Elastic scattering, total cross sections and ρ parameters at the LHC

Jean-René Cudell

November 13, 2017

(mainly) with O.V. Selyugin

LIÈGE université

Analytic S matrix theory

$$\mathcal{A}(s,t,u)$$

elastic amplitude dominated at large *s* by singularities in the complex *j* plane

$$\mathcal{A} = \sum_i C_i s^{\alpha_i(t)}$$

in the case of simple poles

unknown functions of t

$$+ \int d\omega\ c(\omega) s^{\alpha(\omega,t)}$$

simple poles imply cuts

Knowing the discontinuities
of the (C=+1 and C=-1) amplitudes
(the imaginary part)
allows one to reconstruct
the whole amplitude

Real part/s — unknown — Imaginary part/s

$$\rho_\pm \sigma_\pm = \frac{A_\pm(s_0,0)}{2m_p p} + \frac{E}{\pi p}\mathrm{P}\int_{m_p}^{\infty}\left[\frac{\sigma_\pm}{E'(E'-E)} - \frac{\sigma_\mp}{E'(E'+E)}\right] p'\,dE'$$

+=pp; -=$p\bar{p}$, $t=0$

The unitarity of the S matrix
(i.e. $\sigma_{elastic} < \sigma_{tot}$)
implies

$$|A(s,b)|^2 \leq 2 Im(A(s,b))$$

$$(A(s,b) = \mathcal{A}(s,b)/s)$$

Fourier transform of transverse momentum variable
≈partial wave

$$(\ell = b\sqrt{s})$$

This leads to

$$\frac{\sigma_{tot}}{\log^2 s} \to constant \quad \text{as} \quad s \to \infty$$

2 classes of models

Analytic fits: *many parameters*

- pick a number of known analytic functions
- fit the data
- reduce the number of parameters via some assumption (universality, quark counting, etc.)

Physical models: *many assumptions*

- choose a low-energy parametrisation
- extrapolate it to high energy
- implement unitarity

Analytic parametrisations

Usually for t=0

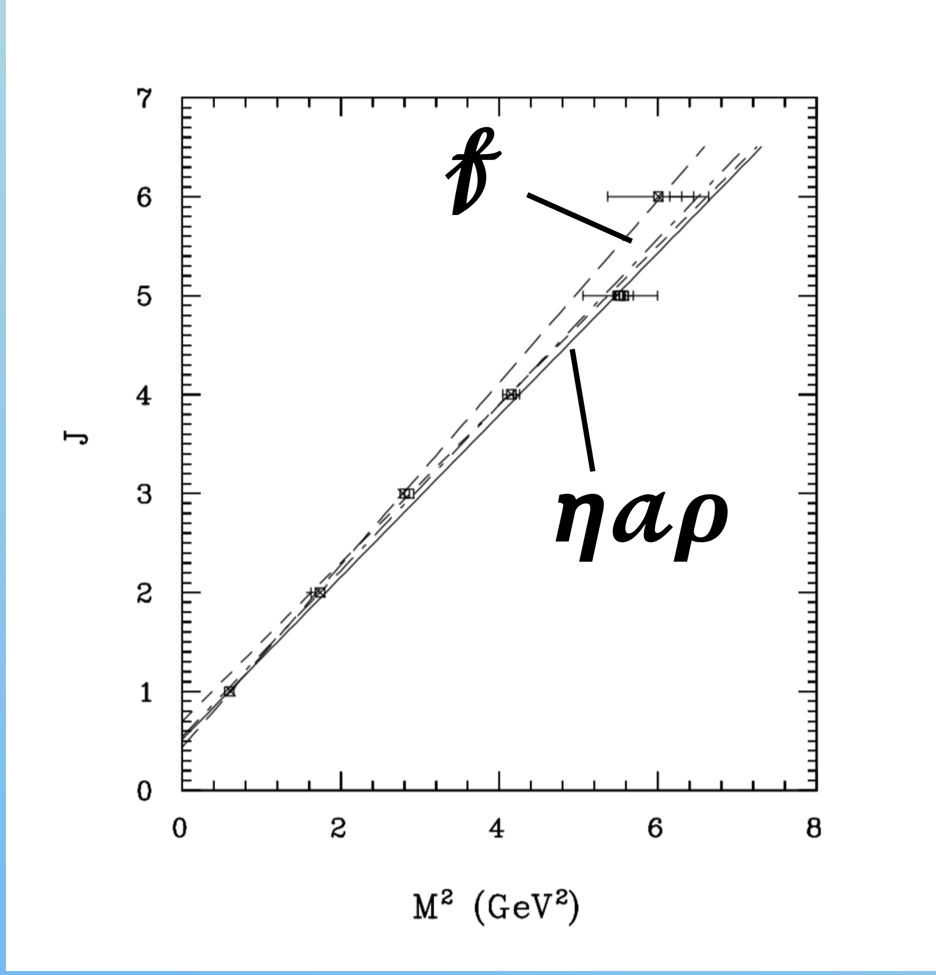
Start with a cross section parametrisation

(about 6 parameters for pp/pp)

Extend it to the real part using dispersion relations

Use $s^\alpha, \log(s), \log^2(s)$

Physical models

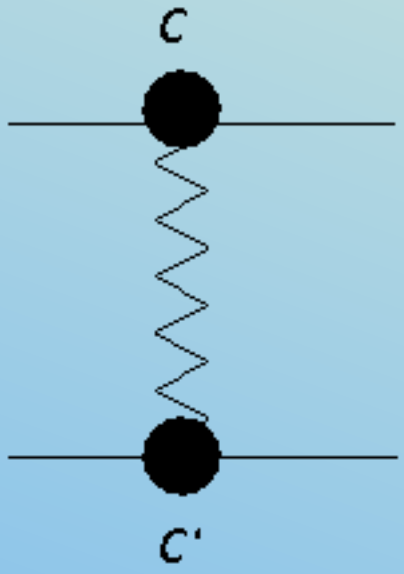


Regge trajectory

$$J = \alpha_0 + \alpha' \ M^2$$

Elastic amplitude

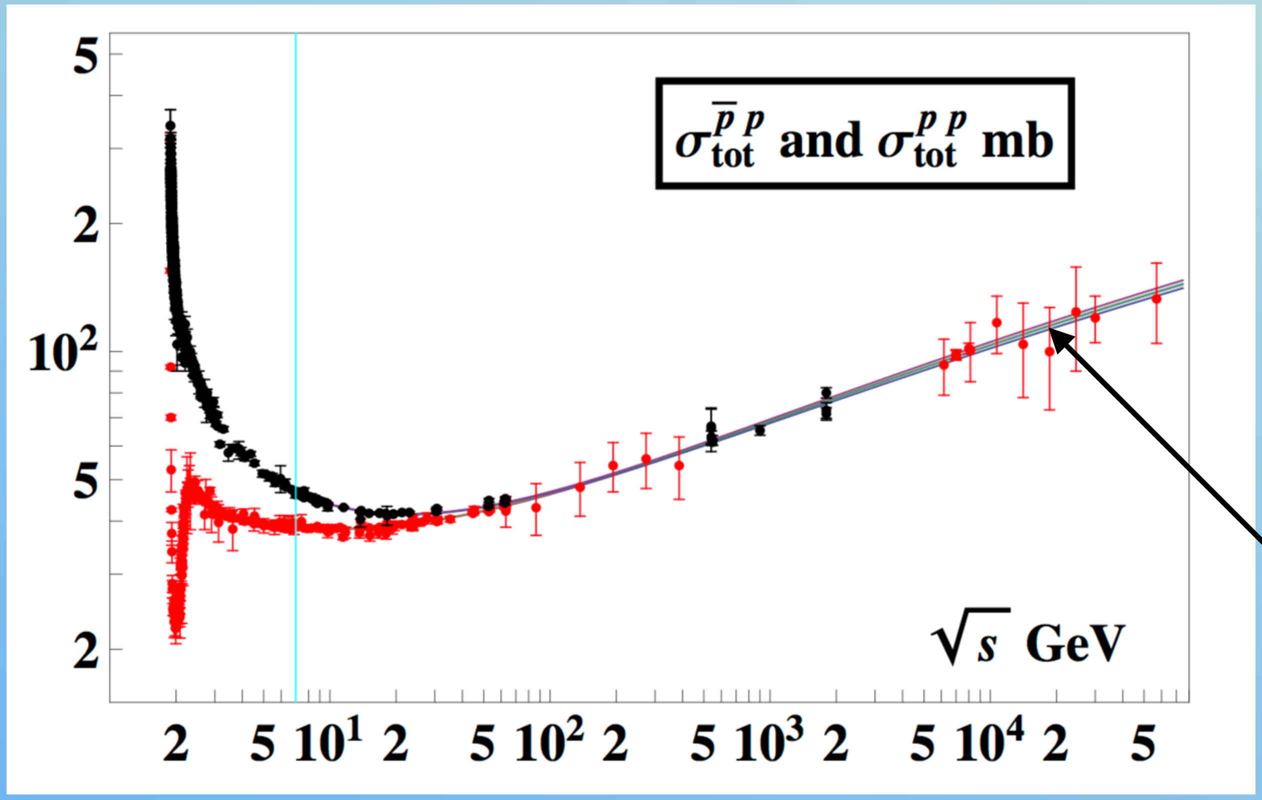
$$\mathcal{A} = c(t)c'(t) \ (is)^{\alpha_0 + \alpha' t}$$



