve for a joint proposed in the Eurocodes.

2 DEVELOPED DESIGN MODEL FOR JOINTS SUBJECTED TO COMBINED M-N

Within this section, a brief description of the analytical procedure developed in (Cerfontaine, 2003) is given.

2.1 *General concept*

In order to develop the improved design procedure according to the component method (which is still valid as the behaviour of the components is independent on the type of loading applied to the whole joint), a new assembly procedure was defined to cover the combined action of bending moments and axial loads. The main difficulty results from the modification of the list of active components within the joints according to the relative importance of the bending moment and axial load, and obviously according to the respective signs of the applied forces.

Two particular features of the component method had also to be carefully considered within the developed procedure:

- *Group effects*: these effects are likely to occur in plate components subjected to transverse bolt forces (i.e. mainly the components end-plate and column flange in bending). Where a bolt force is applied, a yield plastic mechanism may develop in the plate component; if the bolt distances are high, separate yield lines will form in the plate component around the bolts (namely "individual bolt mechanism"), while a single yield plastic mechanism common to several bolts may develop when the distance between the latter decreases (namely "bolt group mechanism" – see Figure 2). Group effects also affect the resistance of other components as the column web in tension and the beam web in tension.

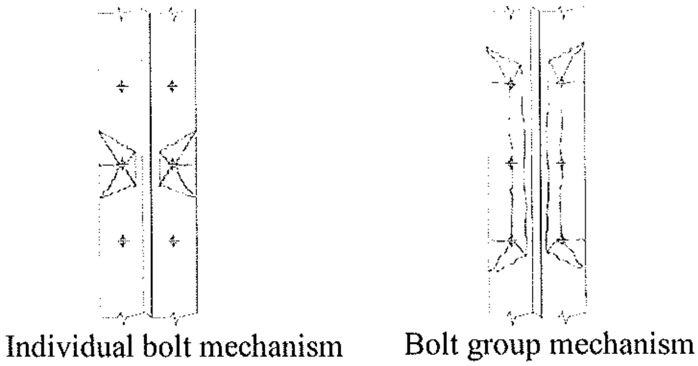

Figure 2. Example of possible plastic mechanism in column flanges or end-plates.

- *Component interactions*: these ones may occur in components "extracted" from the column where three types of stresses interact: shear stresses in the web panel, longitudinal stresses due to axial and bending forces in the column and transversal stresses due to the load-introduction in the joint area (column web in tension, column web in compression and column web in shear).

Within this section, developments are presented for the general case of a bolted end-plate steel connection with N_b bolt rows in which only tension loads may be transferred; in addition, two compression zones located at mid-thickness of the upper and lower beam flanges are identified (respectively noted "upper" or "up" and "lower" or "lo"). The compression zones are constituted of two components: the beam flange and web in compression and the column flange in compression.

So, this leads to a total of $N_b+2 = n$ rows where internal forces may be transferred from the beam to the column. Conventionally, the tension forces are assumed to be positive (or equal to zero) while a compression force has a negative (or zero) value. All the rows are numbered from 1 to n by starting from the upper row. An example of row numbering is given in Figure 4.

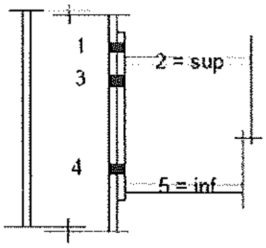

Figure 4. Example of bolt row numbering with an extended end-plate connection.

2.2 *Equilibrium equations for the connection and load eccentricity*

The evaluation of the resistance of the connection based on the static theorem requires at failure an equilibrium between the distribution of internal forces and the external applied loads. For connections subjected to a bending moment M_{Ed} and an axial load N_{Ed}, the equilibrium criteria are:

$$M_{Ed} = \sum_{i=1}^{n} h_i.F_i \text{ and } N_{Ed} = \sum_{i=1}^{n} F_i \quad (2)$$

where F_i is the load in row i and h_i is the corresponding lever arm; the latter is defined as the vertical distance between the reference beam point where M_{Ed} and N_{Ed} are computed and the row itself (h_i values are positive for rows located on the upper side of the reference point).

