Coherent backscattering of bosonic matter-wave in the presence of disorder and interaction

R. Chrétien, J. Dujardin, C. Petitjean and P. Schlagheck

Département de physique, University of Liège, 4000 Liège, Belgium

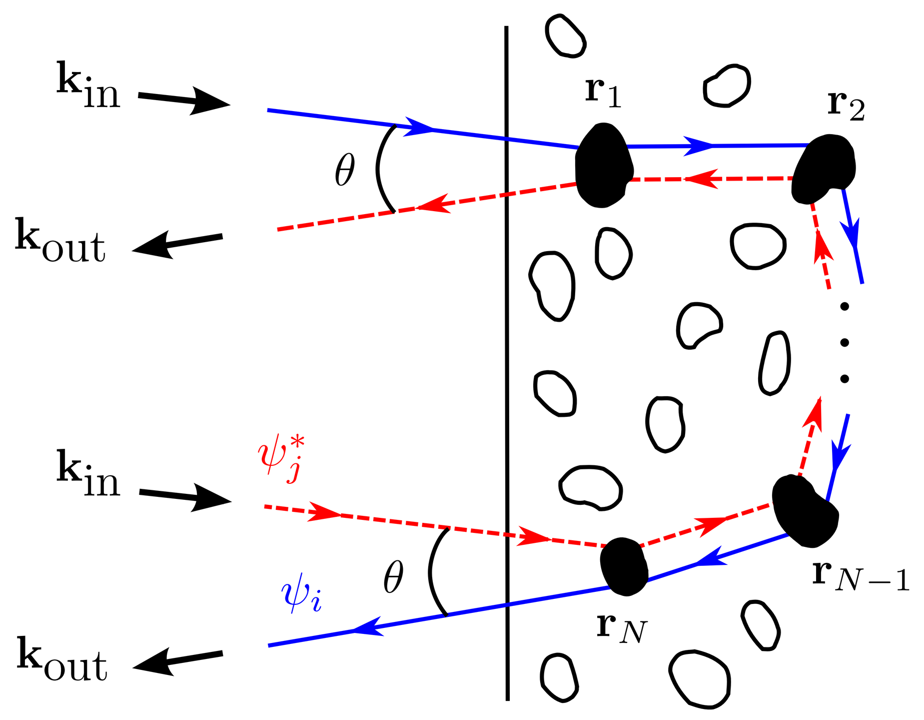
Université de Liège

Abstract

Coherent backscattering, which is an enhancement of the backscattered intensity of a light going through a medium made of point-like scatterers, is known as one of the most robust interference effects. It has been shown, although it is nowadays not fully understood yet, that in the presence of non-linearities, this enhancement turns to an inhibition. We propose to study that effect by means of a system in which we study the transport of a Bose-Einstein condensate through Aharonov-Bohm rings in the presence of interaction and disorder. We find that our system is indeed a good candidate to observe the coherent peak's inversion and is also suitable for more feasible theoretical calculations than in the original case.

Coherent backscattering

Laser light I_0 and wavevector k_{in} going through a sample composed of point-like scatterers at random positions

