Modelling ice flow for large-scale ice-sheet simulations

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Outline

(1) Introduction

(2) Dynamics of ice sheets

(3) Common approximations for large-scale dynamics

(4) Essential mechanisms for large-scale dynamics

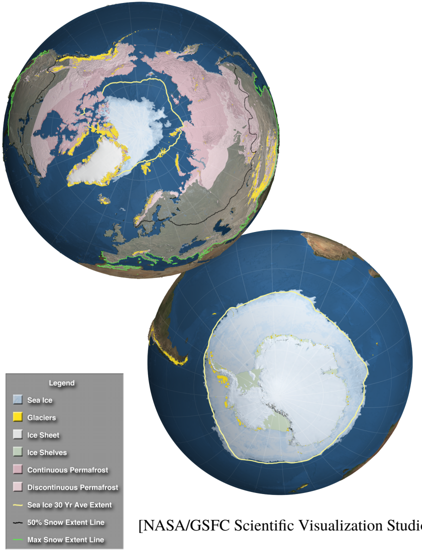
(5) Uncertainty quantification for ice-sheet modelling

(6) Conclusion

Introduction

The Earth's cryosphere

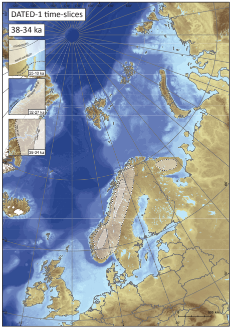
- The cryosphere includes:
  - Sea ice
  - Glaciers
  - Ice sheets
  - Ice caps
  - Permafrost
  - River ice
  - Lake ice



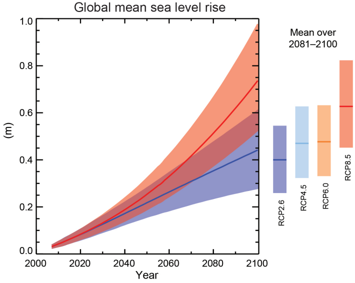
[NASA/GSFC Scientific Visualization Studio]

Why modelling large ice sheets?

- Simulating the evolution of ice sheets during the last glacial periods.



Simulation of the last glacial period [Hughes et al.,2016]



Projected sea-level rise [IPCC, 2013]

- Simulating the future response of present-day ice sheets under climate changes.

Dynamics of ice sheets

How to model ice-sheet flow?

- The motion of an ice sheet is driven by creep deformation induced by the force of gravity and possibly basal sliding.
- On large temporal scales relevant in glaciology, ice motion is usually described as the flow of a highly viscous fluid.

Elephant-foot glacier (Roman Lake, Greenland)

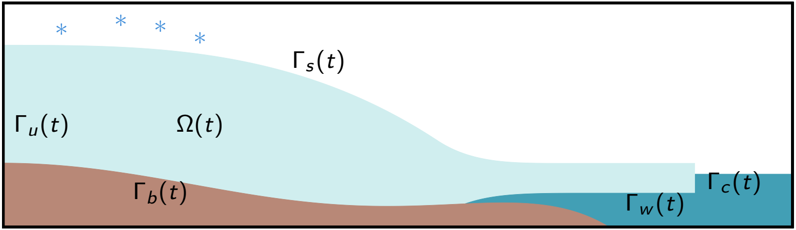
Glacier flow through narrow valleys (Axel Heiberg Island)

The full Stokes flow problem

- Ice motion is described as a quasistationary Stokes flow:

$$
\begin{aligned}
&-\nabla_{\mathbf{x}} p + 2\mathrm{div}_{\mathbf{x}}(\eta \mathbf{D}) + \rho \mathbf{g} = \mathbf{0} && \text{in } \Omega(t) \text{ for } 0 < t < \tau,\\
&\mathrm{div}_{\mathbf{x}} \mathbf{v} = 0 && \text{in } \Omega(t) \text{ for } 0 < t < \tau,\\
&-p\mathbf{n} + 2\eta \mathbf{D}(\mathbf{n}) = \mathbf{0} && \text{on } \Gamma_s(t) \text{ for } 0 < t < \tau,\\
&-p\mathbf{n} + 2\eta \mathbf{D}(\mathbf{n}) = -p_w \mathbf{n} && \text{on } \Gamma_w(t) \cup \Gamma_c(t) \text{ for } 0 < t < \tau,\\
&\begin{cases} \mathbf{v}_t = -\alpha_b \|\mathbf{t}_t\|^{m-1} \mathbf{t}_t \\ (\mathbf{w} - \mathbf{v}) \cdot \mathbf{n} = 0 \end{cases} && \text{on } \Gamma_b(t) \text{ for } 0 < t < \tau.
\end{aligned}
$$

- The free surfaces evolve due to ice dynamics, snow accumulation, ablation, basal melting and calving.