Pomeron

All meson and baryon exchanges lead to falling total cross sections

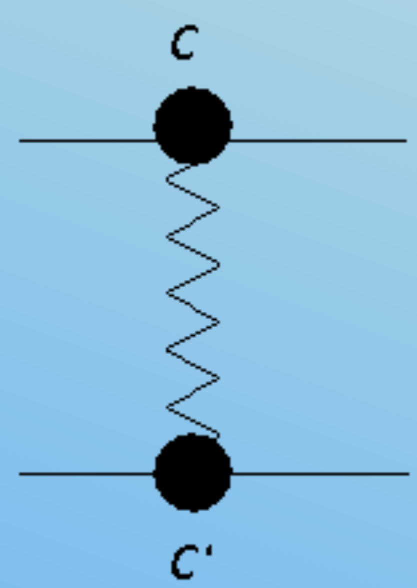
$$\Im m \mathcal{A}/s \propto \sigma_{tot}$$



new contribution

Pomeron amplitude for $\sqrt{s}$ <100 GeV

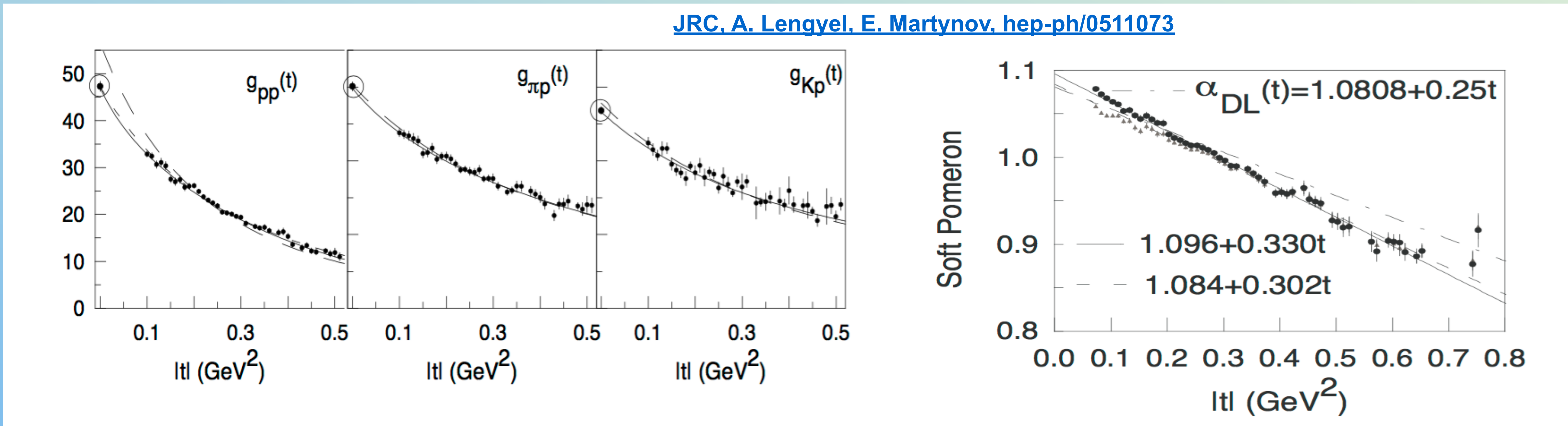
$$\mathcal{A} = c(t)c'(t)\ (is)^{\alpha_0+\alpha' t}$$



$$\alpha_0 \approx 1.1$$
$$\alpha' \approx 0.3\ \mathrm{GeV}^{-2}$$
$$c(t) \approx F_1(t)$$

Glueball exchange?

Get the trajectories and form factors directly from the data



$$5\ \mathrm{GeV} < \sqrt{s} < 100\ \mathrm{GeV}, 0.1\ \mathrm{GeV}^2 < |t| < 0.5\ \mathrm{GeV}^2$$