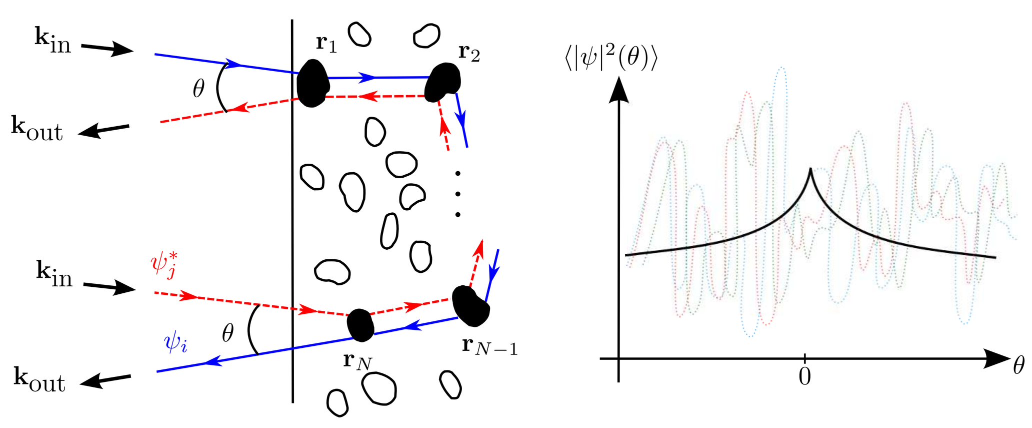


Compute $\langle I(\theta)\rangle \propto |\psi(\theta)|^2$ with

$$\psi = \sum_{\text{paths } i} \psi_i \Rightarrow |\psi|^2 = \sum_{i,j} \psi_i \psi_j^* = \sum_{\text{paths } i} |\psi_i|^2 + \sum_{i \neq j} \psi_i \psi_j^* .$$

No interference ? $\Rightarrow$ Ohm's law

Ensemble average over scatterer's positions yields a specific conic pattern

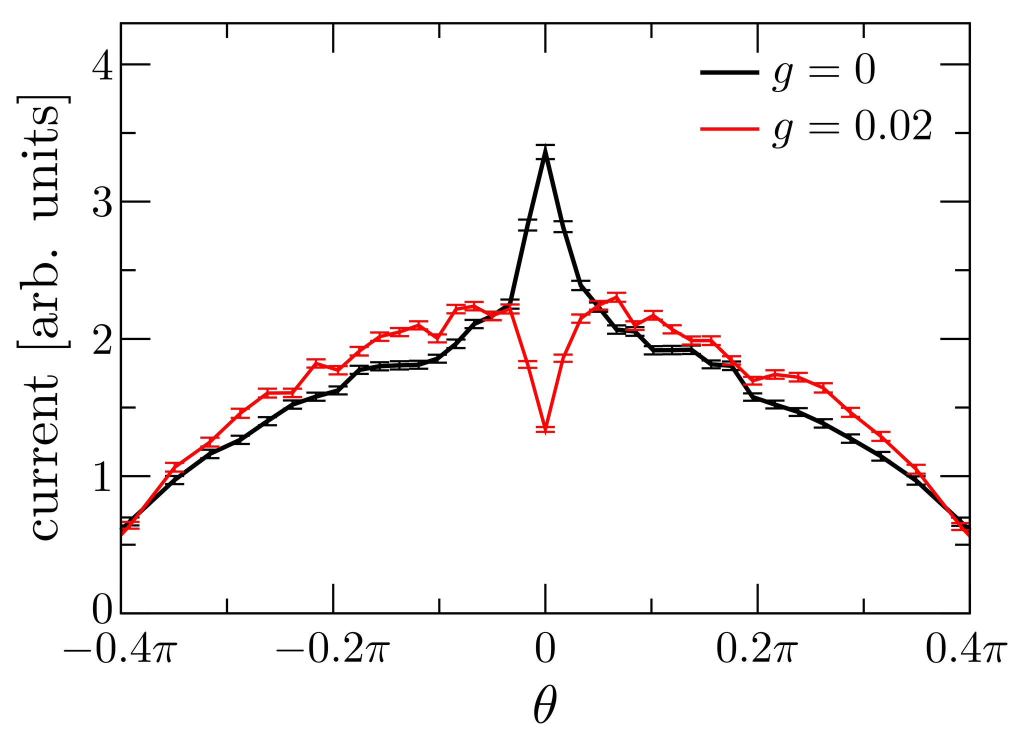


Constructive interferences between time-reversed conjugate paths (same phase acquired due to disorder) around $\theta = 0$ (same path's length).