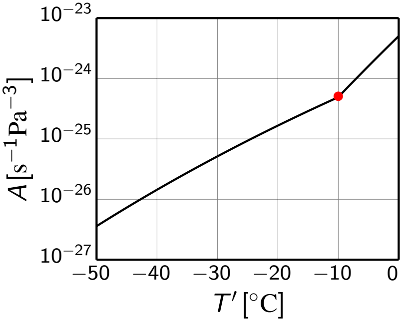


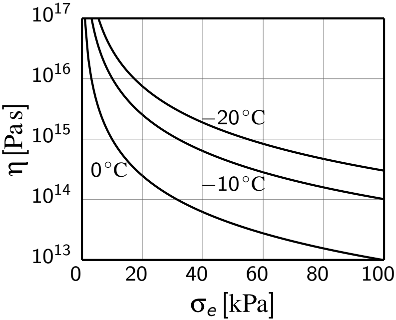
Ice rheology

- Ice is described as a shear-thinning material whose effective viscosity is given as a power law (Glen's power law):

$$\eta(T', de) = \frac{1}{2} A(T')^{-1/n} d_e^{-(1-1/n)} \text{ with } d_e = \sqrt{\frac{1}{2}\mathrm{Tr}\left(\mathbf{D}^2\right)}$$

- Ice viscosity is highly temperature-dependent (one needs to solve a thermal problem).

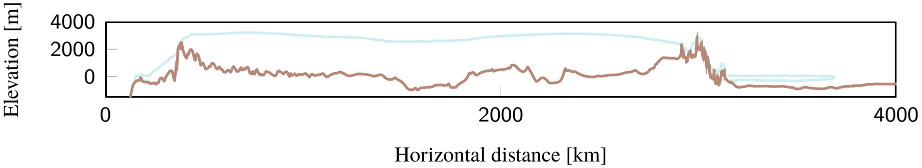




Common approximations for large-scale dynamics

Shallow approximation for ice-sheet flow

- Solving the full Stokes problem is computationally and mathematically challenging.
- Ice flow over large ice sheets is essentially a thin-film flow (aspect ratio $\varepsilon \sim 10^{-3}$ for Antarctica).
- Simplified ice dynamical models are derived from the Stokes equations by dropping higher-order terms in ε.



Transect along the Antarctic ice sheet

The first-order approximation (Blatter-Pattyn model)

- The FOA model is an approximation to the Stokes equations accurate to $O(\varepsilon^2)$.
- Vertical normal stress is hydrostatic $\Rightarrow$ the pressure is decoupled from the velocity.
- $\frac{\partial v_z}{\partial x} \ll \frac{\partial v_x}{\partial z}$ and $\frac{\partial v_z}{\partial y} \ll \frac{\partial v_y}{\partial z}$ $\Rightarrow$ the vertical velocity is decoupled from the horizontal velocity.
- FOA model:

$$4\frac{\partial}{\partial x}\left(\eta\frac{\partial v_x}{\partial x}\right)+2\frac{\partial}{\partial x}\left(\eta\frac{\partial v_y}{\partial y}\right)+\frac{\partial}{\partial y}\left(\eta\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial v_x}{\partial z}\right)=\rho g\frac{\partial h}{\partial x}$$

$$4\frac{\partial}{\partial y}\left(\eta\frac{\partial v_y}{\partial y}\right)+2\frac{\partial}{\partial x}\left(\eta\frac{\partial v_x}{\partial x}\right)+\frac{\partial}{\partial x}\left(\eta\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial v_y}{\partial z}\right)=\rho g\frac{\partial h}{\partial y}$$

The shallow-ice approximation (SIA model)

- The SIA model is a lubrication flow model (shearing across the film thickness balances the pressure gradient). It approximates the FOA model under slow sliding conditions.
- Vertical normal stress is hydrostatic $\Rightarrow$ the pressure is decoupled from the velocity.
- $\frac{\partial v_z}{\partial x} \ll \frac{\partial v_x}{\partial z}$ and $\frac{\partial v_z}{\partial y} \ll \frac{\partial v_y}{\partial z}$ $\Rightarrow$ the vertical velocity is decoupled from the horizontal velocity.
- Ice flow is dominated by shearing. Only p, σ^D_{xz} and σ^D_{yz} are non-negligible in the stress tensor.
- SIA model:

$$\mathbf{v} = \mathbf{v}_b - 2(\rho g)^n \|\nabla_{\mathbf{x}} s\|^{n-1} \int_{b(\mathbf{x},t)}^{z} A(T')(s-z')dz' \nabla_{\mathbf{x}} s.$$