2.3 *Resistance criteria*

According to the static theorem, the resistance of each row, which is equal to the resistance of the weakest component in the row, should never be exceeded. At first sight, it looks easy as long as individual resistances of bolt-rows are considered but it is much more questionable when group effects develop in the connections.

Within the developed procedure, any group of rows noted $[m, p]$, i.e. from row m to row p, in which group effects appear is considered as an equivalent fictitious row with an equivalent lever arm and a group resistance equal to that of the weakest group component. Therefore, the resistance criteria for each of the rows belonging to the $[m, p]$ group may write, for any constitutive component α:

$$\sum_{=m}^{p} F_i \le F_{mp}^{Rd,\alpha} \quad (3)$$

with $m = 1,...,p$ and $p = m, m+1,...,n$

where $F_{mp}^{Rd,\alpha}$ is the resistance of the component α for the group of rows m to p. When m is equal to p, $F_{mp}^{Rd,\alpha}$ designates the individual resistance of the component α for row m. Such a resistance of the group of rows $[m, p]$, noted F_{mp}^{Rd}, is defined as the smallest of the $F_{mp}^{Rd,\alpha}$ values.

This situation is illustrated in Figure 5 for a connection with three bolt rows (1, 2 and 3) but more generally covers the case of any connection with n rows in which group effects would develop in three bolt rows.

2.4 *Definition of the failure criterion for the whole connection*

Each point of the *M-N* resistant curve is obtained by expressing the failure criteria given in Equation 4 for k from 1 to n; this criterion is defined so as to get maximum resistance bending moments by optimising the distributions of the loads amongst the activated bolt rows with account of the group effects.

$$M = h_k.N + \sum_{i=1}^{n} (h_i - h_k).F_i^c \quad (4)$$

Within this expression, two different resistances F_i^c can be attributed to the same row i (F_i^{Rd+} and F_i^{Rd-}). The evaluation procedure of the F_i^{Rd+} and F_i^{Rd-} values is illustrated in Figure 5 for a connection with three bolt rows where the black and white dots respectively show the successive steps for the evaluation of F_i^{Rd+} and F_i^{Rd-}; the objective of the evaluation procedure is to obtain the maximum resistant bending moment by maximising (or minimising if the sign of the bolt row resistance is negative) the loads in the bolt rows which are the most distant from the bolt row "k". More details about the definition of this failure criterion can be obtained in (Cerfontaine, 2003).

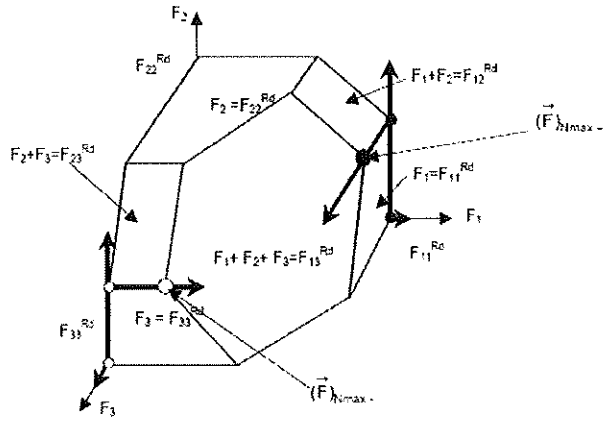

Figure 5. Possible group effects between three bolt rows and successive steps for the evaluation of a connection resistance (black and white dots respectively).

3 VALIDATION OF THE DEVELOPED DESIGN MODEL FOR COMPOSITE JOINTS

3.1 *Extension of the analytical procedure to composite joints*

The analytical model presented in § 2 was initially developed for steel joints. In (Demonceau, 2008), the latter was easily extended to composite joints as the developed model is based on the component method.