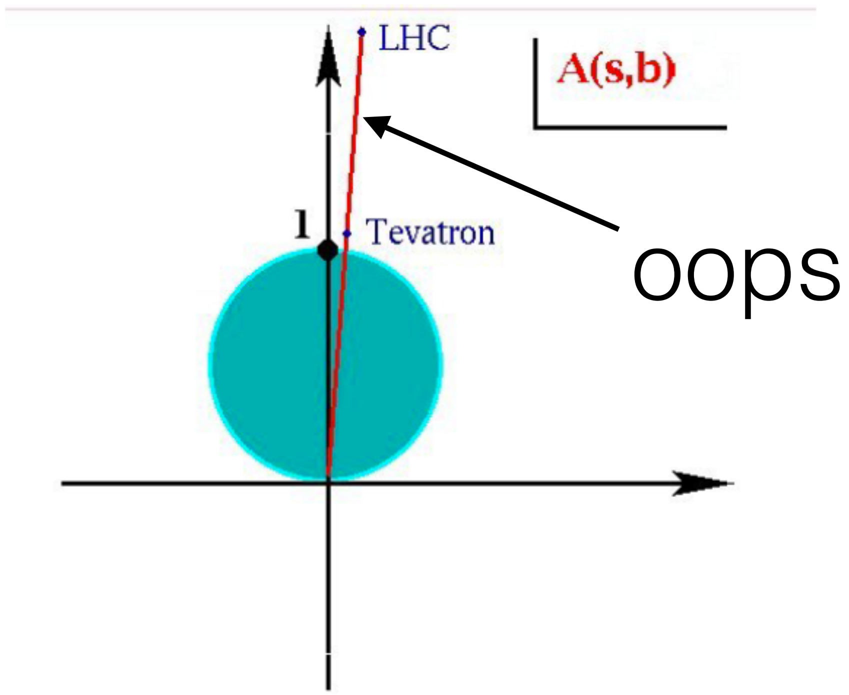
non exponential form factors

linear trajectory

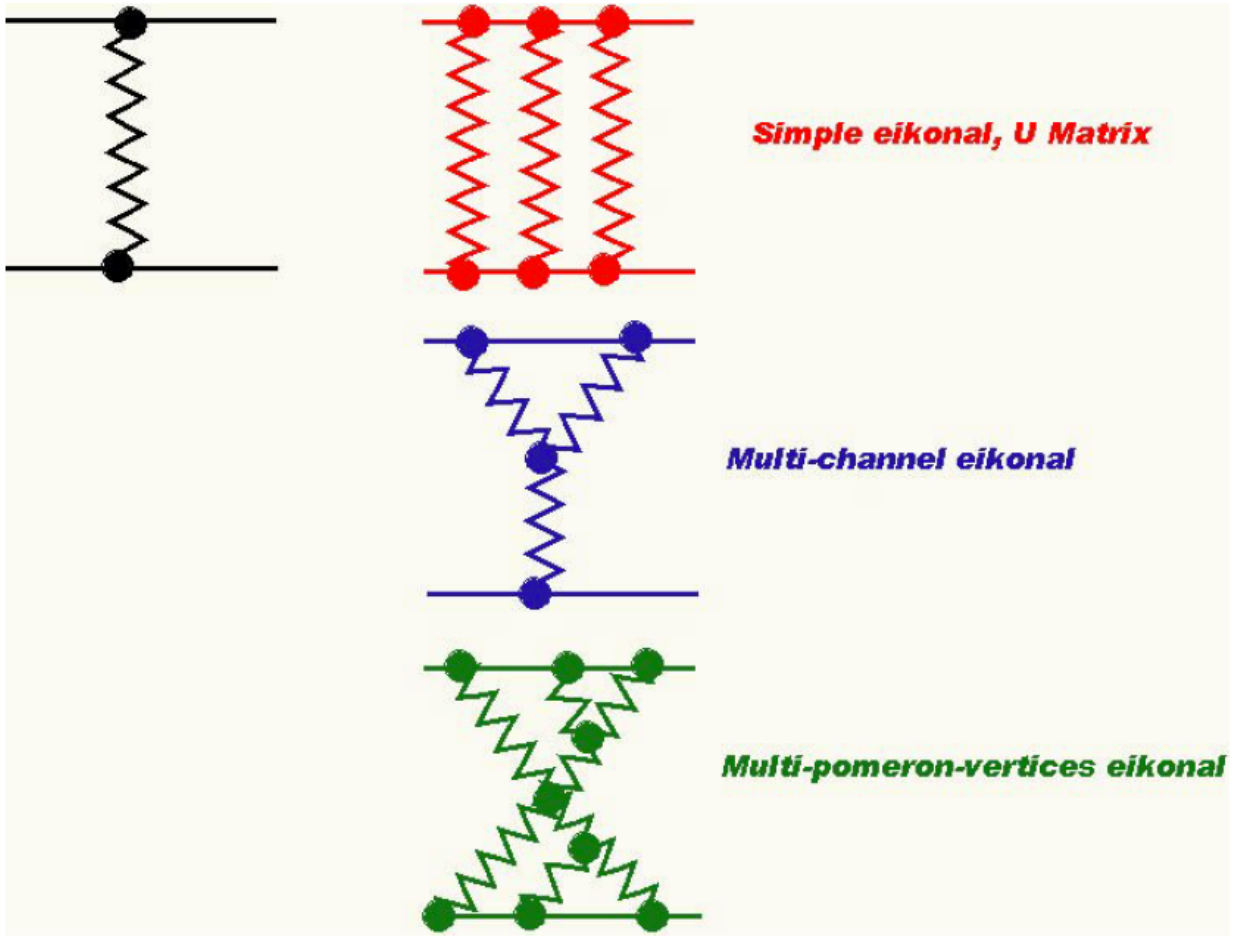
Unitarity

partial wave $A(s, b), \ (\ell = b\sqrt{s})$

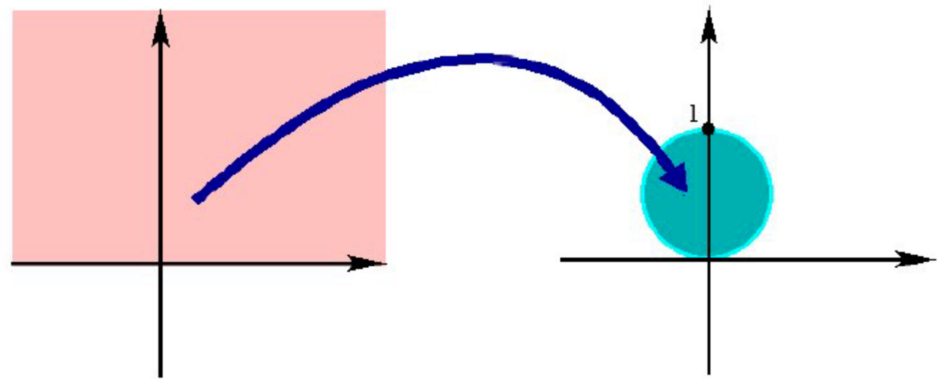
$$|A(s,b)|^2 \leq 2Im\,(A(s,b))$$



Unitarisation



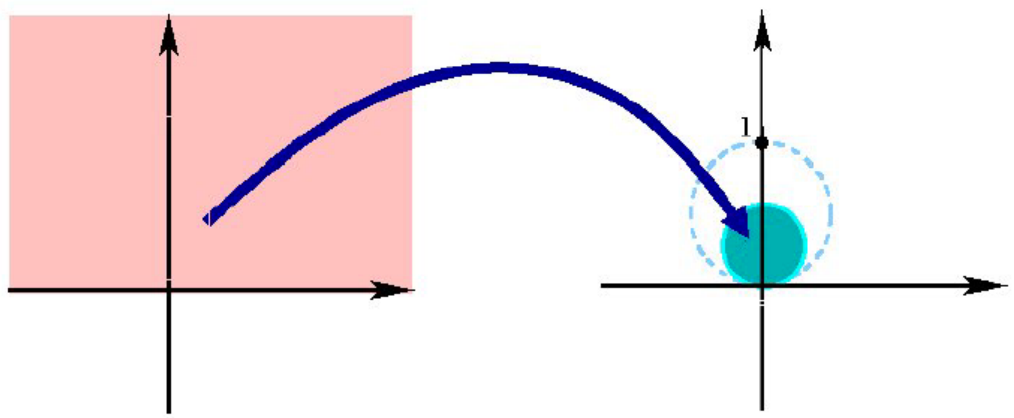
Even in the simplest case we cannot calculate the resummation