The shallow-shelf approximation (SSA model)

- The SSA model is a thin-film model with wall slip. It approximates the FOA model under fast sliding conditions. It is mostly relevant for ice streams and ice shelves.
- Vertical normal stress is hydrostatic $\Rightarrow$ the pressure is decoupled from the velocity.
- $\frac{\partial v_z}{\partial x} \ll \frac{\partial v_x}{\partial z}$ and $\frac{\partial v_z}{\partial y} \ll \frac{\partial v_y}{\partial z}$ $\Rightarrow$ the vertical velocity is decoupled from the horizontal velocity.
- Ice flow is dominated by sliding. The horizontal velocity is constant over the ice thickness $\Rightarrow$ Equations are integrated over the ice thickness.
- SSA model:

$$4\frac{\partial}{\partial x}\left(\bar{\eta}h\frac{\partial v_x}{\partial x}\right)+2\frac{\partial}{\partial x}\left(\bar{\eta}h\frac{\partial v_y}{\partial y}\right)+\frac{\partial}{\partial y}\left(\bar{\eta}h\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\right)=\rho g h\frac{\partial h}{\partial x}$$
$$4\frac{\partial}{\partial y}\left(\bar{\eta}h\frac{\partial v_y}{\partial y}\right)+2\frac{\partial}{\partial x}\left(\bar{\eta}h\frac{\partial v_x}{\partial x}\right)+\frac{\partial}{\partial x}\left(\bar{\eta}h\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\right)=\rho g h\frac{\partial h}{\partial y}$$

Essential mechanisms for large-scale dynamics

Essential feedback mechanisms

- Whatever its level of approximation, an appropriate ice-sheet model should be able to reproduce the essential mechanisms that govern ice-sheet dynamics:

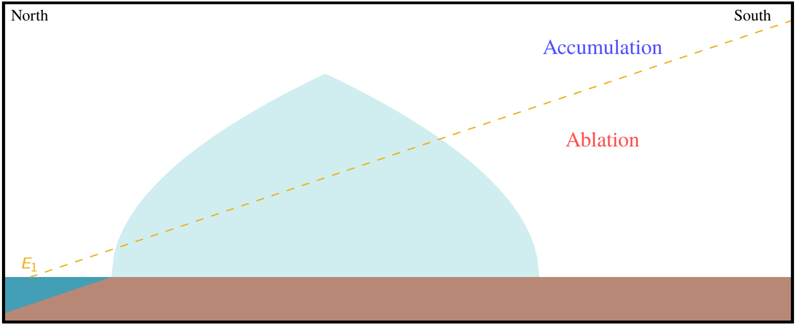
  (1) Melt-elevation feedback: A lowering of the ice-sheet surface induces a positive feedback on ablation [Dijkstra,2013].

  (2) Marine ice-sheet instability (MISI): Marine ice sheets on retrograde slope are instable [Schoof,2007].

  (3) Thermomechanical instability: Changes in basal sliding can induced thermal oscillations [Robel et al.,2013,2014].

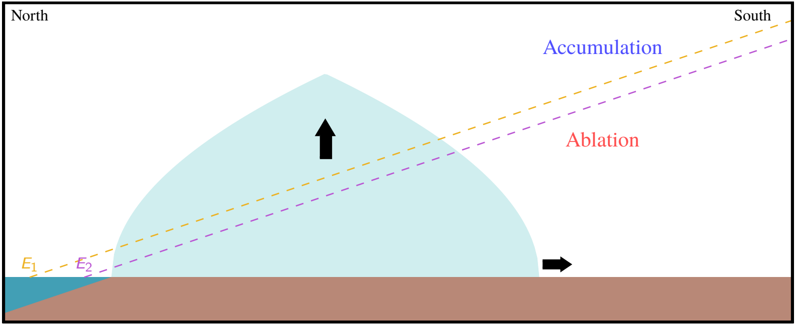
Melt-elevation feedback

- Step 1: Steady state for equilibrium line E_1.