CBS peak inversion

Numerical integration of the Gross-Pitaevskii equation in 2D shows that the interaction strength parameter g plays an important role

[M. Hartung *et al. Phys. Rev. Lett.* **101**, 020603 (2008)]

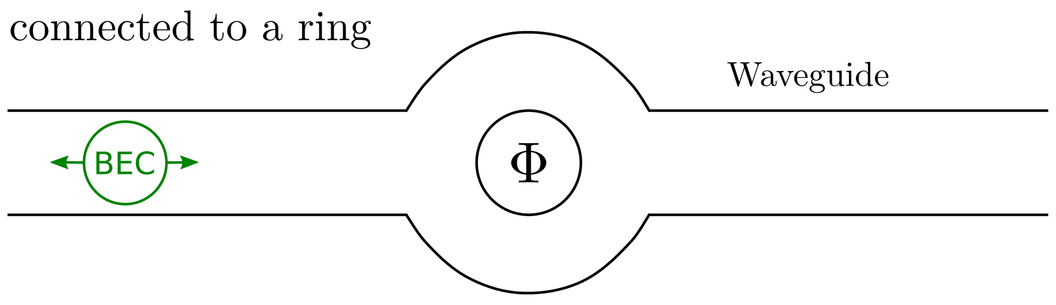


Inverted cone in presence of finite interaction $\rightarrow$ crossover from constructive to destructive interference

Theoretical calculations beyond the Gross-Pitaevskii approach difficultly feasible in 2D

Our model

Our system : BEC coupled to 2 semi-infinite waveguides connected to a ring



Usually described by Gross-Pitaevskii equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) + g|\psi(\mathbf{r})|^2\psi(\mathbf{r}) = \mu\psi(\mathbf{r})$$

$\rightarrow$ Ok if interaction strength g "small enough"

Numerical integration of GP equation and truncated Wigner method

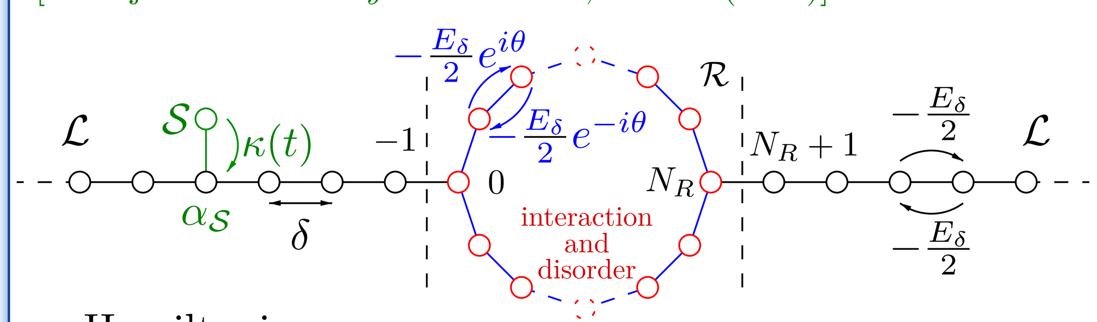
Theoretical description

Ring geometry connected to two semi-infinite homogeneous leads

Perfect condensation of the reservoir ($T = 0\,\text{K}$) with chemical potential μ

Discretisation of a 1D Bose-Hubbard system

[J. Dujardin *et al. Phys. Rev. A* **91**, 033614 (2015)]



Hamiltonian

$$\hat{H} = \hat{H}_\mathcal{L} + \hat{H}_{\mathcal{LR}} + \hat{H}_\mathcal{R} + \hat{H}_\mathcal{S}$$

where

$$\hat{H}_\mathcal{L} = \sum_{\alpha\in\mathcal{L}}\left[E_\delta \hat{a}_\alpha^\dagger \hat{a}_\alpha - \frac{E_\delta}{2}\left(\hat{a}_{\alpha+1}^\dagger \hat{a}_\alpha + \hat{a}_\alpha^\dagger \hat{a}_{\alpha+1}\right)\right]$$

