Probabilistic Reliability Management Approach and Criteria for Power System Short-term Operational Planning

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***Abstract*—This paper develops a probabilistic decision making framework for reliability management in the short-term operational planning context. We build upon our recent work, which proposed a probabilistic reliability management approach and criterion (RMAC) for the latest decision making opportunity of real-time system operation. Here, we transpose the RMAC to the preceding problem instance of short-term operational planning, wherein i) risk is aggravated by the uncertainty on power injections and weather conditions, and, ii) the problem scope concerns choosing 'strategic' actions (*e.g.*, starting additional generating units, granting outage requests for maintenance, *etc.*) to facilitate decision making during the forthcoming real-time system operation. To anticipate on the latter, we formalize the notion of a real-time 'proxy' as a simplified model of the real-time decision making context, adequately accurate for the purpose of operational planning decision making. Stating a first proposal for such a proxy, we mathematically formulate the RMAC for short-term operational planning as a multi-stage stochastic decision making problem and demonstrate its main features by case studies on a modified version of the single area IEEE RTS-96 system.**

I. Introduction

While the modern power system is being jointly reshaped by the societal drive for low carbon generation and the technical advancements in control, computing & communications, the interest in revising reliability management becomes stronger than ever. Indeed, the potential advantages of a probabilistic approach, explicitly taking into account both the novel threats (*e.g.*, wind power generation uncertainty) as well as the novel opportunities (*e.g.*, post-contingency corrective controls) are by now well understood and documented [1]. With this motivation, we recently proposed in [2] a first step towards the construction of a globally coherent probabilistic *reliability management approach and criterion* (RMAC), by focusing on the latest decision making opportunity of real-time operation. In the present work we complement such basis by addressing, in a consistent manner, the context of short-term operational planning [3].

In the context of short-term operational planning, the function of reliability management is to ensure the resources (broadly, generation and transmission capacity as well as operational flexibility) needed to achieve the reliable operation of the power system within the respective planning horizon. Doing so entails anticipating both i) the *exogenous* factors forming the future operational conditions of the system, most notably the weather dependent power injections & demands and component failure rates, and, ii) the *endogenous* real-time decision making strategy of the system operator, potentially resulting in the application of further preventive and/or corrective (*i.e.* post-contingency) control actions upon realization of such operational conditions. The combination of the growing uncertainty in the future operational conditions (due to the developing penetration of renewable sources in the generation sub-system, the ageing of the components of the transmission sub-system, *etc.*) along with the growing complexity in the real-time control strategy (due to the many novel technological opportunities) sets the challenge for reliability management in the short-term operational planning context.

A. Proposal

In response to such challenge, we propose a short-term operational planning Reliability Management Approach and Criterion (RMAC) composed of the three following ingredients:

1) **A reliability target:** it ensures that, under any anticipated operational condition, the objectives of real-time reliability management remain achievable. In other words, it guarantees that the system shall remain operable according to the standard set by the applicable real-time decision making strategy.
2) **A socio-economic objective:** it prescribes to minimise a socio-economic cost function composed of: i) the direct costs associated to the application of 'strategic' operational planning decisions, and, ii) the aggregated risk implied by these operational planning decisions, expressed as the expected value of real-time operation costs (*i.e.*, costs of real-time preventive and/or corrective controls as well as potential service interruption costs) within the planning horizon.
3) **A discarding principle**: it allows to neglect in items 1) and 2) a subset of anticipated operational conditions *only* under the condition that the implied residual risk,

expressed as the expected value of real-time operation costs over that neglected subset, is guaranteed to be below a fixed threshold.

Notice that our proposal explicitly acknowledges the complementary functions of short-term operational planning and real-time operation reliability management, the former serving to facilitate the latter. In this spirit, we propose the short-term operational planning RMAC while abstracting away from the particular decision making strategy (*e.g.*, the preventive N-1, the probabilistic real-time RMAC developed in [2], or other) followed for reliability management in real-time.

To enable such a consideration of the coupling between short-term operational planning and real-time operation reliability management, we formalize the notion of a *proxy* as a simplified model of a 'source' reliability management context (*here*, the context of real-time operation), expressed in such a way that it may be effectively exploited in another 'target' reliability management context (*here*, the context of short-term operational planning), while sufficiently well modeling the considered source-decision-making process for the purpose of the considered target-decision-making-process.

Without any loss in generality, and with an interest in establishing the benefits of a consistent probabilistic approach across all reliability management contexts, in this work we adopt the probabilistic RMAC introduced in [2] for the real-time reliability management context.

B. Related Works

Planning the secure operation of power systems in the short-term, under the uncertainty induced by renewable power generation remains a topic of current interest in the scientific literature. Referring the reader to [4] for a recent extensive documentation of the *state of the art*, we may distinguish three prominent approaches to handling the short-term uncertainty associated to power injections, namely:

i.) the *robust* approach [5]–[8] as well as its adaptive variants [9]–[11];
ii.) the *chance-constrained* approach, most notably advocated in [12]–[18] under diversified problem settings and modeling assumptions, and;
iii.) the *stochastic* approach pursued in [19], [20].

The *robust* approach [5]–[11] offers security against any credible uncertainty realization at the expense of compromising in terms of economic efficiency. The conservativeness can be mitigated by reducing the respective uncertainty space [9]–[11], however doing so efficiently requires taking into account the implications in loss of security. The *chance-constrained* approach [12]–[18] incorporates the probability distribution of uncertain parameters to reduce conservativeness by setting a probabilistic requirement for the compliance with constraints expressing the desired system performance. As in the adaptive robust approach, the efficient practice of the chance-constrained approach would also require taking into account the consequences of any potential constraint violation. The *stochastic* approach [19], [20] of seeking a trade-off between the costs of security and the probability weighted consequences of insecurity appears most suitable to efficiently address the issue. However, while from a theoretical point of view it is more general than the former two approaches, in practice it leads to more complex decision making problems.

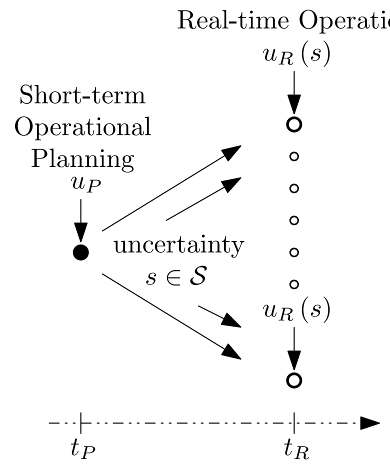

Fig. 1. Short-term Operational Planning Context

In this paper we attempt to complement such related works by introducing a novel approach to tackling uncertainty in the operational planning context. Our approach starts from the framework of *stochastic* optimization, explicitly pursuing the trade-off between the direct (present) and probability weighted indirect (future) socio-economic costs associated to short-term operational planning decisions. To counteract the involved complexity, we formalize the principle of reducing the considered uncertainty set while guaranteeing that the risk implied by such reduction, measured as an accuracy error in approximating the socio-economic costs, remains tolerable. Further, while our proposal for short-term operational planning is formulated irrespectively of the reliability management strategy to be followed in real-time operation, we go beyond the current *state of the art* by planning for the operation of the system as per the RMAC of [2] rather than the N-1 criterion.

II. Reliability Management in the Short-term Operational Planning Context

A. Preliminaries & Compact Notation

To fix ideas, we consider the problem set-up illustrated in Fig.1, corresponding to a single short-term operational planning decision making point, shortly before the start of the respective real-time instance (*e.g.* the hour-ahead planning decision making point). As in [7], this set-up corresponds to the three-stage process of choosing i) operational planning actions, ii) real-time preventive actions, and, iii) real-time corrective (*i.e.* post-contingency) actions. Notice that we jointly represent the latter two decision types by a single bullet in Fig.1 to illustrate the fact that the choice of preventive and/or corrective actions falls within the scope of real-time operation reliability management.

1) The Short-term Operational Planning Context: As already mentioned, the scope of short-term operational planning is to select & apply 'strategic' decisions ahead of the actual instance of real-time operation.

We shall use symbol t_P to denote in time the operational planning decision making point and t_R for the respective instance of real-time operation which is to be covered by the operational planning decisions. Further, we shall employ symbol u_P to denote operational planning decisions, symbol $\mathcal{U}_P$ for the set of candidate decisions amongst which a choice is to be made as well as function $C_P(u_P)$ to represent the direct cost associated to the implementation of a chosen operational planning decision.

Let us also denote by $s \in \mathcal{S}$ the set of scenarios describing the exogenous uncertainty forming the possible operational conditions within the real-time instance under consideration. We focus, without loss of generality, on the uncertainty of power injections & demands as well as of the weather, recalling the effect of the latter factor on the potential realization of contingency events (*i.e.*, component forced outages) [21]. Assuming, to ease notation and facilitate the clarity of the problem statement, the scenario set to be discrete, we shall denote the realization probability of each elementary scenario as $\pi_s, \forall s \in \mathcal{S}$ with $\sum_{s\in\mathcal{S}} \pi_s = 1$.

2) Proxy for real-time reliability management: Real-time operation reliability management serves to counteract the possible occurrence of contingency events over a limited look-ahead horizon, and given the latest realization of power injections, demands and weather conditions.

Let us denote a real-time candidate decision (corresponding to the possible application of several preventive and/or corrective control actions) as $u_R(s) \in \mathcal{U}_R(u_P, s)$ and underline the dependence of the respective set (*i.e.*, the set of all decisions amongst which the operator may choose in real-time) on both the observed uncertainty realization (s) and the already chosen short-term operational planning decisions (u_P). The purpose of a proxy for real-time reliability management is to explicitly model the strategy resulting in the choice of such decisions. In our notation, we can generally express this by means of the constraints & objective of an optimization problem.

More specifically, we begin with introducing function $C_R(u_R(s), s)$ as a minimization objective. This function serves to replicate the order of priority amongst the real-time candidate decisions in the set $\mathcal{U}_R(u_P, s)$. Note that the precise expression of such cost function depends on the applicable real time decision making strategy (again, be it the preventive only N-1, the preventive/corrective N-1, the RMAC of [2], *etc.*). Along with function $C_R(u_R(s), s)$, we shall use constraint set $\mathcal{U}_C(u_P, s)$ to express the desirable system behavior in the real-time operation context, limiting the admissible choice of real-time decisions to the intersection $\tilde{\mathcal{U}}_R(u_P, s, 1) \equiv \mathcal{U}_R(u_P, s) \cap \mathcal{U}_C(u_P, s)$[1]. That is, for any given $u_P \in \mathcal{U}_P$ and $s \in \mathcal{S}$, finding the decisions compliant with the real-time reliability management strategy amounts to solving problem,

$$\min_{u_R(s)} C_R(u_R(s), s) \tag{1}$$

subject to,

$$u_R(s) \in \tilde{\mathcal{U}}_R(u_P, s, 1). \tag{2}$$

We must notice here that the existence of a feasible solution to (1,2) depends on the exogenous operational conditions (s) as well as short-term operational planning decisions (u_P). Acknowledging the possibility that it may turn out to be infeasible without careful choice of $u_p \in \mathcal{U}_P$, we must complete the proxy for real-time reliability management by also modeling the way its constraints would be relaxed, *only if* necessary. For simplicity of the compact notation we shall leave the relaxation principle implicitly represented within cost function $C_R(u_R(s), s)$.

To do so, we will use the notational convention that the upper bound of function $C_R(u_R(s), s)$ within the (reliability constrained) intersection $\tilde{\mathcal{U}}_R(u_P, s, 1)$ is *always smaller* than its lower bound within the (reliability relaxed) relative complement $\tilde{\mathcal{U}}_R(u_P, s, 0) \equiv \mathcal{U}_R(u_P, s) \setminus \mathcal{U}_C(u_P, s)$. In other words, by convention, the value of cost function $C_R(u_R(s), s)$ for any real-time decision complying with constraint set $\mathcal{U}_C(u_P, s)$ will be smaller than its value for any real-time decision not complying with such constraints[2]. Hence, minimizing the real-time cost function $C_R(u_R(s), s)$ should result, *if possible*, in real-time decisions compliant with the reliability management constraints and, *only if not*, in real-time decisions relaxing the reliability management constraints.