The particularity of composite joint configurations is the fact that two main additional components are activated: the slab rebars in tension and the concrete slab in compression.

The component "slab rebars in tension" is already covered by Eurocode 4 (in term of resistance and stiffness). What is assumed within the latter is the fact that this component can be considered as a bolt row and is activated in parallel with the other "tensile" components.

For the component "concrete slab in compression", no rule is actually proposed within Eurocode 4. It is the reason why formulas were developed and validated in (Demonceau, 2003) to characterize this component, which consists in defining a rectangular concrete cross-section with a width $b_{eff,conn}$ and a height z contributing to the joint resistance. The value of z is defined expressing the equilibrium between the components in tension and the concrete slab in compression. As a consequence, when this component is activated within a composite joint subjected to M-N, the value of z (and accordingly, the level arm h_k associated to this component) will continuously change, what results to an incurve segment in the M-N resistance curve of the joint as illustrated in Figure 6 (from point A to point B).

3.2 *Validation of the proposed analytical procedure*

The validity of the proposed analytical procedure presented in the previous section was checked through comparisons to results of experimental tests performed in Stuttgart in the framework of an RFCS project (Kuhlmann et al, 2008).

The comparison of so-the obtained analytical predictions to the experimental test results is presented in Figure 6. On the latter, it can be observed that two analytical curves are reported:

- One named "plastic resistance curve" which is computed with the elastic strengths of the materials and;
- One named "ultimate resistance curve" which is computed with the ultimate strengths of the materials.

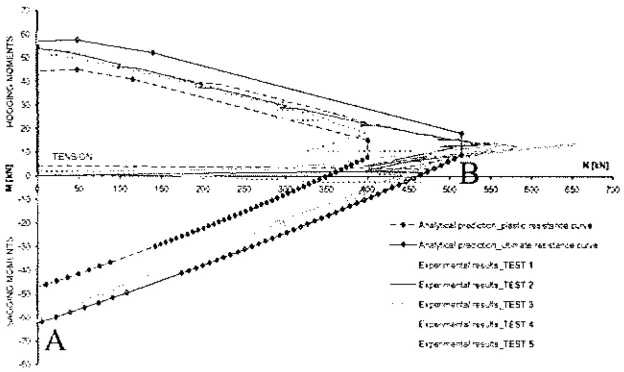

Figure 6. Comparison of the resistance interaction curves

According to Figure 6, the computed analytical curves are in very good agreement with the experimental results. Indeed, the experimental curves are between the plastic and ultimate analytical resistant curves, which is in line with the loading sequence followed during the tests.

In the hogging moment zone, it can be observed that for very small values of tensile loads, the experimental curves are close to the ultimate analytical one which is logical, as a bending moment close to the ultimate resistant bending moment of the joint was first applied to the tested specimen before applying the tensile load. Then, when the tensile load is increasing, the experimental curves first deviate from the ultimate analytical one to finally come back close to the ultimate analytical curve (except for TEST 1). This phenomenon can be explained by the fact that, to pass from the ultimate hogging moment to the ultimate tensile resistant load, different components are activated; indeed, the component which is associated to the ultimate hogging moment is "beam flange in compression" while the one associated to the ultimate tensile load is the component "column flange in bending".

This phenomenon is not observed in the sagging moment zone; indeed, as shown in Figure 6, the experimental curves are close to the ultimate analytical one from the pure bending to the ultimate tensile load. Again, it is in agreement with the component activated from the pure bending to the ultimate tensile load which is the same in the present case, i.e. the component "column flange in bending".

4 CONCLUSIONS

In the present paper, a design model, developed at Liège University, able to predict the M-N resistant curve of steel and steel-concrete composite joints was briefly introduced.

In particular, it was shown how this model was validated through comparisons to recent experimental tests performed on steel-concrete composite beam-to-column joints.

More details about this model are available in (Cerfontaine, 2003) and in (Demonceau, 2008).

5 REFERENCES

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