$$\hat{H}_{\mathcal{LR}} = -\frac{E_\delta}{2}\left(\hat{a}_{-1}^\dagger \hat{a}_0 + \hat{a}_0^\dagger \hat{a}_{-1} + \hat{a}_{N_R}^\dagger \hat{a}_{N_R+1} + \hat{a}_{N_R+1}^\dagger \hat{a}_{N_R}\right)$$

$$\hat{H}_\mathcal{R} = \left[\sum_{\alpha\in\mathcal{R}}(E_\delta + V_\alpha)\hat{a}_\alpha^\dagger \hat{a}_\alpha - \frac{E_\delta}{2}\left(\hat{a}_{\alpha-1}^\dagger \hat{a}_\alpha + \hat{a}_{\alpha+1}^\dagger \hat{a}_\alpha\right) + g\hat{a}_\alpha^\dagger \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\alpha\right]$$

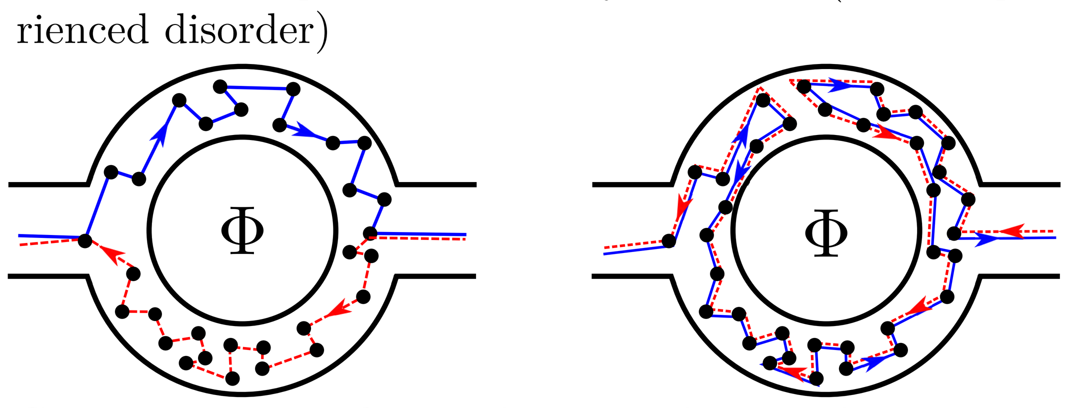
$$\hat{H}_\mathcal{S} = \kappa(t)\hat{a}_{\alpha_\mathcal{S}}^\dagger \hat{b} + \kappa^*(t)\hat{b}^\dagger \hat{a}_{\alpha_\mathcal{S}} + \mu\hat{b}^\dagger \hat{b}$$

with :

- $\hat{a}_\alpha$ ($\hat{b}$) and $\hat{a}_\alpha^\dagger$ ($\hat{b}^\dagger$) the annihilation and creation operators on site α (of the source),
- $E_\delta \propto 1/\delta^2$ the on-site energy,
- V_α the disorder potential at site α,
- g the interaction strength,
- $N \rightarrow \infty$ the number of Bose-Einstein condensed atoms within the source,
- $\kappa(t) \rightarrow 0$ the coupling strength.

CBS within a ring

Time-reversed paths are exactly the same (same experienced disorder)



Constructive interference between those paths

Enhanced backscattering probability $\rightarrow$ Aronov-Altshuler-Spivak oscillations

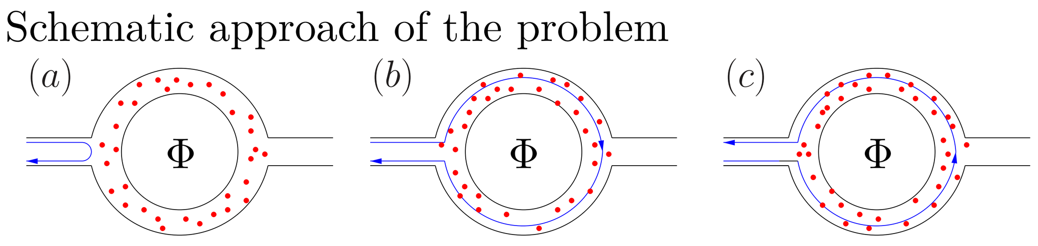
Transmission easier to compute than reflection

Computational resources have been provided by the Consortium des Equipements de Calcul Intensif (CÉCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Grant No. 2.5020.11

Higher order interferences

How to highlight the CBS contribution ?