eikonal: $\mathbf{U}(s,b) = i(1 - \exp(iA(s,b))$

U-matrix: $\mathbf{U}(s,b) = \frac{A(s,b)}{1 - iA(s,b)/2}$

True unitarisation



eikonal

$\mathbf{U}(s,b) = \frac{i}{\omega}(1 - \exp(i\omega A(s,b))$

or U-matrix

$\mathbf{U}(s,b) = \frac{A(s,b)}{1 - i\omega A(s,b)/2}$

Black disk limit

JRC, O. Selyugin & E. Predazzi, arXiv:0812.0735


1
Black disk limit
s
1
Reflective or unitarity limit

Total cross sections @ LHC

$$\Im m \mathcal{A}(s,0)/s \propto \sigma_{tot}$$

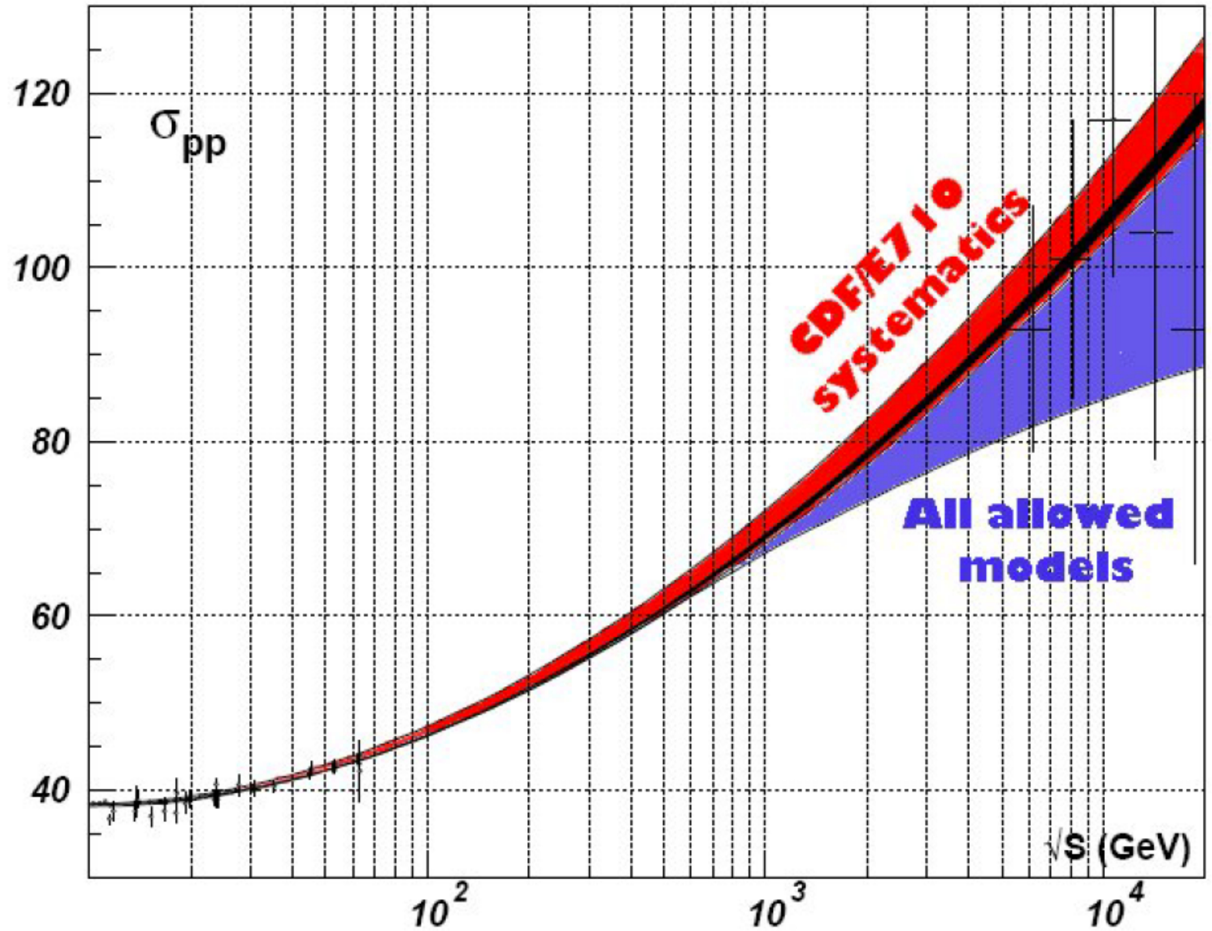
COMPETE fits

qcd.theo.phys.ulg.ac.be/compete

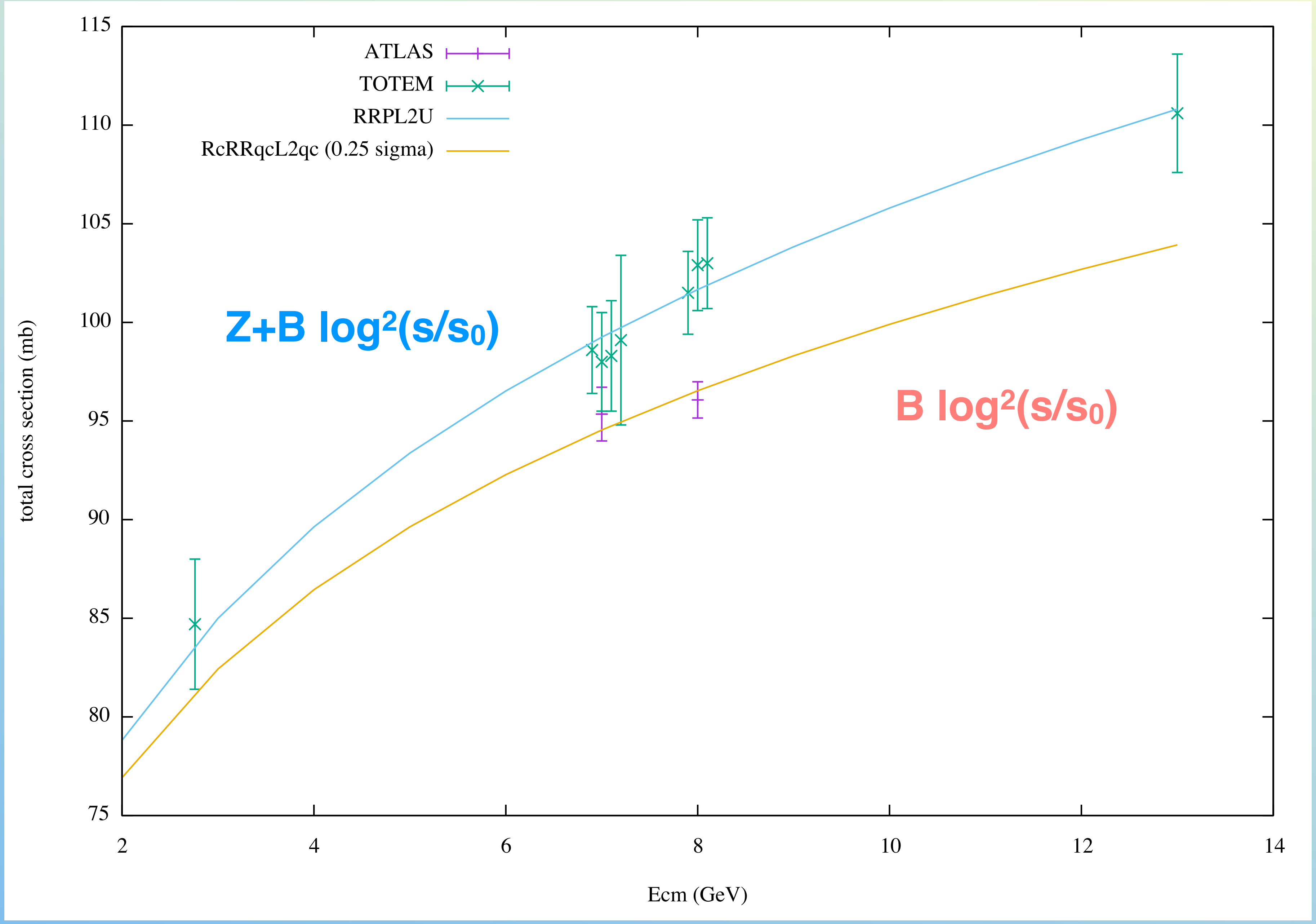
J.R. Cudell, V.V. Ezhela, P. Gauron, K. Kang,4 Yu.V. Kuyanov, S.B. Lugovsky, E. Martynov,, B. Nicolescu, E.A. Razuvaev, and N.P. Tkachenko, PRL

hep-ph/0206172

double poles ($\log(s)$)/ triple poles ($\log^2(s)$)
+ 2 undegenerate lower trajectories