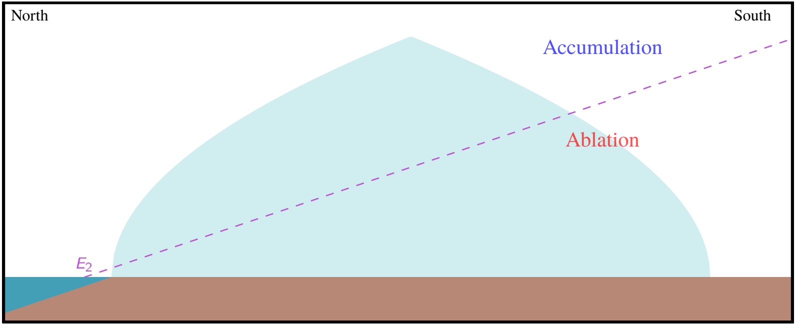
Melt-elevation feedback

- Step 2: Perturbation of the equilibrium line + initiation of ice-sheet growth.



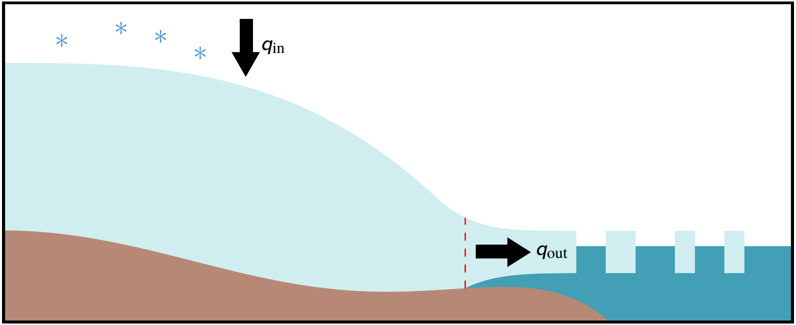
Melt-elevation feedback

- Step 3: Steady state for new equilibrium line E_2.



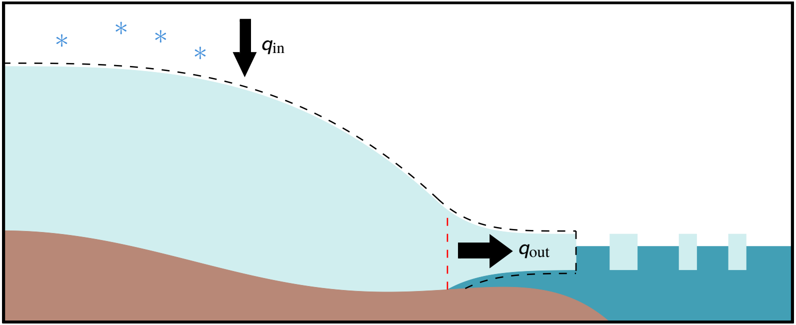
Marine ice sheet instability mechanism

- Step 1: Steady state on an upward sloping bed ($q_{\text{in}} = q_{\text{out}}$).



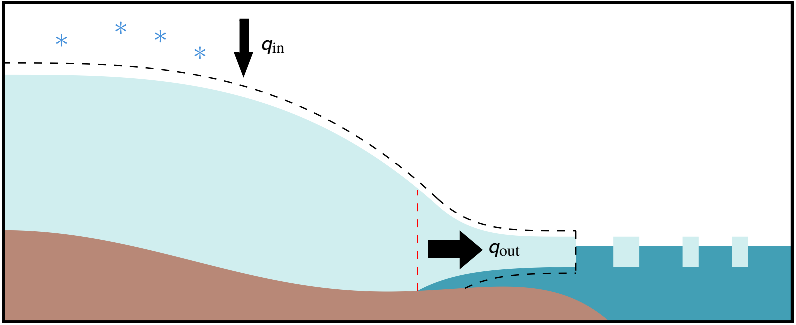
Marine ice sheet instability mechanism

- Step 2: Initiation of grounding line retreat ($q_{\text{in}} < q_{\text{out}}$).



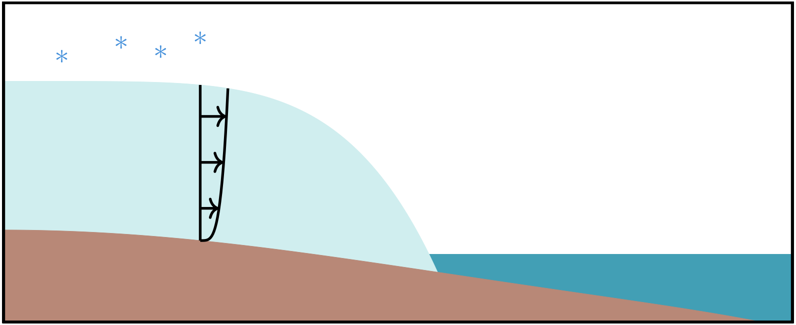
Marine ice sheet instability mechanism

- Step 3: Self-sustained grounding line retreat ($q_{\mathrm{in}} \ll q_{\mathrm{out}}$).