Schematic approach of the problem



[Ihn T., *Semiconductor nanostructures*, Oxford (2010)]

The reflection probability is given by

$$\begin{aligned}\mathcal{R} &= |r_0 + r_1 e^{i\Phi} + r_1 e^{-i\Phi} + \ldots|^2 \\ &= |r_0|^2 + |r_1|^2 + \ldots && (1)\\ &\quad + 4|r_0|\cdot|r_1|\cos\Lambda\cos\Phi + \ldots && (2)\\ &\quad + 2|r_1|^2\cos(2\Phi) + \ldots && (3)\end{aligned}$$

with Λ the disorder-dependent phase accumulated after one turn with $\Phi = 0$.

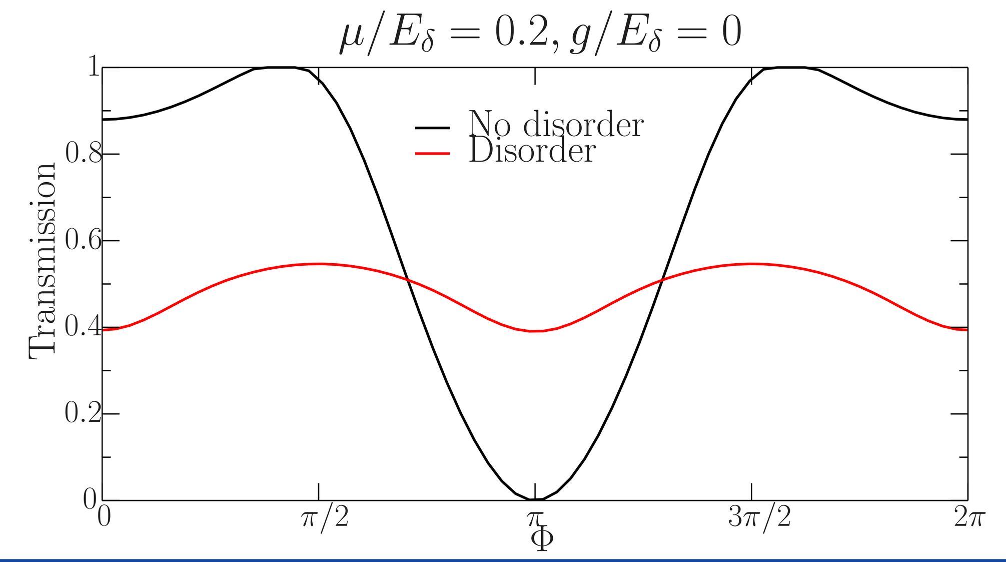
(1) no Φ-dependence, classical contributions

(2) Φ-periodicity, AB contribution, damped to zero when averaged over the disorder