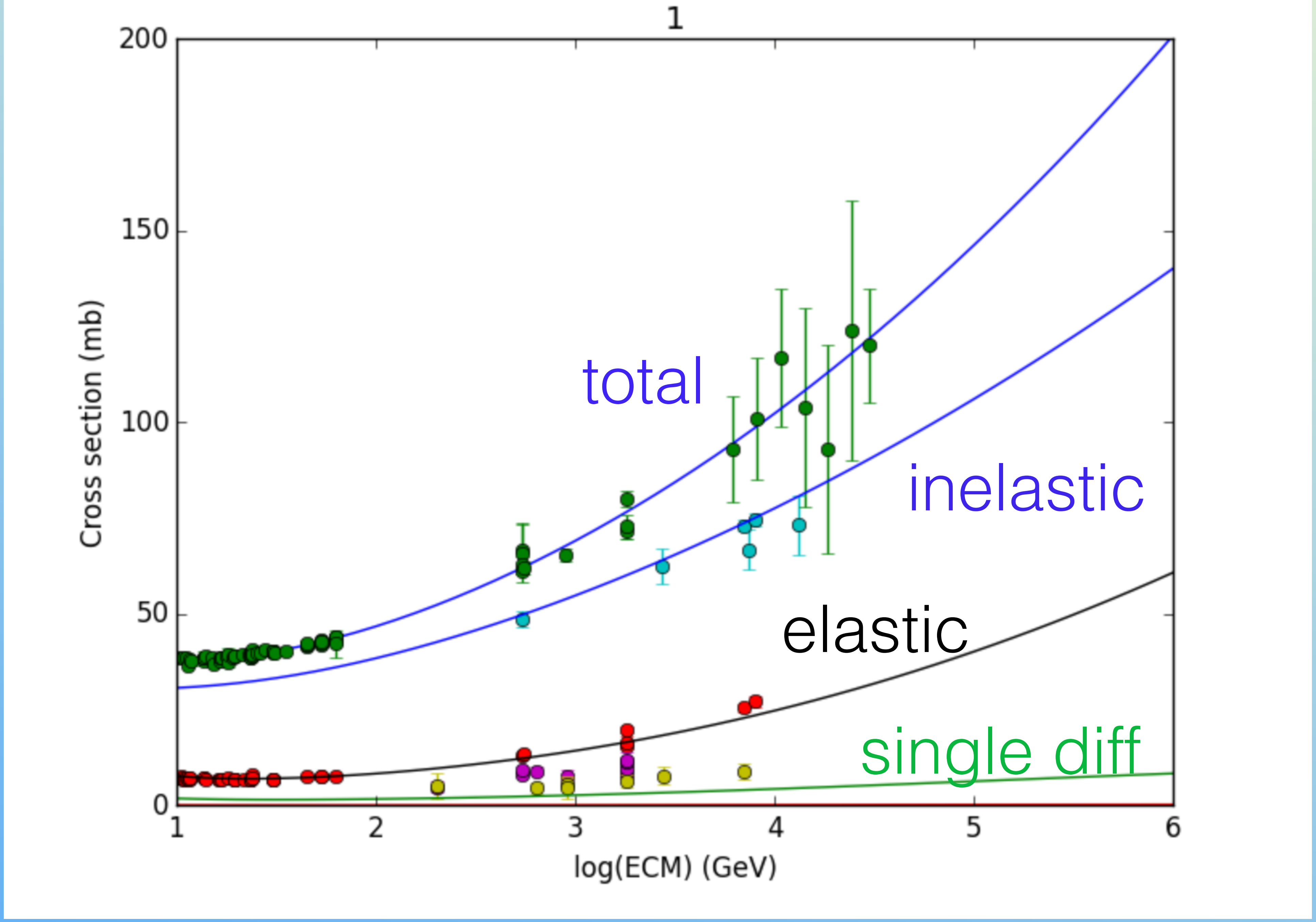


σ^{pp}_{tot}(10 TeV)
84 – 112 mb
σ^{pp}_{tot}(14 TeV)
90 – 117 mb



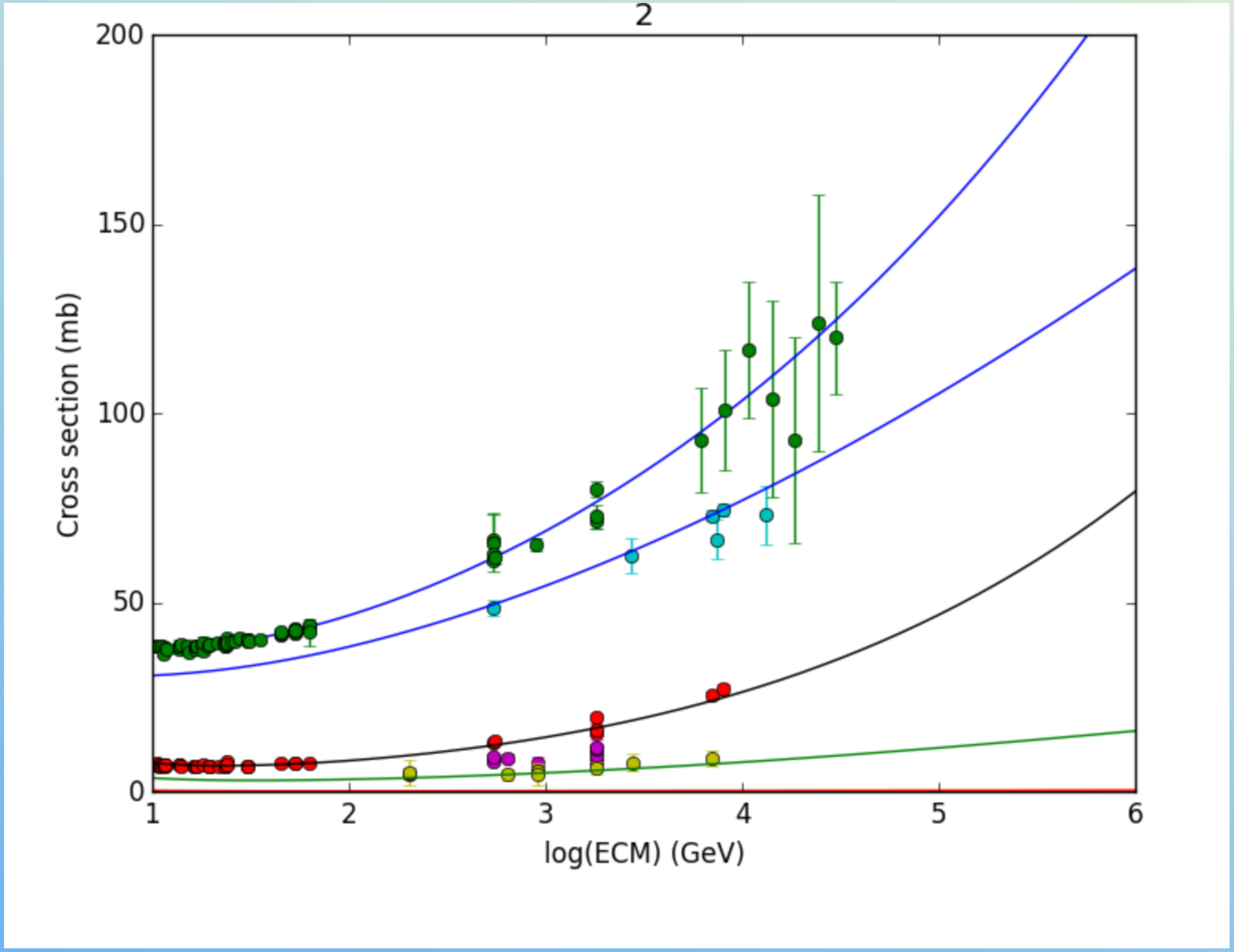
Reminiscent of the TeVatron problem

Simple eikonal (Black disk)



A Vanthieghem & JRC, https://matheo.ulg.ac.be/handle/2268.2/1301

U Matrix (full unitarity)



A Vanthieghem & JRC, https://matheo.ulg.ac.be/handle/2268.2/1301

The elastic cross section @ LHC

$$|\mathcal{A}(s,t)|^2 \propto \frac{d\sigma}{dt}$$

Simplest parametrisation

$$\mathcal{A}(s,t) = \frac{ihs}{4\pi}\log^2\left(\frac{s}{s_0}\right)\left(\frac{s}{s_0}\right)^{\frac{B_1}{2}t+\frac{B_2}{2}t^2} F_1^2(t)$$

+divide each dataset by a factor n_i

	statistical	statistical+ systematic	statistical+ normalisation
χ^2	48337	421	1812
h (GeV^{-2})	0.30	0.30	0.31
B_1 (GeV^{-2})	0.55	0.55	0.58
B_2 (GeV^{-4})	−0.39	−0.39	−0.26
$\sigma_{tot}(7\ TeV)$ (mb)	95.3	95.1	96.8
$\sigma_{tot}(8\ TeV)$ (mb)	98.2	98.0	99.7
TOTEM n_i at 7 TeV			1.03
TOTEM n_i at 8 TeV			1.05,1.06
ATLAS n_i at 7 TeV			0.98
ATLAS n_i at 8 TeV			0.94