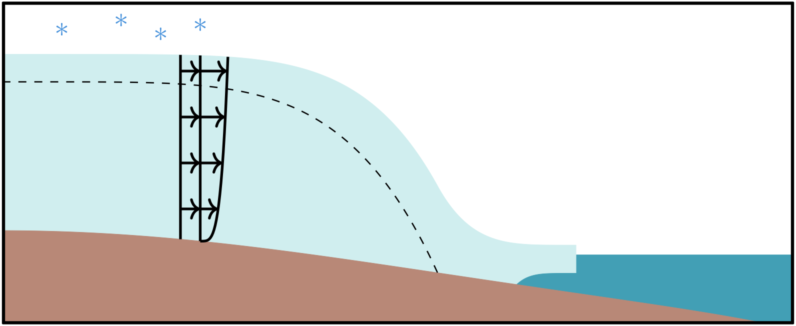
Thermal induced oscillations mechanism

- Step 1: Ice sheet build-up on a frozen bed (binge phase).



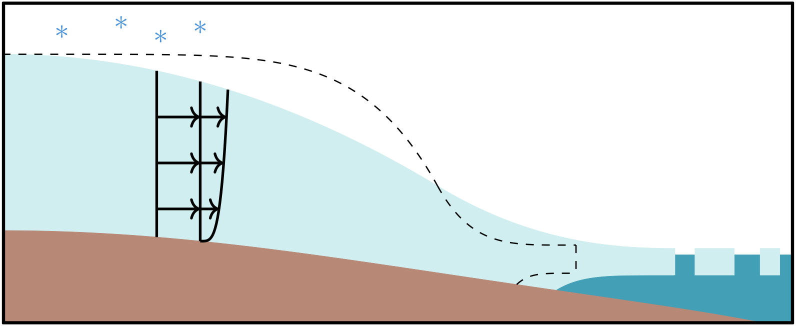
Thermal induced oscillations mechanism

- Step 2: Binge/Purge transition.



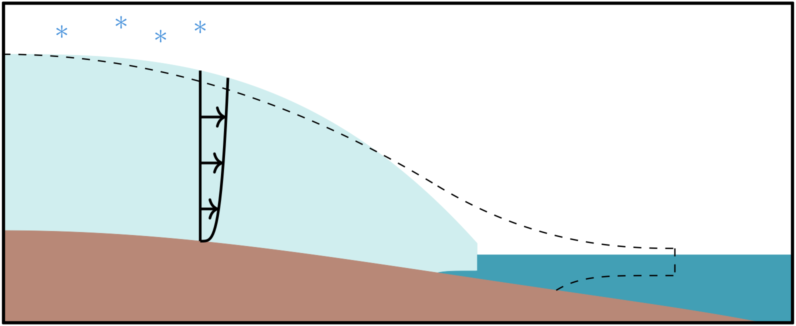
Thermal induced oscillations mechanism

- Step 3: Rapid basal motion (purge phase).



Thermal induced oscillations mechanism

- Step 4: Purge/Binge transition.



Uncertainty quantification for ice-sheet modelling

How are ice-sheet models affected by uncertainties?

Bedrock elevation

Geothermal heat flux

Surface temperature

Surface accumulation

Sub-shelf melting

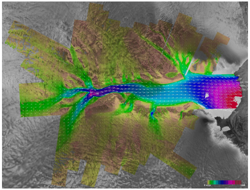
Atmospheric forcing

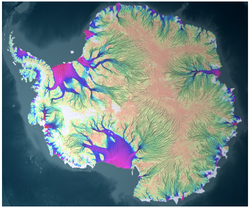
Basal sliding coefficient

. . .

+ uncertainty

$\longrightarrow$





$\longrightarrow$

Change in volume

Change in area

Grounding line position

Ice velocity

Instability analysis

. . .

Input variables

$(x_1, x_2, \ldots, x_m)$

Model

$y = g(x_1, x_2, \ldots, x_m)$

Output variable

y

Conclusion

Conclusion

- Ice sheets are modelled as thin-film flows driven by the force of gravity and basal sliding. These shallow models are the basis for ice-sheet mathematical and numerical models.
- Many challenges still remain in understanding and modelling physical mechanisms appearing in ice sheets such as basal sliding, calving and basal melting underneath ice-shelves.
- Challenges remain in quantifying the role of uncertainties in ice-sheet models on the response and stability of ice-sheets under warming climate conditions.

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