(3) $\Phi/2$-periodicity, AAS contribution, robust to averages over the disorder

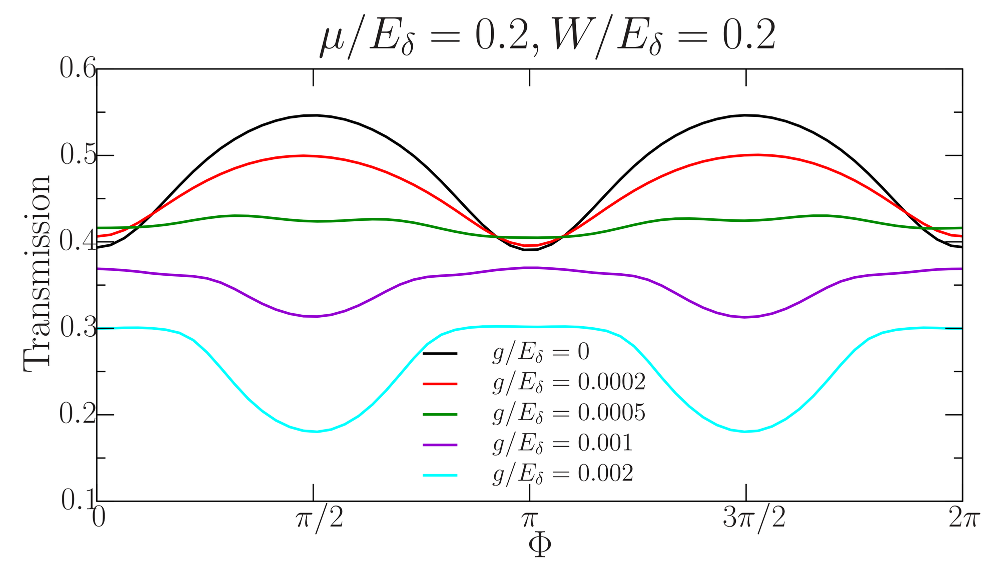
Ensemble average over disorder

$\rightarrow$ Appearance of AAS oscillations

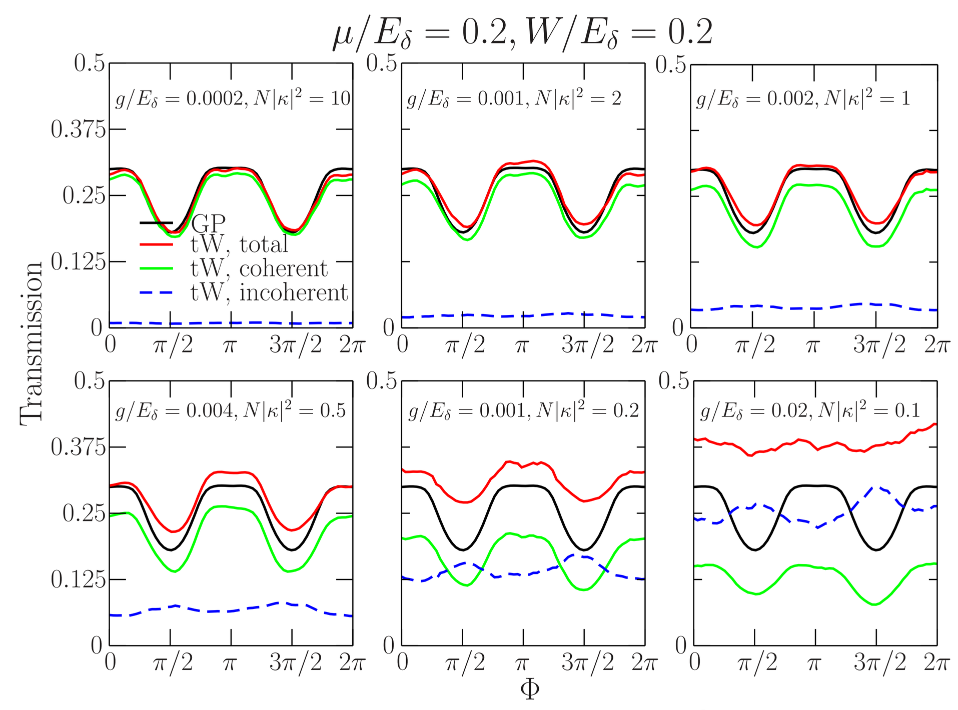


AAS oscillations with interaction

What happens if we set a weak interaction ?



- The oscillations amplitude is reduced
- The minimum at $\Phi = \pi$ becomes a maximum !



- Truncated Wigner simulations confirm the coherent peak inversion for weak interaction
- Presence of dephasing for strong interaction
- Analytical calculations with our 1D model more feasible
- Full diagrammatic theory with interaction (non-linearity)

[T. Hartmann *et. al. Ann. Phys.* (Amsterdam) **327** (2012)]