Accordingly, we compactly express the real-time proxy for reliability management as,

$$\min_{u_R(s), \lambda(s)} C_R(u_R(s), s) \tag{3}$$

subject to,

$$u_R(s) \in \tilde{\mathcal{U}}_R(u_P, s, \lambda(s)) \tag{4}$$

$$\lambda(s) \in \{0; 1\}. \tag{5}$$

Notice that by the solution of (3-5) auxiliary indicator variable $\lambda(s)$ will take the value of one to show that the constraints of the real-time reliability management have been enforced, and the value of zero to indicate that they had to be relaxed.

[1]We use here an auxiliary indicator variable with the value of 1 to show that the constraints of the real-time reliability management are enforced.

[2]Let us remark that, while we merely introduce such property of $\{C_R(u_R(s), s); \mathcal{U}_C(u_P, s)\}$ as a notational convention, ideally it should hold true for a well designed reliability criterion. That is, imposing any reliability constraints on the power system should be justifiable by the avoidance of the (much larger) potential socio-economic consequences of unreliability.

3) Socio-economic impact of short-term operational planning reliability management: Combining the direct costs incurred by short-term operational planning decisions with the expectation of the real-time operation costs as per the respective real-time decision making strategy (3-5), we express the socio-economic impact of any given short-term operational planning reliability management decision as,

$$C_P(u_p)+\sum_{s\in\mathcal{S}}\pi_s\cdot C_R\left(u_R^{\star}(s),s\right) \tag{6}$$

where, $\forall s\in\mathcal{S}$,

$$u_R^{\star}(s),\lambda^{\star}(s)\in\underset{u_R(s),\lambda(s)}{\arg\min}\;C_R\left(u_R,s\right) \tag{7}$$

$$s.t.$$

$$u_R(s)\in\tilde{\mathcal{U}}_R\left(u_P,s,\lambda(s)\right) \tag{8}$$

$$\lambda(s)\in\{0;1\}. \tag{9}$$

We underline here that the second term in (6) expresses the risk implied by the chosen operational planning decision u_P as the expectation over all foreseeable operational conditions of the costs associated to either i) achieving the real-time reliability management objectives (*e.g.*, satisfying the N-1 criterion), for the optimal value ($\lambda^{\star}(s)=1$) in (12)-(14), or, ii) relaxing them. Note also that we have built the socio-economic objective of (6-14) while remaining agnostic with regards to the precise real-time reliability management strategy. We do so to emphasize that, from the short-term operational planning perspective, it suffices to have and employ a proxy of such strategy.

B. Proposed Reliability Management Approach & Criterion

Extending the ideas recently introduced in [2], we propose a probabilistic RMAC for short-term operational planning composed of:

Reliability target We start by formalizing the reliability target of avoiding the realization of situations within which the objectives of real-time reliability management are not achievable. In other words, we seek for short-term operational planning decisions (u_P) ensuring that, under the set of scenarios expressing the foreseeable operational conditions, there exist real-time operation decisions that render the real-time reliability management feasible, as in:

$$\tilde{\mathcal{U}}_R\left(u_P,s,1\right)\equiv\mathcal{U}_R\left(u_P,s\right)\cap\mathcal{U}_C\left(u_P,s\right)\neq\emptyset,\forall s\in\mathcal{S}, \tag{10}$$

or equivalently, $\forall s\in\mathcal{S}$,

$$\lambda^{\star}(s)=1, \tag{11}$$

where,

$$u_R^{\star}(s),\lambda^{\star}(s)\in\underset{u_R(s),\lambda(s)}{\arg\min}\;C_R\left(u_R,s\right) \tag{12}$$

$$s.t.$$

$$u_R(s)\in\tilde{\mathcal{U}}_R\left(u_P,s,\lambda(s)\right) \tag{13}$$

$$\lambda(s)\in\{0;1\}. \tag{14}$$

Notice that such reliability target is motivated by the close temporal coupling between short-term operational planning and real-time operation. Acknowledging this coupling, the purpose of such a reliability target is to verify and establish that the degree of risk-aversion sought during real-time operation is attainable (over all foreseeable operational conditions). In this spirit, the reliability target of the short-term operational planning RMAC is stated independently of the approach to reliability management followed within real-time operation. Doing so, we maintain here the principle of formalizing the short-term operational planning RMAC while abstracting away from the particular specificities of the real-time reliability management strategy.

Discarding principle. For a subset $\mathcal{S}_{RMAC}\subset\mathcal{S}$ and a short-term operational planning decision $u_P\in\mathcal{U}_P$, we express the residual risk associated to operational conditions $s\in\mathcal{S}\setminus\mathcal{S}_{RMAC}$ as,

$$\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}_{RMAC}}(u_P)=\sum_{s\in\mathcal{S}\setminus\mathcal{S}_{RMAC}}\pi_s\cdot C_R\left(u_R^{\star}(s),s\right) \tag{15}$$

where, $\forall s\in\mathcal{S}\setminus\mathcal{S}_{RMAC}$,

$$u_R^{\star}(s),\lambda^{\star}(s)\in\underset{u_R(s),\lambda(s)}{\arg\min}\;C_R\left(u_R(s),s\right) \tag{16}$$

$$s.t.$$

$$u_R(s)\in\tilde{\mathcal{U}}_R\left(u_P,s,\lambda(s)\right) \tag{17}$$

$$\lambda(s)\in\{0;1\}. \tag{18}$$

Such value essentially serves to quantify the error in approximating the socio-economic impact function (6) by computing its expectation over the subset $\mathcal{S}_{RMAC}\subset\mathcal{S}$ only. We thus transpose the discarding principle initially introduced in [2] to the short-term operational planning problem by proposing to neglect from (6) and (10) a subset of operational planning conditions $\mathcal{S}\setminus\mathcal{S}_{RMAC}$ *only if* it can be ascertained that, the implied residual risk is lower than a fixed accuracy threshold ΔE_P, as in,

$$\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}_{RMAC}}(u)\leq\Delta E_P. \tag{19}$$

In (19) we introduce the *discarding threshold* ΔE_P as the *meta-parameter* of the proposed short-term operational planning RMAC. Larger values of such meta-parameter allow for identifying optimal operational planning decisions against smaller subsets $\mathcal{S}_{RMAC}$, gaining in computational simplicity at the expense of solution accuracy and implied risk.

Socio-economic objective To complete the statement of the RMAC for short-term operational planning, we put the socio-economic cost function (6) as a minimization objective together with the reliability target of (10) and the discarding principle of (19), as in,

$$\min_{u_P\in\mathcal{U}_P}C_P(u_p)+\sum_{s\in\mathcal{S}_{RMAC}}\pi_s\cdot C_R\left(u_R^{\star}(s),s\right) \tag{20}$$

subject to, $\forall s \in \mathcal{S}_{RMAC}$,

$$\tilde{\mathcal{U}}_R(u_P, s, 1) \neq \emptyset \tag{21}$$

$$u_R^{\star}(s) \in \underset{u_R(s)\in\tilde{\mathcal{U}}_R(u_P,s,1)}{\arg\min} \quad C_R(u_R(s), s) \tag{22}$$

while,

$$\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}_{RMAC}}(u_P) = \sum_{s\in\mathcal{S}\setminus\mathcal{S}_{RMAC}} \pi_s \cdot C_R(\tilde{u}_R^{\star}(s), s) \leq \Delta E_P \tag{23}$$

where, $\forall s \in \mathcal{S} \setminus \mathcal{S}_{RMAC}$,

$$\tilde{u}_R^{\star}(s), \lambda^{\star}(s) \in \underset{\tilde{u}_R(s),\lambda(s)}{\arg\min} \; C_R(\tilde{u}_R^{\star}, s) \tag{24}$$

subject to,

$$\tilde{u}_R(s) \in \tilde{\mathcal{U}}_R(u_P, s, \lambda(s)) \tag{25}$$

$$\lambda(s) \in \{0; 1\}. \tag{26}$$

We notice here the reliability target of (21) which, as already explained, imposes the feasibility of the real-time reliability management problem (22) over any non-discarded operational condition $s \in \mathcal{S}_{RMAC}$. Accordingly, the second term in the socio-economic objective function (20) only includes the expectation of the function measuring the costs of achieving the real-time decision making strategy. Contrary to this, there is no explicit requirement on the feasibility of the real-time reliability management problem for discarded operational conditions. As such, both the real-time decision making strategy as well as its possible relaxation appear in the statement of the discarding principle (23-26).

III. Modeling requirements & implementation

In this section we briefly discuss the main modeling requirements of the proposed RMAC for short-term operational planning, as well as the modeling approximations and solution approach adopted in our prototype implementation. The formulations of all the models composing our solution approach are analytically presented in mathematical notation in the Appendix.

A. General modeling requirements

Since the short-term operational planning RMAC anticipates on the real-time reliability management strategy, it will integrate the models and data needed to formulate the latter. Referring the reader to [2] for a discussion on the modeling requirements of the real-time RMAC integrated in this work, let us focus here on the additional requirements of the short-term planning RMAC with respect to the models and data exploited by the real-time variant of the RMAC. The main additional requirements concern,

a.) *forecasting* models:
   i. weather conditions at the spatio-temporal resolution and precision required by those real-time models making use of them (recall that outage probabilities, probabilities, costs of service interruption, corrective control failure mode probabilities are in principle weather dependent);
   ii. power injections at the nodal resolution, so as to anticipate the electrical state of the system in real-time;
   iii. a probability model for the joint process of weather conditions & power injections[3] modeling marginal distributions as well as the significant stochastic (temporal and spatial) dependencies among the different exogenous factors;

b.) *real-time proxy* models:
   i. candidate real-time preventive and corrective actions, including cost-functions, admissible ranges of control, coupling constraints among successive real-time controls and coupling constraints of these latter with the operational planning decisions;
   ii. constraints and objective(s) of the real-time reliability management strategy;
   iii. the way to relax the aforementioned constraints and/or objective(s) *if-and-only-if* the real-time reliability management strategy turns out unattainable.

B. Mathematical models adopted in this implementation

In large-scale power systems, the exact solution of the complete optimization problem (20 – 26) becomes quickly out of reach, even in the single period context of Fig. 1. Developing scalable & tractable approximations to such problem is thus a topic which merits considerable attention.

In our prototype implementation, we employ a discrete set of uncertain operational conditions while representing the randomness in the active power output of renewable generation as well as in the weather conditions. More specifically, each elementary scenario $(s \in \mathcal{S})$ is assumed to *jointly* define the active power injection (in MW) per each wind power generator and the state of the weather, classified as either *normal* or *adverse*. We associate to each elementary scenario $(s \in \mathcal{S})$ the respective realization probability (π_s), with $\sum_{s\in\mathcal{S}} \pi_s = 1$.

Given such scenario set as input, we decompose the short-term operational planning RMAC of (20 – 26) into the functions of *assessment*, *discarding* and *control*. As represented by the illustration in Fig. 2 and the pseudo-code in Algo. 1, we make use of such decomposition to address the complete problem (20 – 26) by iteratively growing the subset of non-discarded operational conditions $\left(\mathcal{S}^i_{RMAC}\right)$ while updating the corresponding optimal operational planning decisions $\left(u_p^{\star,i}\right)$. Initializing the sub-set of non-discarded scenarios with the *most probable* operational conditions, at each iteration we use:

- the RMAC *control* function to update the planning decisions $\left(u_p^{\star,i}\right)$ with respect to all non-discarded operational conditions $\left(s \in \mathcal{S}^i_{RMAC}\right)$,
- the RMAC *assessment* function to update the corresponding residual risk estimation

[3] Notice that in principle weather conditions and power injections are not are not mutually independent, since both renewable power generation and (domestic) electricity demand strongly depend on the weather.

