

Bayesian inference for material parameter identification in elastoplasticity

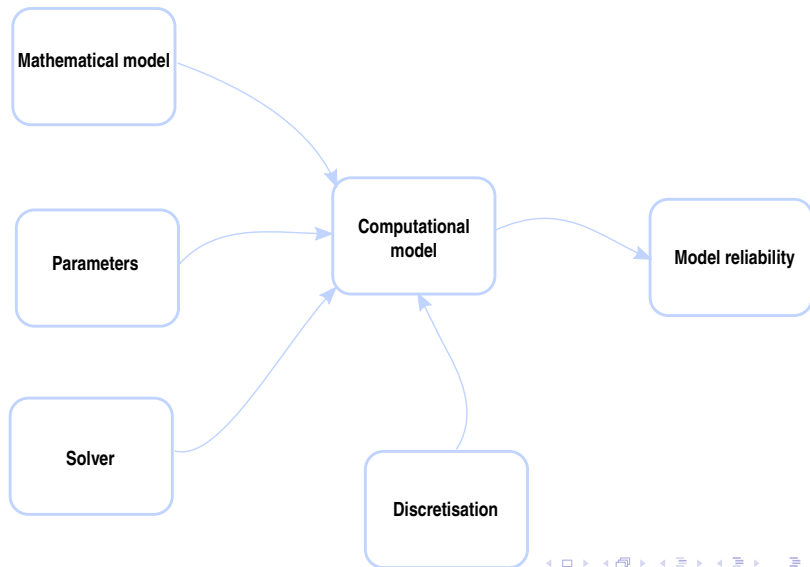
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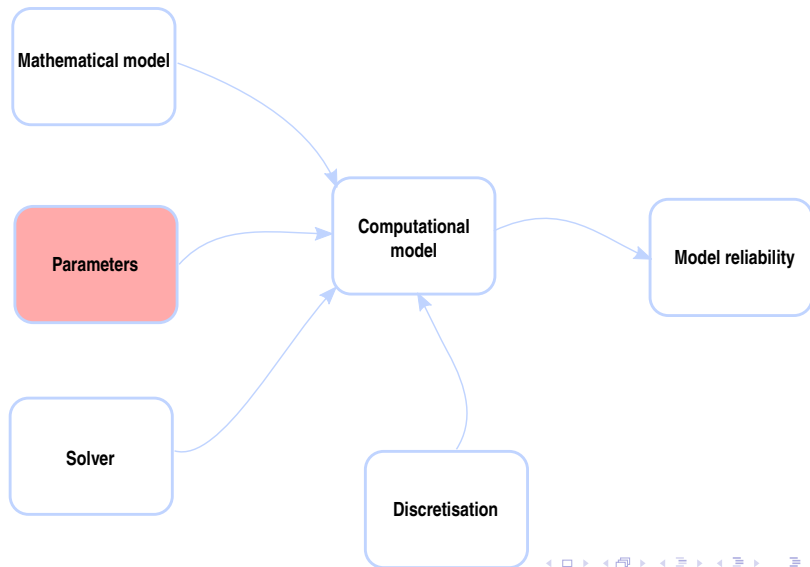
September 9, 2016



Introduction



Introduction



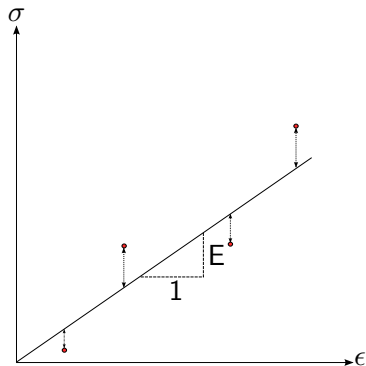
Error minimisation

Least squares method (conventional approach):