O. Selyugin & JRC, arXiv:1710.08696

More sophisticated: Selyugin's High Energy Generalised Structure (HEGS)

- Eikonal unitarisation
- Vector and tensor form factors from GPD's
- Coulomb region
- $s \leftrightarrow u$ crossing symmetry
- normalisation coefficients for each dataset

$n_{TOTEM} \approx 0.9$, $n_{ATLAS} \approx 1$

Small |t| description

O. Selyugin, arXiv:1410.6305

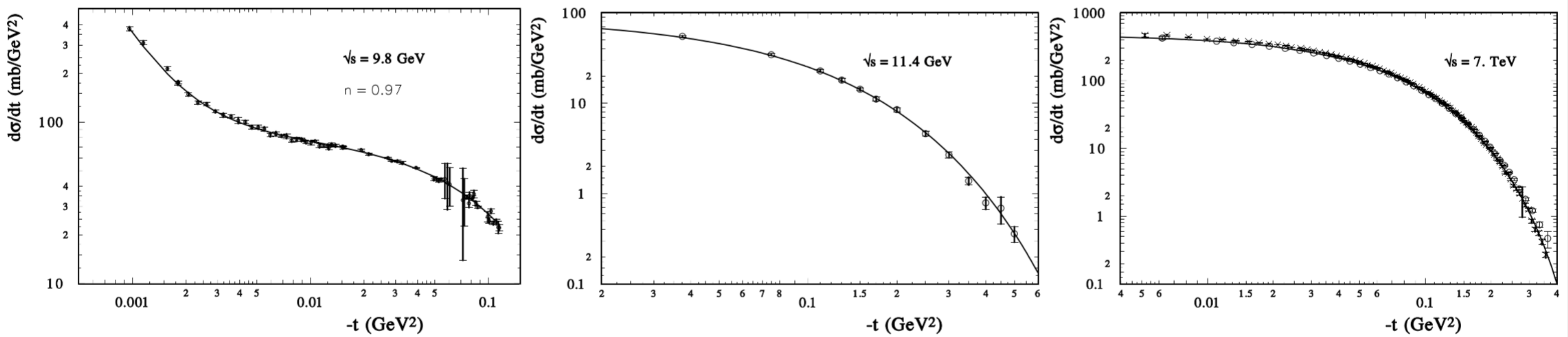
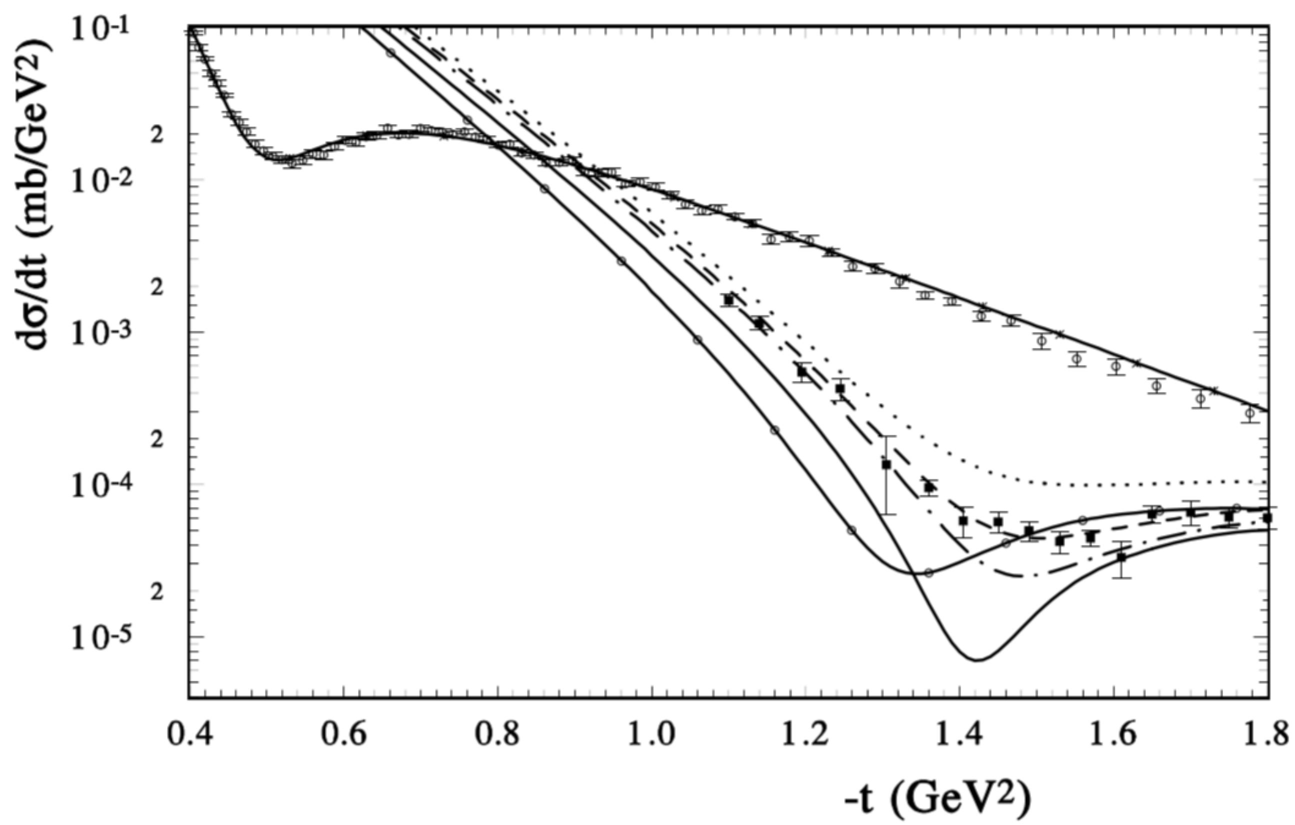


FIGURE 1. $d\sigma/dt$ a) for pp (left panel) at $\sqrt{s} = 9.8$ GeV and b) $p\bar{p}$ (central panel) at $\sqrt{s} = 11.3$ GeV and c) pp (right panel) at $\sqrt{s} = 7$ TeV [cross - TOTEM data [10], circles - ATLAS data [11].

Large |t| description



O. Selyugin, arXiv:1609.08847

Figure 2: The model calculation of the diffraction minimum in $d\sigma/dt$ of pp at $\sqrt{s}$ = 13.4; 16.8; 19.4; 30.4; 52.8; 7000 GeV; (lines correspondingly - dots; short dash; dot-dash; solid; solid+circles; solid+ants); the squares - the data at $\sqrt{s} = 16.82$ GeV, and the circles - the data at $\sqrt{s} = 7$ TeV [32].