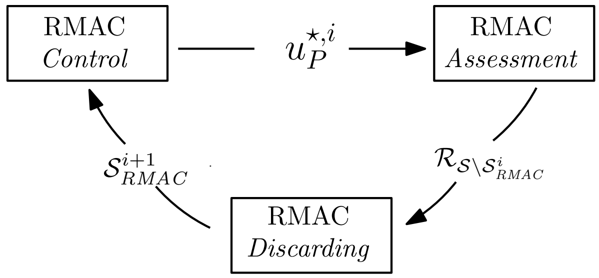

Fig. 2. RMAC iterative decomposition approach

$\left(\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}^i_{RMAC}}(u_p^{\star,i})\right)$ with respect to discarded operational conditions $\left(s \in \mathcal{S} \setminus \mathcal{S}^i_{RMAC}\right)$, and,

- the RMAC *discarding* function, to enlarge the set of non-discarded operational conditions $\left(\mathcal{S}^i_{RMAC} \subset \mathcal{S}^{i+1}_{RMAC} \subseteq \mathcal{S}\right)$ *if-and-only-if* the residual risk value is found to be non-negligible as per the applicable discarding threshold value (ΔE_P).

Algorithm 1 Short-term Operational Planning RMAC

initialise

$i = 0,$

$\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}_{RMAC}}(u_p^{\star,0}) >> \Delta E_P,$

$S^1_{RMAC} = \hat{s},$

while $\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}_{RMAC}}(u_p^{\star,i}) \geq \Delta E_P$ **do**

  $i \leftarrow i + 1$

  function RMAC CONTROL($\mathcal{S}^i_{RMAC}$)

    ↻ *updates operational planning decisions,*

    $u_p^{\star,i} \in \mathcal{U}_P,$

  function RMAC ASSESSMENT($u_P^{\star,i}, \mathcal{S}^i_{RMAC}$)

    ↻ *updates implied residual risk,*

    $\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}^i_{RMAC}}(u_P^{\star,i}),$

  function RMAC DISCARDING($\mathcal{R}_{\mathcal{S}\setminus\mathcal{S}^i_{RMAC}}(u_P^{\star,i})$)

    ↻ *updates non-discarded scenarios,*

    $\mathcal{S}^{i+1}_{RMAC} \supset \mathcal{S}^i_{RMAC}.$

1) RMAC control: The RMAC *control* problem concerns identifying operational planning decisions to render real-time operation feasible for all uncertainty realizations within a given sub-set of non-discarded operational conditions.

Casting this problem in a post market-clearing context, we model the modification of the commitment status of thermal generating units with respect to the market outcome, as well as the procurement of upward and downward active power re-dispatch flexibility (*i.e.*, the ability to activate such re-dispatch controls in real-time) as short-term operational planning candidate decisions $(\mathcal{U}_P)$. To emphasize on the inter-relation between short-term operational planning and real-time operation, we restrict the set of candidate real-time recourse decisions to the utilization of the already procured re-dispatch flexibility, either in a preventive or in a corrective (*i.e.* post-contingency) manner. In other words, the Security Constrained Optimal Power Flow (SCOPF) problem formulation under consideration includes three decision stages, namely,

i. short-term operational planning: procurement of re-dispatch flexibility under uncertainty on power injections, weather conditions, contingency occurrence and post-contingency corrective control behavior;
ii. real-time preventive: active power re-dispatch under uncertainty on contingency occurrence and post-contingency corrective control behavior;
iii. real-time post-contingency corrective: active power re-dispatch following the occurrence of a contingency.

We employ the approximation of the real-time RMAC contingency discarding problem by means of a conservative upper-bound on the contingency severity function (*see* § II.B in [2]) to define scenario specific sub-sets of non-discarded contingencies $(\mathcal{C}_{RMAC}(s)\ \forall s \in \mathcal{S}_{RMAC})$ and the DC approximation of the power flow equations. Accordingly, for each non-discarded operational condition $(s \in \mathcal{S}_{RMAC})$, we consider a set of linear inequality constraints expressing power flow limits in preventive mode, as well as at the intermediate and corrective post-contingency stages over the respective non-discarded contingency subset $(\mathcal{C}_{RMAC}(s))$. On top of such limitations we also impose, again per non-discarded operational condition $(s \in \mathcal{S}_{RMAC})$, the real-time RMAC chance-constrained reliability target restricting the choice of post-contingency corrective control actions according to their implied failure probability. We model the failure probability of post-contingency corrective controls as in [22]. Similarly to [2] we penalize any such potential failure conservatively via the scalar product of load demand and value of lost load.

Finally, we assume that any change in the commitment status of a generating unit is to be remunerated according to a fixed fee in monetary units. Concerning re-dispatch flexibility, we consider a marginal reservation fee (in monetary units per unit of re-dispatchable capacity) as payable at the operational planning decision making point. We also consider a marginal activation fee (in monetary units per unit of re-dispatched energy) as payable upon the utilization of such flexibility during real-time operation.

The above lead to a mixed-integer linear programing (MILP) approximation of the short-term operational planning RMAC control problem which is presented in detailed mathematical notation as appendix A.

2) RMAC assessment: As already mentioned, the short-term operational planning RMAC *assessment* problem concerns quantifying the (residual) risk implied by a given combination of operational planning decision(s) and scenario (sub)set. Further, doing so necessitates anticipating the control actions to be chosen as per the real-time reliability management strategy. More specifically, the assessment scope includes modeling:

- the decisions resulting from the application of the real-time RMAC per each operational condition in the concerned (sub)set;
- the decisions made *while relaxing* the real-time RMAC, *if-and-only-if* achieving its constraints & objectives is not attainable for a certain operational condition in the concerned (sub)set.

For our prototype implementation, we rely on analytical decision-making *proxy* models to anticipate on such choice. More specifically, we formulate the real-time variant of the RMAC control as a SCOPF problem while employing the same physical models and assumptions as those integrated in the short-term planning variant of RMAC control. Appendix B introduces such SCOPF problem formulation in detailed mathematical notation. Further, we assume that in any instance wherein the real-time RMAC problem turns out to be infeasible, the real-time variant of the discarding principle would have to be relaxed by reducing the set of contingency events to be explicitly covered by preventive and/or post-contingency corrective control actions. We treat any such case by i.) solving the modified SCOPF problem formulation presented in appendix C so as to identify which security constraints should be relaxed while minimizing the respective relaxation probability[4], and, ii.) subsequently re-solving the original real-time RMAC SCOPF formulation of appendix B while manually relaxing the constraints corresponding to the identified contingencies. To measure the implied risk, we account the impact of such relaxation conservatively by multiplying the adopted relaxation probability with the scalar product of load demand and value of lost load.

3) RMAC discarding: The function of RMAC discarding is to expand, at each iteration, the sub-set of non-discarded operational conditions $\mathcal{S}^i_{RMAC}$ so as to eventually discard a sub-set of operational conditions which collectively imply negligible residual risk. As illustrated in Fig. 2, this is an intermediary function between assessment and control, exploiting the results of the former (residual risk quantification) in order to provide inputs to the latter (subset of scenarios to be "covered" by operational planning decisions).

The prototype implementation of the RMAC discarding function builds on top of the approach originally developed in [6]. Adapting such ideas as per the purposes of the proposed RMAC, at each iteration we expand the set of non-discarded scenarios with the single scenario contributing the most to the residual risk quantity and do so until the discarding principle holds true. In other words, we keep at each iteration the additional single scenario that implies, in expectation, the highest cost of real-time operation reliability management, as in,

$$\mathcal{S}^{i+1}_{RMAC} = \mathcal{S}^i_{RMAC} \cup s^{i+1} \tag{27}$$

where,

$$s^{i+1} \in \underset{s \in \mathcal{S} \setminus \mathcal{S}^i_{RMAC}}{\arg\max} \; \pi_s \cdot C_R\left(u^{\star,i}_R, s\right). \tag{28}$$

Let us briefly remark that, with respect to the worst-case scenario screening ideas originally introduced in [6] to facilitate a robust approach to planning under uncertainty, the RMAC discarding proposed here corresponds to a probabilistic risk management doctrine, hence differentiates between scenarios on the basis of realization probability times implied cost rather than potential degree of constraint violation. Further, rather than seeking for all potentially "problematic" scenarios, it seeks for those scenarios necessary to achieve the proposed *discarding principle*. The latter feature may well be advantageous in both i) allowing to discard a greater number of scenarios, hence reducing the computational burden of the operational planning SCOPF problem, and ii) allowing to resolve situations involving conflicting scenarios, whose joint consideration would render the operational planning SCOPF problem infeasible, in a consistent manner.

IV. Demonstrative case studies

In order to demonstrate the features of our proposed approach and motivate further work related to its modeling requirements and implementation challenges, we discuss in this section a set of case studies based on the single area version of the well know IEEE RTS–96 benchmark [23].

A. Set-up for case studies

The demand values used in all case studies refer to the time interval $8:00-9:00$ pm at the first day of the year, for an annual peak of 3135 MW and the respective values of lost load per demand are defined as per [24]. To facilitate a meaningful study of our proposal we relied on the additional assumptions and data originally developed in [25] and made publicly available by the authors at [26]. More specifically, we have adopted from these sources:

- the addition of 9 wind farms with a total capacity of 3900 MW, as listed in table I and illustrated in the single line diagram of Fig. 3;
- the description of the respective power injection uncertainty by means of a nodal "central forecast", as well as 10 scenarios of credible nodal realizations. Fig. 4 illustrate the central forecast (a) and realization scenarios (b) for the time period in question under the favorable wind profile and an assumed wind energy penetration level of 35%, while table II lists the corresponding realization probabilities;
- the reduction of long-term and emergency flow limits of all transmission lines and transformers by 20% with respect to the values listed in table 12 of [23];
- the modernized cost coefficients of all thermal generating units.

[4]Notice that such relaxed SCOPF formulation may only turn out to be infeasible in the event that, given the power injection forecast error realization, even the pre-contingency operation of the system (without any security constraint) is unattainable. In any such case, we regard that the monetized impact of the concerned operational condition is equal to the socio-economic cost of shedding the whole system load demand.

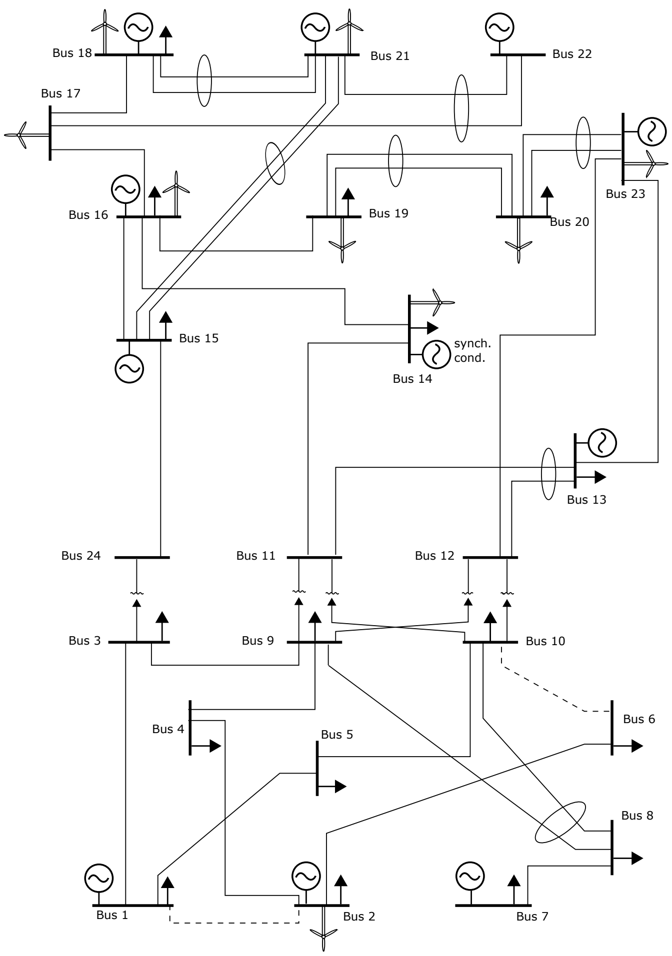

Fig. 3. Modified version of IEEE RTS-96 area A

TABLE I
WIND POWER CAPACITY ADDITION PER NODE

Node	*Capacity (MW)*	*Node*	*Capacity (MW)*
2	300	19	600
14	300	20	600
16	600	21	300
17	600	22	300
18	300		

In order to complete the set of uncertain operational conditions $\mathcal{S}$, we arbitrarily assume that the ratio of the probability of realizing the adverse weather state to the probability of realizing the normal weather state is equal to 0.25, unless otherwise specified. In the absence of relevant data, we neglect in our case study parameters the dependence of renewable power injections on weather conditions to consider any combination of the aforementioned realizations of power injections and weather states as jointly credible. This amounts to a set of $|\mathcal{S}| = 20$ distinct scenarios for the short-term operational planning RMAC. The first (last) ten scenarios in such set correspond to the original ten wind power injection scenarios

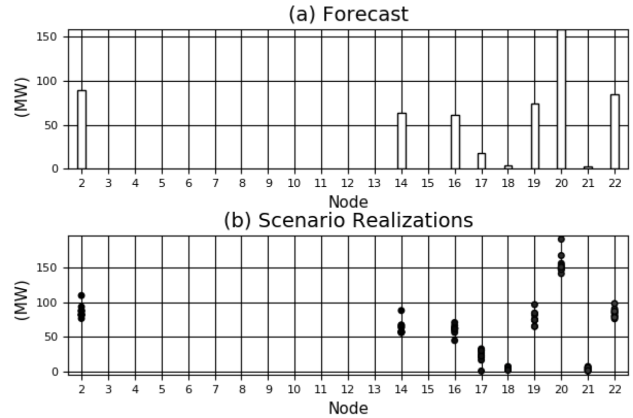

Fig. 4. Wind power injections

TABLE II
WIND POWER INJECTION SCENARIO REALIZATION PROBABILITIES

Scenario	*Probability*	*Scenario*	*Probability*
1	0.02	6	0.15
2	0.16	7	0.0864
3	0.1073	8	0.0009
4	0.2418	9	0.1255
5	0.1073	10	0.0009

under the realization of the normal (adverse) state of the weather.