$$\sigma = E\epsilon$$

$$J = \frac{1}{2} \sum_{i=1}^N (\sigma_i - E\epsilon_i)^2$$

$$\bar{E} = \operatorname{argmin}_E J(E)$$



Frequentist inference



Frequentist inference



10



6

Frequentist inference

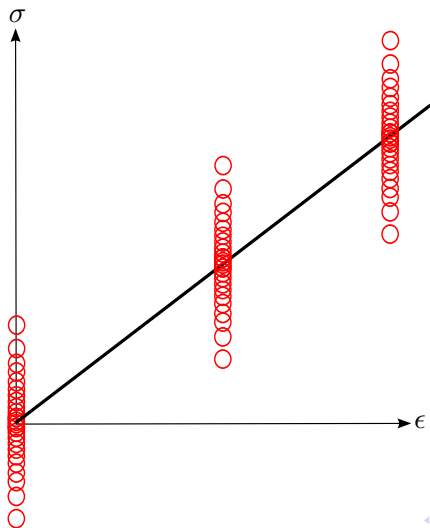


$$Pr(\text{head}) = \frac{10}{16}$$

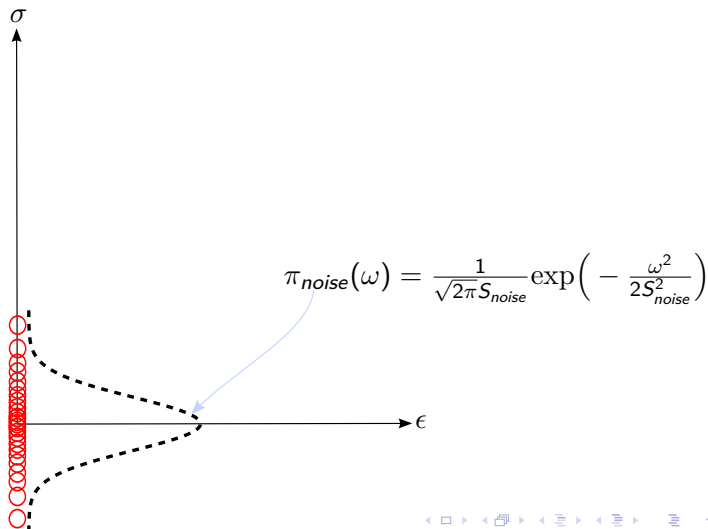


$$Pr(\text{tail}) = \frac{6}{16}$$

Frequentist inference: Young's modulus identification



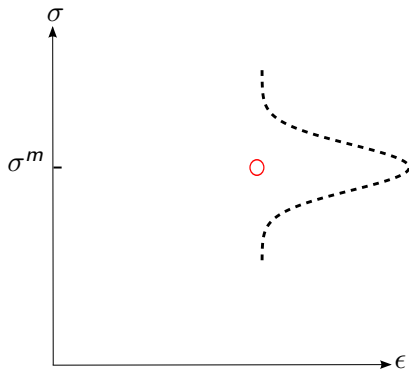
Frequentist inference: Young's modulus identification



Frequentist inference: Young's modulus identification

$$\sigma^m = E\epsilon + \Omega$$

$$\Omega \sim \pi_{noise}(\omega)$$



Frequentist inference: Young's modulus identification

Method of maximum likelihood (ML):

$$\pi(\sigma^m | E) = \frac{1}{\sqrt{2\pi}S_{noise}} \exp\left(-\frac{(\sigma^m - E\epsilon)^2}{2S_{noise}^2}\right)$$

Frequentist inference: Young's modulus identification

Method of maximum likelihood (ML):

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and for M measurements:

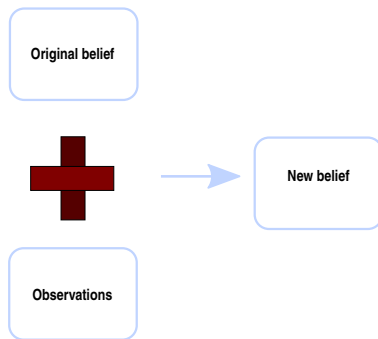
$$\pi(\boldsymbol{\sigma}^m | E) = \frac{1}{(2\pi S_{noise}^2)^{\frac{M}{2}}} \exp\left(-\frac{\sum_{i=1}^M (\sigma_i^m - E\epsilon_i)^2}{2S_{noise}^2}\right)$$

$$\boldsymbol{\sigma}^m = [\sigma_1, \sigma_2, \dots, \sigma_M]$$

Bayesian inference

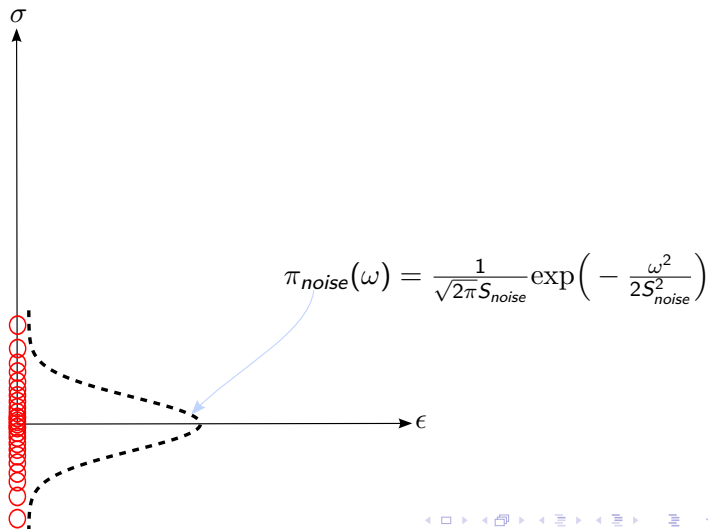


Bayesian inference



$$\pi(\textit{cause}|\textit{effect}) = \frac{\overbrace{\pi(\textit{cause})}^{\textit{prior}} \times \overbrace{\pi(\textit{effect}|\textit{cause})}^{\textit{likelihood}}}{\underbrace{\pi(\textit{effect})}_{\textit{evidence}}}$$