Small |t|

O. Selyugin, arXiv:1505.02426 doi: 10.1103/PhysRevD.91.113003

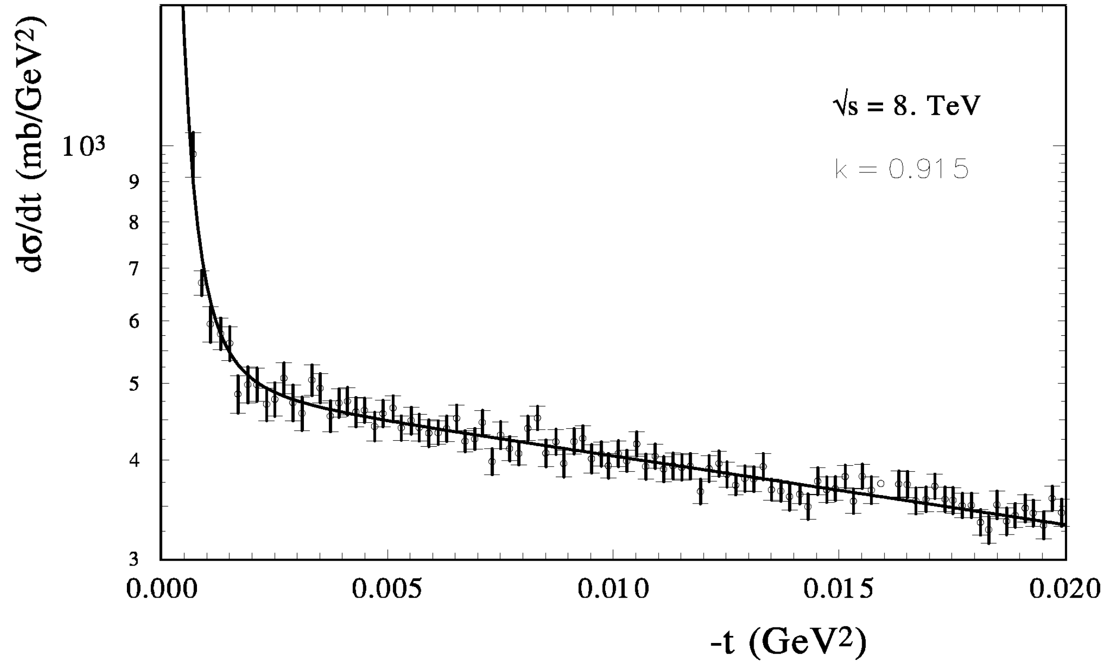


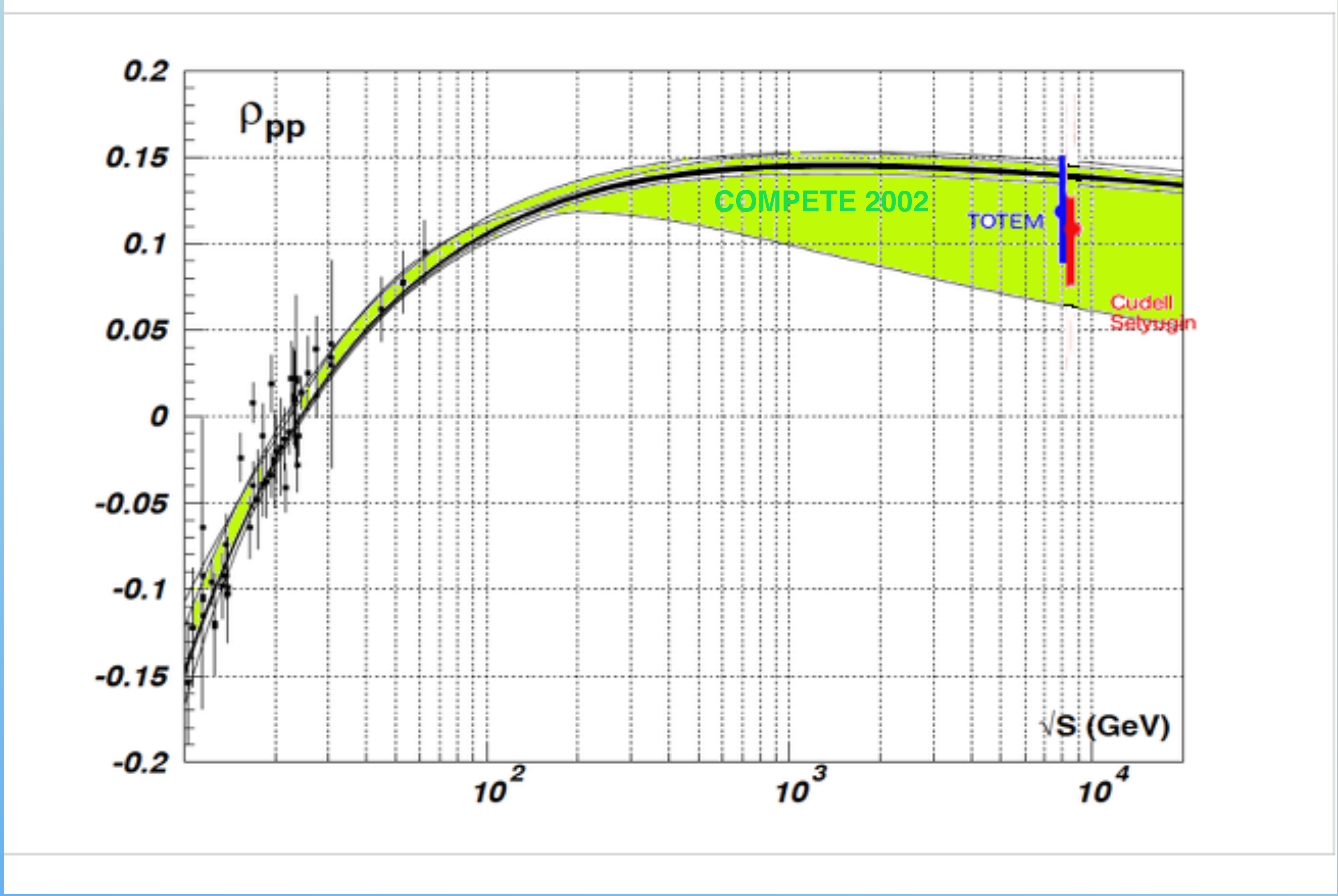
TABLE V. The obtained [85] and predicted sizes of $\sigma_{tot}(s)$ (in mb) and $\rho(t=0,s)$.

$\sqrt{s}$, GeV	$\sigma_{\text{tot-exp}}$	σ_{tot}	$\rho(t=0,s)$
19.42	38.98 ± 0.4	39.58 ± 0.8	-0.005 ± 0.0006
22.96	39.42 ± 0.4	39.89 ± 0.8	-0.005 ± 0.0006
52.8	42.85 ± 0.7	43.15 ± 0.5	0.074 ± 0.005
541	62.72 ± 0.2	62.72 ± 0.2	0.128 ± 0.005
1800	77.3 ± 0.38	77.3 ± 0.38	0.127 ± 0.02
7000	98.0 ± 2.6	97.16 ± 0.5	0.121
7000	$96.4 \pm 2.$	97.16 ± 0.5	0.121
8000	$101. \pm 2.1$	99.4 ± 0.5	0.12
14000	$104 \pm 26.$	108.76 ± 0.5	0.1176
30000	$120. \pm 15$	122.7 ± 0.5	0.11
57000	133 ± 23	135.4 ± 0.5	0.11