Concerning the uncertainties integrated within the RMAC for real-time operation we begin by considering all single component failures as well as the common mode failures suggested in the original system specification [23] as credible. To model the weather dependency of outage realization probabilities, we follow the fundamental work of Billinton [21] while assuming, for the benefit of demonstration, that 70% of the failures occur under adverse weather conditions, and that the normal (adverse) weather state would be realized during 85 % (15 %) of the hours of a typical year. We also assume that the probability of failure of post-contingency corrective control is weather independent and equal to 0.01.

The instance of the real-time RMAC employed in all of our case studies has been specified according to an indicative contingency discarding threshold of $\Delta E_{Rt} = 250$ € and a probabilistic reliability target $\varepsilon_{Rt} = 10^{-5}$. The former value is, as per the employed pessimistic approximation, consistent with considering the loss of any single transmission line and cable as a non-discarded contingency upon realisation of the normal weather state. Upon realization of adverse weather, it implies the expansion of the non-discarded contingency subset with 6 (common mode) double line outages, as well as with the transformer outages.

Further, to measure the cost of the TSO reliability management decisions we define the reference commitment status and active power dispatch of each generating unit by simulating a single period perfectly competitive market for electrical energy, constrained by the long-term transmission

TABLE III
REFERENCE COST VALUES (€)

Approach	Planning Cost	Exp. Recourse Cost – SCOPF	Residual Risk	Total Cost
s-SCOPF	379.75	2981.54	0	3361.29

capacity ratings under the *no-outage* network state only. We represent the wind power injections as deterministic in the context of this market clearing problem, by means of the aforementioned "central forecast" values. Appendix D presents the mathematical statement of this MILP problem for the sake of completeness.

We should finally elaborate on the numerical parameters employed in the objective function of the short-term operational planning RMAC. We account for modifications on the commitment status of any generating unit at the respective start-up cost. The reservation of re-dispatch flexibility is accounted for at a marginal cost (in €/MW), which we compute by dividing the respective fixed running cost by the capacity of each unit. For the re-dispatch utilization fee (€/MWh), we employ the largest marginal cost coefficient amongst all segments of the respective piece-wise linear generation cost curve.

B. Reference values on the test system

Prior to analyzing the features of the proposed probabilistic RMAC for short term operational planning, we should establish a frame of reference on the considered test system.

We begin by summarizing in table III the results of the classical *stochastic* approach (*s-SCOPF*) to short-term operational planning, which we obtained by solving the RMAC SCOPF formulation (29 – 50, in appendix A) without discarding any uncertainty realization (that is, with $\mathcal{S}_{RMAC} \equiv \mathcal{S}$). Such an approach would by default seek to guarantee the feasibility of the real-time reliability management strategy under any uncertainty realization ($s \in \mathcal{S}$). As reported in the last column of table III, assuming that set ($\mathcal{S}$) completely describes the uncertainty, the residual risk implied by the stochastic approach is *by default* equal to zero.

Further, Fig. 5 presents reference results on the variation of the *s-SCOPF* cost function with respect to the ratio of the adverse to normal weather state probability. The left-most point in this curve corresponds to the results in table III and the default adverse to normal weather probability ratio of 0.25. As anticipated, these results indicate that while the probability of realizing the adverse weather state increases and the probability of realizing the normal weather state decreases symmetrically, the *s-SCOPF* approach would imply greater costs of planning and securely operating the system. This was indeed anticipated since the real-time recourse stages corresponding to adverse weather realizations are in principle more costly due to the tighter reliability constraints from the adaptive contingency list of the real-time RMAC.

Finally, 6 completes the basis of reference results by plotting the cost resulting from the application of the *s-SCOPF* under

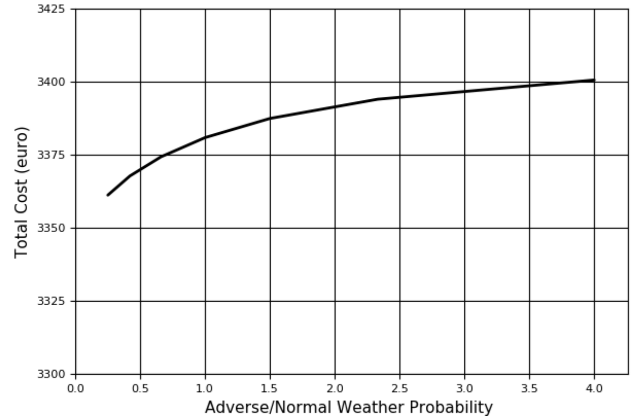

Fig. 5. *s-SCOPF* Cost *vs* Adverse to Normal Weather Probability Ratio

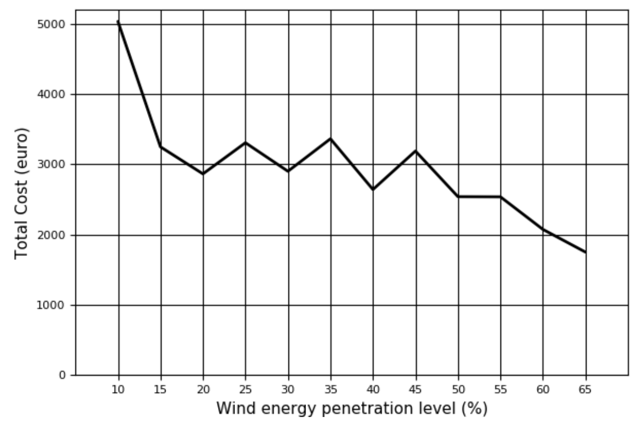

Fig. 6. *s-SCOPF* Cost *vs* Wind Energy Penetration Level

variable levels of daily wind energy penetration in the range $[10, 65]\%$. Recall that the values in table III relate to a daily wind energy penetration of 35% and thus correspond to the middle point of *x-axis* in figure 6. To obtain the remaining values, we have followed the models accompanying the original set of data [25], [26] in scaling per scenario the nodal wind power injection realizations according to the ratio of the modified to the default daily wind energy penetration level. The general trend of the *s-SCOPF* cost decreasing for increasing levels of wind energy penetration relates mostly to the availability of cheaper re-dispatch controls while wind is displacing thermal generation in the market clearing outcome[5].

C. Planning as per the proposed RMAC

We present in this subsection a set of exemplary results of short-term operational planning as per the proposed RMAC. First, let us concentrate on the proposed RMAC discarding principle. To analyze its potential utility, we have selected a range of values for the so-called discarding threshold parameter ΔE_P and solved the corresponding RMAC assessment, discarding & control problems.

[5]The analysis in [25] considering a broader version of the original dataset reports a similar trend in a multi-period stochastic unit commitment context.

TABLE IV
RMAC COSTS *vs* DISCARDING THRESHOLD (€)

Discarding Threshold	*Planning Cost*	*Exp. Recourse Cost* – SCOPF	*Residual Risk*	*Total Cost*
0	379.75	2981.54	0	3361.29
20	379.61	2963.61	18.94	3362.17
500	378.27	2541.20	434.41	3353.87
750	353.68	2458.31	586.81	3398.80
1250	265.45	1862.20	1242.71	3370.36
1750	265.45	1593.81	1511.10	3370.36

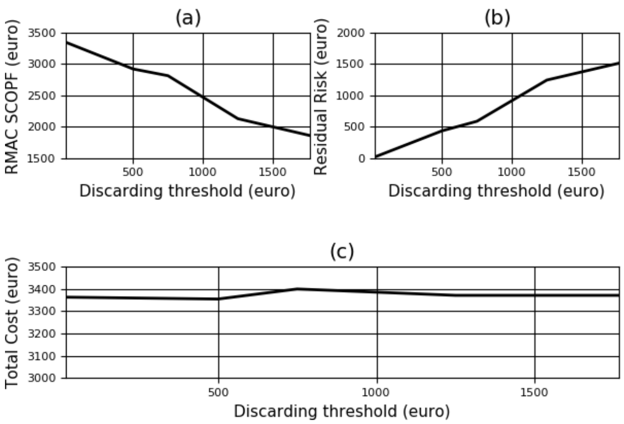

Fig. 7. RMAC Cost & Residual Risk *vs* Discarding threshold

In table IV we start from the *s-SCOPF* solution of zero residual risk and detail the costs associated to the RMAC application for increasing values of the discarding threshold parameter. The table lists the first-stage and recourse components of the RMAC SCOPF cost function (in the 2nd and 3rd columns respectively) as well as a total cost value computed by additionally accounting for the implied residual risk. Complementary illustrations of the RMAC SCOPF cost function, the residual risk value and the total cost value are presented as Fig.7 (a) through (c).

As can be seen from the listed values, lower discarding threshold values generally translate into greater cost values for the RMAC control SCOPF problem while further limiting its implied residual risk (by taking into consideration more uncertainty realizations). In this manner, the discarding threshold serves as a "tuning" parameter defining the level of security implied by the application of the RMAC. We further showcase this by presenting in table VII the total number of non-discarded scenarios (2nd column) as well as the composition of the non-discarded scenario subset (3rd column) corresponding to the studied discarding threshold values. We find that the cardinality of the non-discarded subset increases with a reduction in the discarding threshold value.

It is also is relevant to comment on the difference in the solutions obtained for the discarding threshold values of 1750 €and 1250 €. Table VII suggests that in the latter case the set of non-discarded scenarios is expanded by the additional scenario 3. Further, from table IV we deduce that such additional scenario necessitates no modification in the operational planning decisions, since the changes in the expected recourse component of the SCOPF objective and residual risk between these two cases are symmetrical. The interpretation is that while the already identified planning decisions result to a feasible instance of real-time reliability management upon occurrence of the scenario of concern, its associated economic impact is considerably high and so it should be kept into consideration in order to i) maintain its feasible status, and, ii) possibly identify a better trade-off between planning and expected operational costs (*i.e*, summation of the expected recourse cost component of the SCOPF and residual risk).

TABLE V
RMAC NON-DISCARDED SUBSETS

Discarding Threshold (€)	*Cardinality*	*Composition*
0	20	1–20
20	15	1–7; 9; 12–17; 19
500	9	2–7; 9; 14; 15
750	8	2–7; 9; 14
1250	5	2–6
1750	4	2; 4–6

Alternatively, the expansions of the non-discarded scenario set implied by further reducing the discarding threshold parameter indeed bring about modifications in the operational planning decisions. We notice with interest the total cost corresponding to the discarding threshold value of 500 €. Such total cost value is both lower than the total cost corresponding to a discarding threshold value of 750 €, which is a solution obtained while seeking to guarantee the feasibility of real-time reliability management under less scenarios, and also lower than the optimal cost value of the *s-SCOPF* listed in table III, obtained while seeking to guarantee the feasibility of real-time reliability management under more scenarios. The former finding showcases the potential sub-optimality while neglecting a larger part of the uncertainty set through a greater discarding threshold value.

To clearly explain the second observation we resort to more detailed results on the solution obtained under the 500 € discarding threshold value. The second column of table VI lists whether each discarded uncertainty realization corresponds to an operational condition wherein the real-time operation reliability management strategy is achievable or not. The third and fourth column of this table present complementary information on the probability and risk implied by each uncertainty realization. We can identify that the proposed RMAC for short-term operational planning achieves a lower total cost value with respect to the *s-SCOPF* by tolerating the low risk of having to relax the real-time reliability management strategy upon occurrence of several low-probability weather & power injection realizations. In contrast, any relaxation to the real-time reliability management strategy is non-tolerable by the *s-SCOPF*.

It is finally noteworthy that most realizations under which

TABLE VI
DISCARDED SCENARIOS, $\Delta E_p = 500$ (€)

$s \notin \mathcal{S}_{RMAC}$	*Rt-RMAC*	*Probability*	*Risk (€)*
8		0.00073	0.80
10		0.00073	1.27
18	feasible	0.00018	0.24
19		0.02509	62.79
20		0.00018	0.33
1		0.01600	55.65
11		0.00400	31.40
12		0.03200	82.10
13	relaxed	0.02145	73.09
15		0.02145	105.98
16		0.03000	92.65
17		0.01727	80.51

the relaxation of the real-time criterion is tolerated by the short-term operational planning RMAC correspond to the adverse weather state (scenarios 11-17 in table VI). Recalling that for each one of these scenarios, there is also a non-discarded scenario with the same injection pattern and the normal weather state, the critical factor here is evidently the relatively low likelihood of the adverse weather state. The interpretation is that, due such relatively low likelihood, the risk of not preparing the system to face the adverse weather conditions (*i.e.*, to be able to withstand more contingency events such as common mode outages) is negligible according to the standard of the short-term operational planning RMAC. Motivated by this example, we discuss further the adaptability to the uncertain weather state in the following subsection.

D. Adaptability to the uncertain weather state

In this subsection we gradually modify the ratio of the adverse weather state to the normal weather state realization probability so as to study the adaptability of the proposed RMAC with regard to the uncertain weather state. Fixing for the purposes of this analysis the discarding threshold parameter value to ΔE_P=500 €, we consider 7 different values in the range [0.25,4].

Recalling the already introduced Fig. 6 which plots the relevant reference result on the *s-SCOPF*, Fig. 8 presents the corresponding curves obtained while applying the proposed RMAC for operational planning. More specifically, the solid line in Fig. 8 presents the optimal cost value of the RMAC SCOPF problem while the dashed line corresponds to the total cost associated to the application of the RMAC, accounting as well for the residual risk value. As previously, the results corresponding to the assumed default ratio of 0.25 appear in the leftmost part of both curves. We confirm the anticipated trend of such curves increasing with the increase in the adverse weather realization probability, in a similar manner to the *s-SCOPF* cost.

The adaptation of the RMAC to such an increase can be clearly identified by examining the data presented in table VII. The second column of such table lists the cardinality of the subset of non-discarded operational conditions as per the

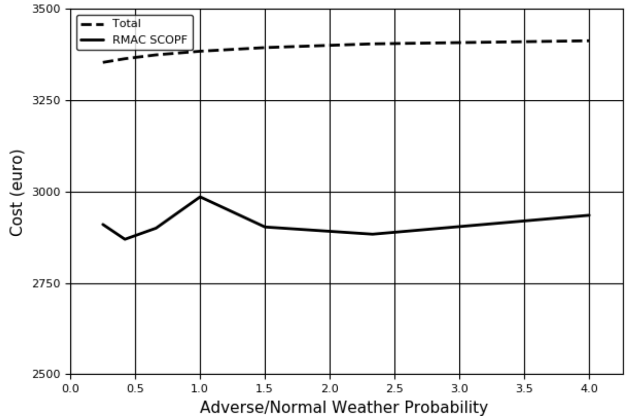

Fig. 8. RMAC Costs *vs* Adverse to Normal Weather Probability Ratio

TABLE VII
NON-DISCARDED SCENARIOS *vs* WEATHER STATE PROBABILITY RATIO

Adverse/Normal Probability	*Non-Discarded Scenarios (#)*		
	Total	*Normal Weather*	*Adverse Weather*
0.25	9	7	2
0.42	10	7	3
0.66	11	7	4
1	11	5	6
1.5	12	5	7
2.33	11	3	8
4	9	2	7

RMAC for the problem instances under consideration. The third and fourth columns further decompose such subset by listing the amount of non-discarded scenarios corresponding to the normal and adverse weather state respectively. We notice the gradual increase (decrease) in the amount of non-discarded scenarios concerning adverse (discarded scenarios concerning normal) weather conditions as the cardinality of the non-discarded scenario sub-set remains relatively stable. This is due to the fact that the gradual increase in the probability of realizing the adverse weather states progressively makes its implied risk non-negligible. Upon such conditions, the subset of non-discarded scenarios of the short-term operational planning RMAC would mostly include the adverse weather scenarios, facilitating the reliable operation of the system in real-time. This result exemplifies again the role of the discarding threshold as the parameter establishing the sought degree of risk-aversion at the short-term operational planning stage.

To complete this preliminary investigation, we focus through Fig. 9 on the relationship between the weather state probabilities and the expected severity associated to the potential failure of the post-contingency corrective control actions in real-time. As seen in Fig. 9, the increase of the severity expectation with the increase of the adverse weather state probability is rather notable. Examining our detailed results, we have identified that the reason behind such increase is two-fold. Primarily, the severity expectation increases due to the increase in the realization probability of contingency events.

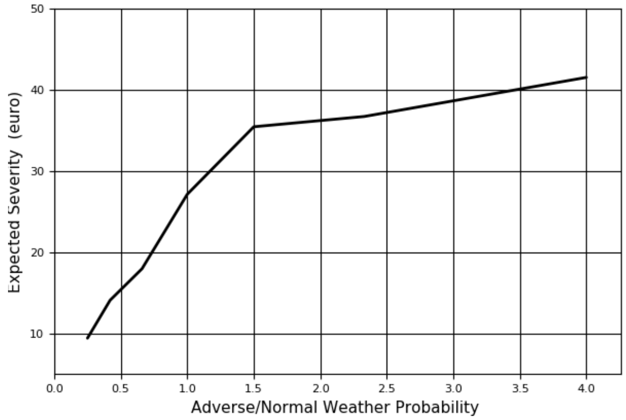

Fig. 9. Expected Severity *vs* Adverse to Normal Weather Probability Ratio

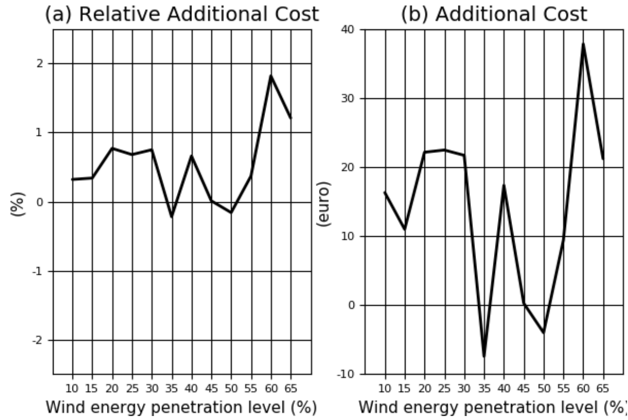

Fig. 11. RMAC Cost & Residual Risk *vs* *s-SCOPF* Cost

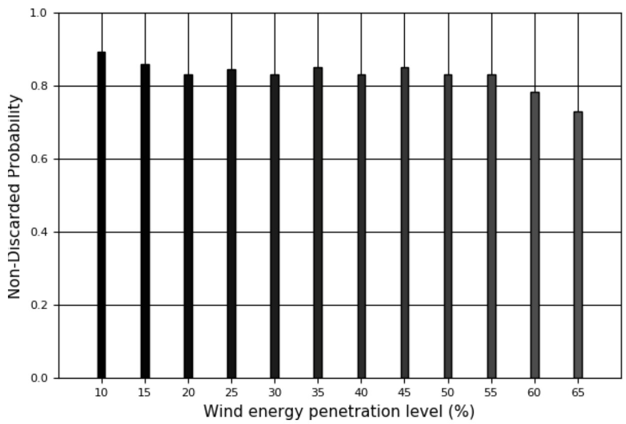

Fig. 10. Non-discarded probability *vs* wind energy penetration level

Secondarily, the severity expectation is further increased due to the fact that, in the adverse weather scenarios, post-contingency corrective controls are more heavily relied upon in order to effectively manage the greater and more complex set of contingency events under consideration.

E. Adaptability to the level of wind energy penetration

Through our final set of numerical results we investigate the adaptability of the proposed probabilistic RMAC to the level of wind energy penetration. We do so in similar manner to producing the reference curve of Fig.6, that is by modifying the wind energy daily penetration level in the range $[10, 65]\%$ and accordingly scaling the wind power injection realizations. Moreover, we also maintain constant the discarding threshold parameter to the value of 500 € for the purpose of this analysis.

We begin with showcasing the functionality of the discarding principle by means of the bar chart in Fig. 10. Such chart presents the probability of the non-discarded scenario sub-set ($\mathcal{S}_{RMAC}$) for the varying wind energy penetration levels under consideration, including the considered default level of 35%. The dependence of the non-discarded probability, hence of the cardinality and composition of the non-discarded scenario subset, to the wind energy penetration level is evident. Further, examining the leftmost against the rightmost points, we may notice that the non-discarded probability is considerably greater (smaller) for lower (higher) wind energy penetration levels.

It is of relevance to recall here our reference values of Fig.6, demonstrating a reduction of the *s-SCOPF* cost as increasing wind energy penetration levels result in the availability of more and cheaper dispatchable resources to plan and reliably operate the system. The performance of the RMAC illustrated in Fig. 10 is in complete accordance with such reference results. Situations within which the operator has to resort to more costly control actions may generally be interpreted as corresponding to increased system stress (for instance due to increased loading and/or reduced availability of control means and/or unavailability of transmission elements, either planned or unplanned, *etc.*). The adaptability of the proposed RMAC for operational planning to the level of wind energy penetration (and the implications thereof for system operation) is prominent here, as Fig. 10 shows that its application would imply neglecting a smaller (larger) part of the uncertainty space accordingly. We should also observe here that the five most costly *s-SCOPF* problem instances from Fig. 6 correspond to the five problem instances with the largest non-discarded probability, Fig. 10, as per the proposed RMAC.

To conclude this analysis, we briefly turn our attention to the costs implied by the application of the proposed RMAC. Fig. 11 presents the additional costs implied by the RMAC (including the associated residual risk) with respect to the previously introduced costs of the *s-SCOPF* approach, both in relative (a) and absolute (b) terms. While our analysis merely serves the purpose of demonstrating the features of our proposal and the findings discussed here should only be treated as indicative, we note that the relative cost differences are typically smaller (greater) for the problem instances wherein the *s-SCOPF* costs are higher (lower). It is useful to recall here that such cost differences arise from the fact that the RMAC discards a part of the uncertainty space. As already showcased in earlier our results, the uncertainty space discarded by the RMAC should be smaller (greater) for the more (less)

TABLE VIII
RMAC COSTS *vs* WIND ENERGY PENETRATION

Penetration Level (%)	*Planning Cost (€)*	Exp. Recourse *Cost (€)* – SCOPF	*Residual Risk (€)*	*Total Cost (€)*
10	1449.31	3189.48	408.61	5047.40
15	322.65	2509.86	425.52	3258.03
20	203.25	2198.89	482.54	2884.67
25	353.19	2495.23	479.04	3327.45
30	204.25	2226.11	490.96	2921.32
35	378.27	2541.20	434.41	3353.87
40	269.74	1949.75	438.00	2657.48
45	413.42	2358.14	415.79	3187.35
50	275.39	1839.12	419.60	2534.11
55	275.49	1848.43	421.44	2545.37
60	275.13	1372.14	463.08	2110.34
65	276.99	1100.55	393.65	1771.20

costly operational planning problem instances. Thus, achieving smaller (greater) relative cost differences under the more (less) costly problem instances should be an expected feature of the proposed RMAC.