The **ρ** parameter

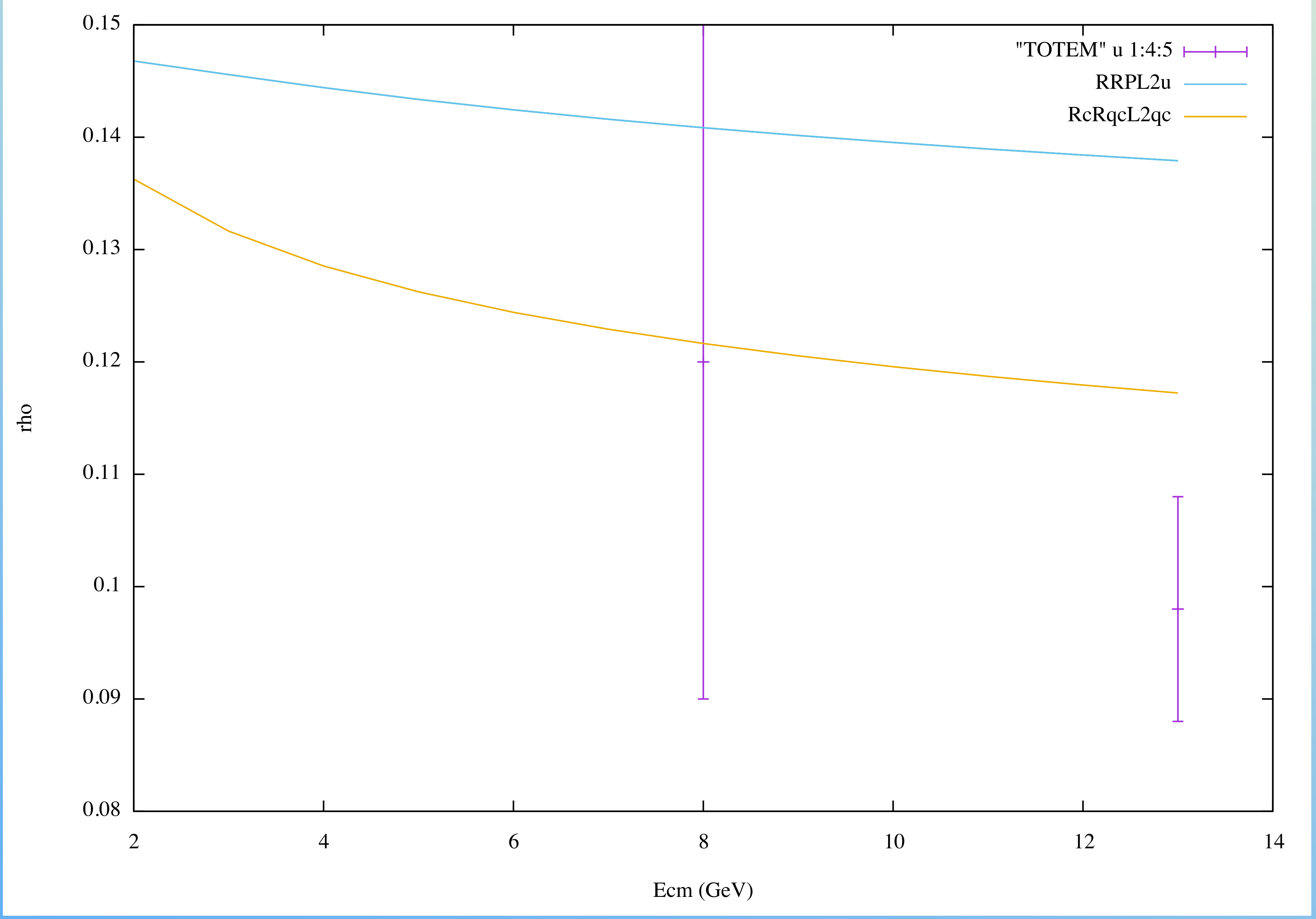
$$\frac{\Re e \mathcal{A}(s,0)}{\Im m \mathcal{A}(s,0)} = \rho(s,0)$$

TOTEM 2015



0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
ρpp
COMPETE 2002
TOTEM
Cudell
Selyugin
√S (GeV)
10^2
10^3
10^4

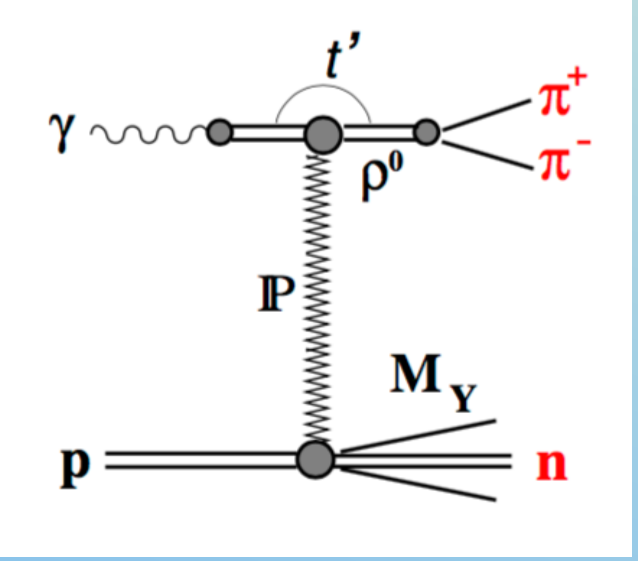
13 TeV?

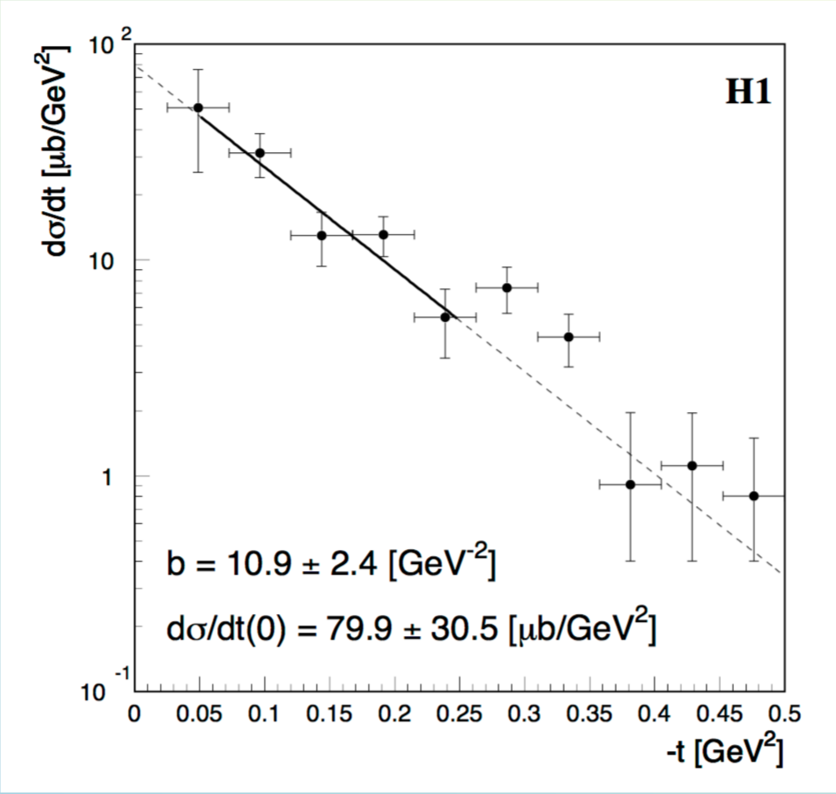

0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
rho
2
4
6
8
10
12
14
Ecm (GeV)
"TOTEM" u 1:4:5
RRPL2u
RcRqcL2qc

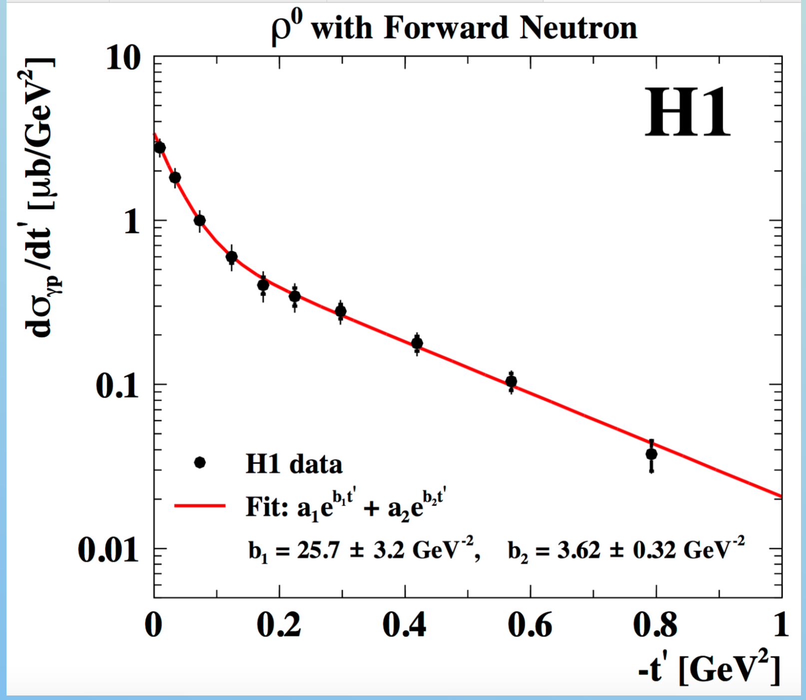
Conclusions

- What is the source of the disagreement between TOTEM and ATLAS?
    - Overall normalisation?
    - Single-diffractive background?
- COMPETE central values are meaningless.
- Models are flexible enough not to favour either dataset.

H1 neutrons







$x_L > 0.95$, $\theta n < 0.75$ mrad