Finally, and for the sake of completeness, we list in table VIII the cost-breakdown associated to the short-term operational planning RMAC application under the different studied levels of wind energy penetration. An interesting finding from our detailed results suggests that the ranking of the studied problem instances as per i) the RMAC total cost, composed of the RMAC-SCOPF costs and residual risk, ii) the RMAC-SCOPF cost only and iii) the *s-SCOPF* cost remains equivalent. This suggests that, provided that the discarding threshold parameter is well-chosen, the approximation achieved by using the proposed RMAC should remain indicative of the relative difference in the criticality of different operational planning problem instances.

V. CONCLUDING DISCUSSION

This section concludes with several considerations on the proposed probabilistic approach to reliability management through planning and operation. We begin with an overview of the work reported here, briefly recapitulating our proposal as well as the findings from the numerical investigations in the context of its prototype implementation. We continue by assessing the stakes for implementation upgrades in terms of the employed physical models and assumptions, as well as in terms of computational simplicity and tractability. Further, we also introduce the broader general scope of the multiperiod look-ahead class of operational planning problems to trace future work pertaining to the generalization of the probabilistic RMAC proposal.

A. Overview of motivation, proposal & main findings

Short-term operational planning serves to prepare the operation of the system by ensuring the potentially needed resources as early as required ahead of real-time. Over the recent years, we have witnessed the growth of uncertainties sparking the growing interest of the scientific community on two questions, specifically the *conceptual* question of (re)defining the criteria for reliability management within a risk-aware planning perspective, and the *dimensionality* question of facing the larger and larger uncertainty space.

We proposed in this work a probabilistic reliability management (*i.e.*, assessment and control) framework as an attempt to contribute to jointly answering these two questions. Our proposal concentrates on tackling the uncertainties manifested between planning and operation (*e.g.*, power injections, weather conditions, *etc.*) by relying on i) the *reliability target* of rendering real-time operation achievable as per its respective reliability management strategy, and, ii) the *discarding principle* of explicitly considering only a part of the uncertainty space for decision making, provided that the residual risk implied by those neglected realizations remains acceptably low. On top of these fundamental building blocks, our approach adopts a *socio-economic* cost function as a minimization objective in a stochastic optimization context.

To facilitate the clear identification of the properties of this proposal, we developed a prototype implementation and presented demonstrative case studies on an academic benchmark. The results are indicative of the inherent flexibility and adaptability in our approach. Indeed, neglecting a subset of uncertainty realizations offers the flexibility not only to simplify the decision making problem in question but also to potentially avoid costly operational planning decisions while tolerating low risk anticipated relaxations of the real-time reliability criterion. Doing so in a risk-aware manner, while limiting residual risk, offers adaptability. The reported results exemplify such adaptability with respect to the qualitative features of both i) the uncertainty space (*see* § IV-D under variable weather state realization probabilities), and, ii) the operational situation (*see* § IV-E under variable wind energy penetration levels).

B. Beyond the proof-of-concept implementation

While the presented results serve to establish the general soundness of the proposed RMAC for short-term operational planning, going beyond the *proof-of-concept* implementation used to facilitate this study requires further progress in terms of modeling, as well as dedicated attention to reducing computational complexity and achieving algorithmic scalability.

Starting from uncertainty modeling, we relied on a publicly available description of wind power injections via a discretized set of realizations, along with a discrete set of weather states of assumed probability distribution. We have neglected any dependence of the former random variable on the latter, as well as other uncertainties from our case study set-up. To improve on this basic model, the first step would be to expand the scope of uncertainty modeling by including several weather dependent exogenous variables (such as, but not limited to, the load demand, the value of lost load, *etc.*). As soon as the uncertainty modeling scope is well identified, considerable effort should be anticipated to represent all such continuous random processes as well as their spatial and temporal dependencies. We believe such effort to be rather worthwhile, in order to fully exploit the benefits of the proposal set forward in this paper.

Further, we used relatively simple physical models of the power system (involving quasi-static and linear assumptions) to foster the interpretability of results. With the same motivation, as well as with the intention to establish the interest in the consideration of real-time proxies, the real-time reliability management strategy as well as its relaxation principle were represented analytically as SCOPF problems of mixed integer linear type. Enhancing the practical relevance of the modeled set-up while reducing its associated computational burden would both be necessary to enable pursuing our proposal further. Let us point here to the potential of using machine learning techniques to develop real-time reliability management proxies and exploiting such proxies within an upgraded implementation of the RMAC for short-term operational planning. Referring the interested reader to [27] for preliminary results on the development of real-time reliability management proxies through machine learning, we should further acknowledge the possibility to develop different such proxies of the real-time for the different functions of the operational planning RMAC.

Indeed, in the context of the operational planning RMAC assessment function, the relevant outcome is the cost resulting from the application of the real-time reliability management strategy (or, *if-need-be*, its relaxation) upon occurrence of any operational condition. While in the prototype implementation the RMAC assessment function also computes the real-time feasibility indicator as well as preventive and/or post-contingency corrective decisions corresponding to each uncertainty realization, these variables have been merely treated as by-products to facilitate the analysis and discussion of results. It is therefore in principle admissible to further specify the scope of a real-time proxy for short-term operational planning RMAC assessment as limited to the estimation of the costs implied by the real-time reliability management strategy and thus endogenise the modeling of the respective decisions. Such further specification may be beneficial regarding both the accuracy and the computational performance of the proxy model. In a similar manner, we may envision the operational planning RMAC control function relying on a different proxy for modeling i) the feasibility of the real-time reliability management strategy as a function of a non-discarded operational condition and operational planning decisions, and, ii) the real-time preventive and/or corrective control decisions.

The exploration of such ideas for upgrading the algorithmic implementation of the proposed RMAC for short-term operational planning remains a topic for further research.

C. Towards look-ahead mode reliability management for operational planning

In parallel to the aforementioned modeling progress and algorithmic implementation upgrades, future work concerns extending the discussed ideas to the more general class of look-ahead mode multi-stage problems. Look-ahead mode multi-stage operational planning concerns making, as early in advance as required, decisions that have effects over a prolonged temporal horizon. The respective uncertainty is progressively resolved within the temporal horizon, giving rise to recourse decision making opportunities. This problem class finds much more applications in power system operational planning, such as the week-ahead verification of planned outages for maintenance, the determination of available transmission capacities for the day-ahead market clearing, the post-market clearing adjustments of the unit commitment and dispatch in day-ahead, *etc.*.

Using the day-ahead reliability management problem instance as an example, planning decisions are typically being made around 8 – 12 hours before the start of the respective daily horizon. Such daily horizon is furtherly decomposable into a sequence of instances of the real-time reliability management problem. The stakes for reliability management in such a context are raised both by the properties of the random processes of concern and by the sequential transitions between the different real-time reliability management problem instances within the planning horizon. The uncertainties pertaining to the power systems operational conditions over a prolonged horizon are (typically) not only resolved in a gradual manner but also temporally correlated. Moreover, real-time reliability management decisions corresponding to sequential problem instances are joined by the coupling constraints describing the existence of a feasible transition from the anterior to the posterior problem instance. The proper representation of both such features results in a large-scale multi-stage stochastic decision making problem. To the best of the authors' knowledge, the effective resolution of such multi-stage look-ahead reliability management problem remains open in the relevant literature [4].

In our prospective research we shall attempt to develop the multi-stage look-ahead variant of the RMAC for operational planning. At the conceptual level, this implies the replacement of the single real-time problem instance considered in this work with the notion of a *trajectory*, corresponding to a sequence of real-time problem instances spanning the temporal horizon of concern. Accordingly, we conclude by briefly framing the look-ahead mode: i) *reliability target* of ensuring that the objectives of real-time reliability management are achievable under any anticipated trajectory, and, ii) *socio-economic objective* of minimizing the planning costs along with the cost expectation over all anticipated trajectories, and, iii) *discarding principle* of reducing the problem size by discarding those anticipated trajectories that imply negligible residual risk.

APPENDIX

Nomenclature

The main mathematical symbols used in this appendix are defined as follows. Others may be defined as needed within the text.

Indices:

c	Index of contingencies.
d	Index of demands.
g	Index of dispatchable generating units.

k	Index of piece-wise linear dispatchable generation cost curve segments.
ℓ	Index of transmission elements (*i.e.* lines, cables and transformers).
n	Index of nodes.
s	Index of power injection & weather scenarios.
w	Index of wind power generators.

Sets:

$\mathcal{C}$	Set of contingencies.
$\mathcal{C}_{RMAC}(s) \subseteq \mathcal{C}$	Subset of non-discarded RMAC contingencies (*i.e.*, covered by the SCOPF formulation) under scenario $s \in \mathcal{S}$.
$\mathcal{D}$	Set of demands.
$\mathcal{D}_n \subseteq \mathcal{D}$	Subset of demands connected at node n.
$\mathcal{G}$	Set of dispatchable generating units.
$\mathcal{G}_n \subseteq \mathcal{G}$	Subset of dispatchable generating units connected at node n.
$\mathcal{K}$	Set of piece-wise linear dispatchable generation cost curve segments.
$\mathcal{L}$	Set of transmission elements.
$\mathcal{N}$	Set of nodes.
$\mathcal{S}$	Set of weather & wind power injection scenarios.
$\mathcal{S}_{RMAC} \subseteq \mathcal{S}$	Subset of non-discarded RMAC weather & wind power injection scenarios (*i.e.*, covered by the SCOPF formulation).
$\mathcal{W}$	Set of wind power generators.
$\mathcal{W}_n \subseteq \mathcal{W}$	Subset of wind power generators connected at node n.

Parameters:

$a_{\ell,c}(s)$	Binary parameter taking a zero value if transmission element $\{\ell \in \mathcal{L}\}$ is unavailable under contingency c and scenario s.
c_g^f	Fixed running cost (in monetary units) of generating unit g.
$c_{g,k}$	Marginal running cost (in monetary units per unit) of generating unit g at the segment k of its piecewise linear cost curve.
c_g^R	Re-dispatch flexibility reservation marginal cost (in monetary units per unit) of generating unit g.
c_g^r	Re-dispatch flexibility activation marginal cost (in monetary units per unit) of generating unit g.
c_g^0	Startup cost (in monetary units) of generating unit g.
P_g^{max}	Capacity of generating unit g.
P_g^{min}	Minimum stable output of generating unit g.
ΔP_g^-	Ramp-down limit of generating unit g in corrective mode.
ΔP_g^+	Ramp-up limit of generating unit g in corrective mode.
P_d	Active power demand of load d.
$P_w(s)$	Active power injection of wind generator w under scenario s.
v_d	Value of lost load of demand d.
f_ℓ^{max}	Long-term thermal rating of transmission element ℓ.
r_ℓ	Ratio of the short-term thermal rating to the long-term thermal rating of transmission element ℓ ($r_\ell \geq 1$).
X_ℓ	Reactance of transmission element ℓ.
$\beta_{n,\ell}$	Element of the flow incidence matrix, taking a value of one if node n is the sending node of element ℓ, a value of minus one if node n is the receiving node of element ℓ, and a zero value otherwise.
$\pi_c(s)$	Probability of occurrence of contingency c under scenario s.
π_{fcc}	Probability of failure of corrective control.
$\tilde{\pi}_(s)$	Probability of occurrence of scenario s.
ε_{Rt}	Tolerance level of the real-time reliability target.
M	A large constant.

Continuous Variables:

P_g^M	Dispatch of generating unit g as per the market clearing.
$P_{g,0}^+(s)$	Preventive ramp-up of generating unit g under scenario s.
$P_{g,0}^-(s)$	Preventive ramp-down of generating unit g under scenario s.
$P_{g,c}^+(s)$	Corrective ramp-up of generating unit g following contingency c under scenario s.
$P_{g,c}^-(s)$	Corrective ramp-down of generating unit g following contingency c under scenario s.
R_g^+	Upward re-dispatch flexibility provided by generating unit g.
R_g^-	Downward re-dispatch flexibility provided by generating unit g.
$f_{\ell,0}(s)$	Power flowing through transmission element ℓ under the pre-contingency state and scenario s.
$f_{\ell,c}^{IS}(s)$	Power flowing through transmission element ℓ at the intermediate stage following contingency c and prior to the application of corrective control under scenario s.
$f_{\ell,c}(s)$	Power flowing through transmission element ℓ following contingency c and the successful application of corrective control under scenario s.
$\theta_{n,0}(s)$	Voltage angle at node n under the pre-contingency state.
$\theta_{n,c}^{IS}(s)$	Voltage angle at node n at the intermediate stage following contingency c and prior to the application of corrective control under scenario s.
$\theta_{n,c}(s)$	Voltage angle at node n following contingency c and the succesful application of corrective control under scenario s.

Nb: All continuous variables are non-negative with the exception of the transmission element flow variables, and angle variables.

Binary Variables:

on_g	Indicating the decision to start-up generating unit g as per the short-term operational planning RMAC SCOPF.
off_g	Indicating the decision to shut-down generating unit g as per the short-term operational planning RMAC SCOPF.
v_g	Indicating the commitment status of generating unit g as per the market clearing.
u_g	Indicating the commitment status of generating unit g as per the short-term operational planning RMAC SCOPF.
γ_c	Auxiliary variable employed for the relaxation of post-contingency constraints in the relaxation of the real-time RMAC SCOPF.
$\zeta_c(s)$	Indicating the use of post-contingency corrective control following contingency c under scenario s.

A. Short-term Operational Planning RMAC SCOPF statement

The MILP formulation of the short-term operational planning RMAC SCOPF problem statement is as shown in (29 – 50).

N.b: Superscript (*) is used where appropriate (e.g., with the variable corresponding to the market dispatch of generating units) to denote a variable whose value has been fixed as per the solution of a different problem, treated here as a parameter.

$$\begin{aligned}\min\Bigg\{&\sum_{g\in\mathcal{G}}\left[c_g^0\left(on_g+off_g\right)+c_g^R\left(R_g^+ + R_g^-\right)\right]\\&+\sum_{s\in\mathcal{S}_{RMAC}}\tilde{\pi}_s\Bigg\{\sum_{g\in\mathcal{G}}\left[c_g^r\cdot\left(P_{g,0}^+(s)+P_{g,0}^-(s)\right)\right]\\&\qquad+\sum_{c\in\mathcal{C}_{RMAC}(s)}\pi_c(s)\cdot\sum_{g\in\mathcal{G}}\left[c_g^r\cdot\left(P_{g,c}^+(s)+P_{g,c}^-(s)\right)\right]\\&\qquad+\sum_{c\in\mathcal{C}_{RMAC}(s)}\pi_c(s)\cdot\zeta_c(s)\cdot\pi_{fcc}\cdot\sum_{d\in\mathcal{D}}v_d\cdot P_d\Bigg\}\Bigg\},\end{aligned} \tag{29}$$

subject to, for all generating units $g \in \mathcal{G}$:

$$u_g - on_g \leq v_g^\star, \tag{30}$$
$$- u_g - off_g \leq -v_g^\star, \tag{31}$$
$$R_g^+ - u_g \cdot \left(P_g^{\max} - \cdot P_g^{M\star}\right) \leq 0, \tag{32}$$
$$R_g^- - u_g \cdot v_g^\star \cdot \left(P_g^{M\star} - P_g^{\min}\right) \leq 0, \tag{33}$$

for all non-discarded scenarios $s \in \mathcal{S}_{RMAC}$:

$$\sum_{c \in \mathcal{C}_{RMAC}(s)} \zeta_c(s) \cdot \pi_c(s) \cdot \pi_{fcc} \leq \varepsilon_{Rt}, \tag{34}$$

for all nodes $n \in \mathcal{N}$ & non-discarded scenarios $s \in \mathcal{S}_{RMAC}$:

$$\sum_{g \in \mathcal{G}_n} \left[u_g \cdot P_g^{M\star} + \left(P_{g,0}^+(s) - P_{g,0}^-(s)\right)\right] - \\ - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,0}(s) = \sum_{d \in \mathcal{D}_n} P_d - \sum_{w \in \mathcal{W}_n} P_w(s), \tag{35}$$

for all transmission elements $\ell \in \mathcal{L}$ & non-discarded scenarios $s \in \mathcal{S}_{RMAC}$:

$$f_{\ell,0}(s) - \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,0}(s) = 0, \tag{36}$$
$$f_{\ell,0}(s) \leq f_\ell^{\max}, \tag{37}$$
$$- f_{\ell,0}(s) \leq f_\ell^{\max}, \tag{38}$$

for all generating units $g \in \mathcal{G}$, non-discarded contingencies $c \in \mathcal{C}_{RMAC}(s)$ & non-discarded scenarios $s \in \mathcal{S}_{RMAC}$:

$$0 \leq \left(P_{g,0}^+(s) + P_{g,c}^+(s)\right) \leq R_g^+, \tag{39}$$
$$0 \leq \left(P_{g,0}^-(s) + P_{g,c}^-(s)\right) \leq R_g^-, \tag{40}$$
$$0 \leq P_{g,c}^+(s) \leq \zeta_c(s) \cdot \Delta P_g^+, \tag{41}$$
$$0 \leq P_{g,c}^-(s) \leq \zeta_c(s) \cdot \Delta P_g^-, \tag{42}$$

for all nodes $n \in \mathcal{N}$, non-discarded contingencies $c \in \mathcal{C}_{RMAC}(s)$ & non-discarded scenarios $s \in \mathcal{S}_{RMAC}$:

$$\sum_{g \in \mathcal{G}_n} \left[u_g \cdot P_g^{M\star} + \left(P_{g,0}^+(s) - P_{g,0}^-(s)\right)\right] - \\ - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c}^{IS}(s) = \sum_{d \in \mathcal{D}_n} P_d - \sum_{w \in \mathcal{W}_n} P_w(s), \tag{43}$$

$$\sum_{g \in \mathcal{G}_n} \Big[u_g \cdot P_g^{M\star} + \left(P_{g,0}^+(s) - P_{g,0}^-(s)\right) + \left(P_{g,c}^+(s) - P_{g,c}^-(s)\right)\Big] - \\ - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c}(s) = \sum_{d \in \mathcal{D}_n} P_d - \sum_{w \in \mathcal{W}_n} P_w(s), \tag{44}$$

for all transmission elements $\ell \in \mathcal{L}$, non-discarded contingencies $c \in \mathcal{C}_{RMAC}(s)$ & non-discarded scenarios $s \in \mathcal{S}_{RMAC}$:

$$f_{\ell,c}^{IS}(s) - a_{\ell,c}(s) \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c}^{IS}(s) = 0, \tag{45}$$
$$f_{\ell,c}^{IS}(s) \leq a_{\ell,c}(s) \cdot r_\ell \cdot f_\ell^{\max}, \tag{46}$$
$$- f_{\ell,c}^{IS}(s) \leq a_{\ell,c}(s) \cdot r_\ell \cdot f_\ell^{\max}, \tag{47}$$
$$f_{\ell,c}(s) - a_{\ell,c}(s) \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} = 0, \tag{48}$$
$$f_{\ell,c}(s) \leq a_{\ell,c}(s) \cdot f_\ell^{\max}, \tag{49}$$
$$- f_{\ell,c}(s) \leq a_{\ell,c}(s) \cdot f_\ell^{\max}. \tag{50}$$

The summation appearing in the first row of objective function (29) measures, over all dispatchable generating units, the direct costs associated to i.) modifying the commitment status of any unit, and ii.) procuring re-dispatch flexibility to be potentially used in real-time. The summation in the second to fourth row of objective function (29) corresponds to the expectation of real-time reliability management costs over the sub-set of non-discarded operational conditions. The first component of such expectation (second row), expresses the expected costs of re-dispatching generating units in preventive mode while the second component (third row) corresponds to corrective control, and is payable upon occurrence of the respective contingency. The final term (fourth row) conservatively penalizes the potential failure of the chosen corrective actions, at the cost of shedding the whole system load demand. Notice that variable $\zeta_c(s)$ also appears in (41–42) to indicate the use of post-contingency corrective controls.

Constraints (30 - 31) are used in order to identify and charge the modifications imposed on the status of the dispatchable generating units with respect to the market clearing outcome. Further, inequalities (32– 33) set the upper bounds on the upward (downward) re-dispatch flexibility positive variables as allowed by the market clearing outcome and the maximum capacity (minimum stable output) of each dispatchable unit.

For each non-discarded scenario $s \in \mathcal{S}_{RMAC}$, the chance-constrained reliability target of the real-time RMAC is expressed by (34). Recall again that variable $\zeta_c(s)$ denotes the use of post-contingency corrective controls for the respective contingency, which as mentioned may fail with probability π_{fcc}. Notice also that the summation is expressed over the scenario-specific, subset of non-discarded contingencies. Expressions (35–38) are the "classical" DC power flow equality, and flow limit inequality constraints for the preventive stage, again per non-discarded scenario $s \in \mathcal{S}_{RMAC}$. The restriction of only using the procured upward and downward re-dispatch flexibility in pre- and post-contingency stages is expressed by inequality constraints (39,40) respectively.

For each non-discarded scenario $s \in \mathcal{S}_{RMAC}$ and corresponding non-discarded contingency $c \in \mathcal{C}_{RMAC}(s)$ inequality constraints (41-42) are generation ramping restrictions linking the preventive and corrective active power outputs. The group of equalities and inequalities (43, 45–47) impose the power flow linear approximation and active power flow limits at the intermediate stage following the occurence of a contingency and the use of post-contingency corrective control. Finally, the similar type of physical restrictions referring to the post-contingency corrective stage (*i.e.*, given the successful use of corrective controls) is denoted by (44, 48–50).

B. *Real-time RMAC SCOPF statement*

The MILP formulation of the real-time RMAC SCOPF problem statement is as shown in (51 – 68).

N.b.: Superscript $(^\star)$ is used where appropriate (e.g., with the variable corresponding to the market dispatch of generating units) to denote a variable whose value has been fixed as per the solution of a different problem, treated here as a parameter.

N.b.(2): While, as explained in the main body of this paper, all variables & several parameters of this problem are defined with respect to a given realization of operational conditions $s \in \mathcal{S}$, we avoid the use of such symbol here for the benefit of notational simplicity.

$$\min\Bigg\{\sum_{g \in \mathcal{G}} c_g^r \cdot \Big[\left(P_{g,0}^+ + P_{g,0}^-\right) + \sum_{c \in \mathcal{C}_{RMAC}(\bar{s})} \pi_c \cdot \left(P_{g,c}^+ + P_{g,c}^-\right)\Big] \\ + \sum_{c \in \mathcal{C}_{RMAC}} \pi_c \cdot \zeta_c \cdot \pi_{fcc} \cdot \sum_{d \in \mathcal{D}} v_d \cdot P_d,\Bigg\} \tag{51}$$

subject to,

$$\sum_{c\in\mathcal{C}_{RMAC}(\bar{s})} \pi_c \cdot \zeta_c \cdot \pi_{fcc} \leq \varepsilon_{Rt}, \tag{52}$$

for all nodes $n \in \mathcal{N}$:

$$\left\{ \sum_{g\in\mathcal{G}_n} \left[u_g^\star \cdot P_g^{M\star} + \left(P_{g,0}^+ - P_{g,0}^-\right)\right] - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell}\cdot f_{\ell,0} = \sum_{d\in\mathcal{D}_n} P_d - \sum_{w\in\mathcal{W}_n} P_w \right\}, \tag{53}$$

for all transmission elements $\ell \in \mathcal{L}$:

$$f_{\ell,0} - \frac{1}{X_\ell}\sum_{n\in\mathcal{N}_n} \beta_{n,\ell}\cdot\theta_{n,0} = 0, \tag{54}$$

$$f_{\ell,0} \leq f_\ell^{\max}, \tag{55}$$

$$-f_{\ell,0} \leq f_\ell^{\max}, \tag{56}$$

for all generating units $g \in \mathcal{G}$ *& non-discarded contingencies* $c \in \mathcal{C}_{RMAC}$:

$$0 \leq \left(P_{g,0}^+ + P_{g,c}^+\right) \leq R_g^{+\star}, \tag{57}$$

$$0 \leq \left(P_{g,0}^- + P_{g,c}^-\right) \leq R_g^{-\star}, \tag{58}$$

$$0 \leq P_{g,c}^+ - \zeta_c \cdot \Delta P_g^+ \leq 0, \tag{59}$$

$$0 \leq P_{g,c}^- - \zeta_c \cdot \Delta P_g^- \leq 0, \tag{60}$$

for all nodes $n \in \mathcal{N}$ *& non-discarded contingencies* $c \in \mathcal{C}_{RMAC}$:

$$\left\{ \sum_{g\in\mathcal{G}_n} \left[u_g^\star \cdot P_g^{M\star} + \left(P_{g,0}^+ - P_{g,0}^-\right)\right] - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell}\cdot f_{\ell,c}^{IS} = \sum_{d\in\mathcal{D}_n} P_d - \sum_{w\in\mathcal{W}_n} P_w \right\}, \tag{61}$$

$$\left\{ \sum_{g\in\mathcal{G}_n} \left[u_g^\star \cdot P_g^{M\star} + \left(P_{g,0}^+ - P_{g,0}^-\right) + \left(P_{g,c}^+ - P_{g,c}^-\right)\right] - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell}\cdot f_{\ell,c} = \sum_{d\in\mathcal{D}_n} P_d - \sum_{w\in\mathcal{W}_n} P_w \right\}, \tag{62}$$

for all transmission elements $\ell \in \mathcal{L}$ *& non-discarded contingencies* $c \in \mathcal{C}_{RMAC}$:

$$f_{\ell,c}^{IS} - a_{\ell,c}\cdot\frac{1}{X_\ell}\sum_{n\in\mathcal{N}_n} \beta_{n,\ell}\cdot\theta_{n,c}^{IS} = 0, \tag{63}$$

$$f_{\ell,c}^{IS} \leq a_{\ell,c}\cdot r_\ell \cdot f_\ell^{\max}, \tag{64}$$

$$-f_{\ell,c}^{IS} \leq a_{\ell,c}\cdot r_\ell \cdot f_\ell^{\max}, \tag{65}$$

$$f_{\ell,c} - a_{\ell,c}\cdot\frac{1}{X_\ell}\sum_{n\in\mathcal{N}_n} \beta_{n,\ell}\cdot\theta_{n,c} = 0, \tag{66}$$

$$f_{\ell,c} \leq a_{\ell,c}\cdot f_\ell^{\max}, \tag{67}$$

$$-f_{\ell,c} \leq a_{\ell,c}\cdot f_\ell^{\max}. \tag{68}$$

The first row objective function (51) correspond to the costs of using the re-dispatch flexibility in preventive mode, and the expected costs of further using this resource for post-contingency corrective control. The term appearing in the second row of (51) relates to the potential failure of corrective control. Notice that, as anticipated, the objective function of the real-time RMAC SCOPF (51) is an instance of the summation appearing in the second to fourth rows of the short-term RMAC SCOPF (29). The same accordance holds true for the constraint sets of these two problems.

More specifically, the real-time reliability target (52) is indeed an instance of constraint (29) of the short-term planning problem. Further, the preventive (53-56), intermediate post-contingency (61, 63–65) and corrective post-contingency flow constraints (62, 66–68) correspond with constraint groups (35-38), (43, 45–50) and (48–50) respectively. Finally, the use of the re-dispatch flexibility is limited here by (57,58) analogously to (39,40).

C. Real-time RMAC relaxation SCOPF statement

The MILP formulation of the real-time RMAC SCOPF relaxation problem statement is as shown in (69 – 89).

N.b.: Superscript $(^\star)$ *is used where appropriate (e.g., with the variable corresponding to the market dispatch of generating units) to denote a variable whose value has been fixed as per the solution of a different problem, treated here as a parameter.*

N.b.(2): While, as explained in the main body of this paper, all variables & several parameters of this problem are defined with respect to a given realization of operational conditions $s \in \mathcal{S}$, *we avoid the use of such symbol here for the benefit of notational simplicity.*

$$\min \sum_{c\in\mathcal{C}_{RMAC}} \pi_c \cdot \gamma_c \tag{69}$$

subject to,

$$\sum_{c\in\mathcal{C}_{RMAC}} \pi_c \cdot \zeta_c \cdot \pi_{fcc} \leq \varepsilon_{Rt}, \tag{70}$$

for all nodes $n \in \mathcal{N}$:

$$\left\{ \sum_{g\in\mathcal{G}_n} \left[u_g^\star \cdot P_g^{M\star} + \left(P_{g,0}^+ - P_{g,0}^-\right)\right] - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell}\cdot f_{\ell,0} = \sum_{d\in\mathcal{D}_n} P_d - \sum_{w\in\mathcal{W}_n} P_w \right\}, \tag{71}$$

for all transmission elements $\ell \in \mathcal{L}$:

$$f_{\ell,0} - \frac{1}{X_\ell}\sum_{n\in\mathcal{N}_n} \beta_{n,\ell}\cdot\theta_{n,0} = 0, \tag{72}$$

$$f_{\ell,0} \leq f_\ell^{\max}, \tag{73}$$

$$-f_{\ell,0} \leq f_\ell^{\max}, \tag{74}$$

for all generating units $g \in \mathcal{G}$ *& non-discarded contingencies* $c \in \mathcal{C}_{RMAC}(\bar{s})$:

$$0 \leq \left(P_{g,0}^+ + P_{g,c}^+\right) \leq R_g^+, \tag{75}$$

$$0 \leq \left(P_{g,0}^- + P_{g,c}^-\right) \leq R_g^-, \tag{76}$$

$$0 \leq P_{g,c}^+ - \zeta_c \cdot \Delta P_g^+ \leq 0, \tag{77}$$

$$0 \leq P_{g,c}^- - \zeta_c \cdot \Delta P_g^- \leq 0, \tag{78}$$

for all nodes $n \in \mathcal{N}$ & non-discarded contingencies $c \in \mathcal{C}_{RMAC}$:

$$\Bigg\{ \sum_{g \in \mathcal{G}_n} \Big[u_g^\star \cdot P_g^{M\star} + \big(P_{g,0}^+ - P_{g,0}^- \big) \Big] - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c}^{IS} - \gamma_c \cdot M \leq \sum_{d \in \mathcal{D}_n} P_d - \sum_{w \in \mathcal{W}_n} P_w \Bigg\}, \tag{79}$$

$$\Bigg\{ \sum_{g \in \mathcal{G}_n} \Big[u_g^\star \cdot P_g^{M\star} + \big(P_{g,0}^+ - P_{g,0}^- \big) \Big] - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c}^{IS} - \gamma_c \cdot M \leq - \sum_{d \in \mathcal{D}_n} P_d + \sum_{w \in \mathcal{W}_n} P_w \Bigg\}, \tag{80}$$

$$\Bigg\{ \sum_{g \in \mathcal{G}_n} \Big[u_g^\star \cdot P_g^{M\star} + \big(P_{g,0}^+ - P_{g,0}^- \big) + \big(P_{g,c}^+ - P_{g,c}^- \big) \Big] - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c} - \gamma_c \cdot M \leq \sum_{d \in \mathcal{D}_n} P_d - \sum_{w \in \mathcal{W}_n} P_w \Bigg\}, \tag{81}$$

$$\Bigg\{ \sum_{g \in \mathcal{G}_n} \Big[u_g^\star \cdot P_g^{M\star} + \big(P_{g,0}^+ - P_{g,0}^- \big) + \big(P_{g,c}^+ - P_{g,c}^- \big) \Big] - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c} - \gamma_c \cdot M \leq - \sum_{d \in \mathcal{D}_n} P_d + \sum_{w \in \mathcal{W}_n} P_w \Bigg\}, \tag{82}$$

for all transmission elements $\ell \in \mathcal{L}$ & non-discarded contingencies $c \in \mathcal{C}_{RMAC}$:

$$f_{\ell,c}^{IS} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c}^{IS} = 0, \tag{83}$$

$$f_{\ell,c}^{IS} - a_{\ell,c} \cdot (r_\ell \cdot f_\ell^{\max} + \gamma_c \cdot M) \leq 0, \tag{84}$$

$$- f_{\ell,c}^{IS} - a_{\ell,c} \cdot (r_\ell \cdot f_\ell^{\max} + \gamma_c \cdot M) \leq 0, \tag{85}$$

$$f_{\ell,c} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c} = 0, \tag{86}$$

$$f_{\ell,c} - a_{\ell,c} \cdot f_\ell^{\max} \cdot (1 - \gamma_c) \leq 0, \tag{87}$$

$$- f_{\ell,c} - a_{\ell,c} \cdot f_\ell^{\max} \cdot (1 - \gamma_c) \leq 0, \tag{88}$$

for all non-discarded contingencies $c \in \mathcal{C}_{RMAC}$:

$$\zeta_c + \gamma_c \leq 1. \tag{89}$$

With respect to the original problem statement (51 – 68), we minimize here the probability of those contingency events that need to also be discarded within the subset $\mathcal{C}_{RMAC}$, originally defined as per the real-time RMAC contingency discarding principle. We use auxiliary binary variable γ_c as an indicator for such contingencies to be further discarded. Specifically, this auxiliary variable is employed to facilitate and indicate the relaxation of intermediate and corrective post-contingency power balance and line flow restrictions (79–80, 83–85) and (81–82 , 86–88) using a sufficiently large constant (M). We should finally explain that constraint (89) serves to allow, per non-discarded contingency, either the use of corrective control or the relaxation of post-contingency constraints only.

D. Market Clearing statement

The MILP formulation (90 – 98) has been used in our case studies in order to set initial commitment status and dispatch values for all thermal generating units as per a single period perfectly competitive electricity market.

N.b.: Superscript (M) is used to designate that all variables in this problem refer to the market clearing only.

$$\min \left(\sum_{g \in \mathcal{G}} \left(c_g^0 + c_g^f \right) \cdot v_g + \sum_{k \in \mathcal{K}} c_{g,k} \cdot p_{g,k} \right), \tag{90}$$

subject to, for all generating units $g \in \mathcal{G}$:

$$P_g^M - \sum_{k \in \mathcal{K}} p_{g,k} = 0, \tag{91}$$

$$P_g^M - v_g \cdot P_g^{\max} \leq 0, \tag{92}$$

$$- P_g^M + v_g \cdot P_g^{\min} \leq 0, \tag{93}$$

for all generating units $g \in \mathcal{G}$ & cost-curve segments $k \in \mathcal{K}$:

$$p_{g,k} - v_g \cdot p_{g,k}^{\max} \leq 0, \tag{94}$$

for all nodes $n \in \mathcal{N}$:

$$\sum_{g \in \mathcal{G}_n} P_g^M - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_\ell^M = \sum_{d \in \mathcal{D}_n} P_d - \sum_{w \in \mathcal{W}_n} P_w(s_0), \tag{95}$$

for all transmission elements $\ell \in \mathcal{L}$:

$$f_\ell^M - \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_n^M = 0, \tag{96}$$

$$f_\ell^M \leq f_\ell^{\max}, \tag{97}$$

$$- f_\ell^M \leq f_\ell^{\max}. \tag{98}$$

We should briefly explain that the first term of (90) measures start-up and fixed running costs, while all dispatchable units are assumed to be initially off and available to be committed. The summation appearing as a second term of the same function measures the marginal running cost of dispatchable generation, assuming a piece-wise linear cost function of $|\mathcal{K}|$ segments. Expressions (91 – 94) are for the active power generation of all dispatchable units, in total and across the segments of the piece-wise linear cost curve. Finally, expressions (95–98) are the nodal power balance and line flow constraints considering the forecast power injections (denoted here as s_0) and the *no-outage* state of the network.

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