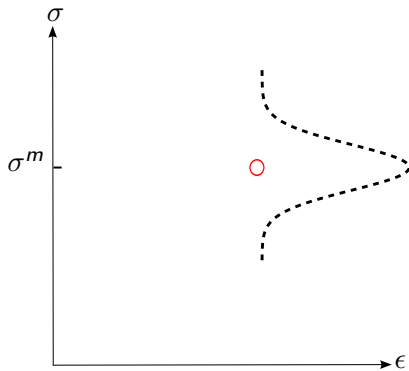
Bayesian inference: Young's modulus identification



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Bayesian inference: Young's modulus identification

Bayes' formula:

$$\pi(E|\sigma^m) = \frac{\pi(E)\pi(\sigma^m|E)}{\pi(\sigma^m)}$$

Bayesian inference: Young's modulus identification

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$$\pi(E|\sigma^m) = \frac{\pi(E)\pi(\sigma^m|E)}{\pi(\sigma^m)} \implies \frac{\pi(E)\pi(\sigma^m|E)}{C}$$

Bayesian inference: Young's modulus identification

Bayes' formula:

$$\pi(E|\sigma^m) = \frac{\pi(E)\pi(\sigma^m|E)}{\pi(\sigma^m)} \implies \frac{\pi(E)\pi(\sigma^m|E)}{C}$$

$$\boxed{\pi(E|\sigma^m) \propto \pi(E)\pi(\sigma^m|E)}$$

Bayesian inference: Young's modulus identification

$$\pi(E|\sigma^m) \propto \exp\left(-\frac{(E - \bar{E})^2}{2S_E^2}\right) \exp\left(-\frac{(\sigma^m - E\epsilon)^2}{2S_{noise}^2}\right)$$

Bayesian inference: Young's modulus identification

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and for M measurements:

$$\pi(E|\boldsymbol{\sigma}^m) \propto \prod_{i=1}^M \exp\left(-\frac{(E - \bar{E})^2}{2S_E^2}\right) \exp\left(-\frac{(\sigma_i^m - E\epsilon_i)^2}{2S_{noise}^2}\right)$$

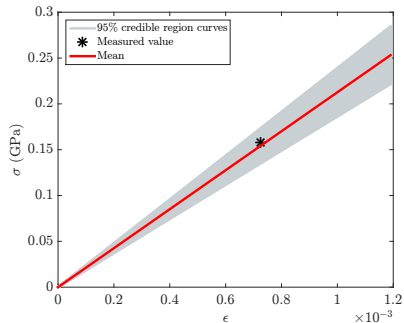
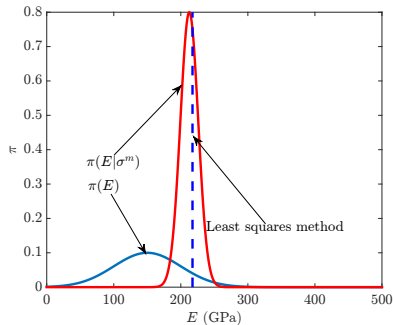
$$\boldsymbol{\sigma}^m = [\sigma_1, \sigma_2, \dots, \sigma_M]$$

Bayesian inference: Young's modulus identification

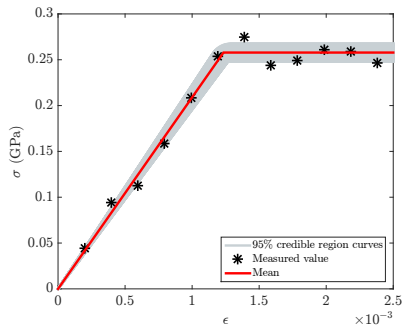
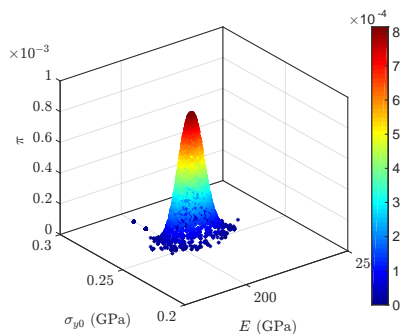
$$\pi(E|\boldsymbol{\sigma}^m) \propto \exp\left(-\frac{(E - \mu)^2}{2S_{post}^2}\right)$$

with $\mu = f(\sigma_i, \bar{E}, S_{nosie}, S_E, \epsilon_i)$
 $S_{post} = f(\sigma_i, \bar{E}, S_{nosie}, S_E, \epsilon_i)$

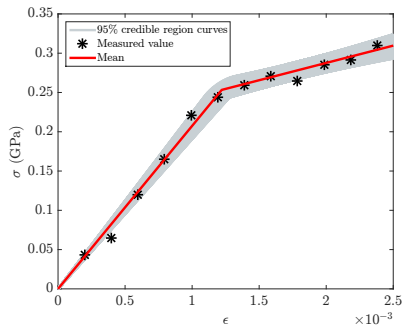
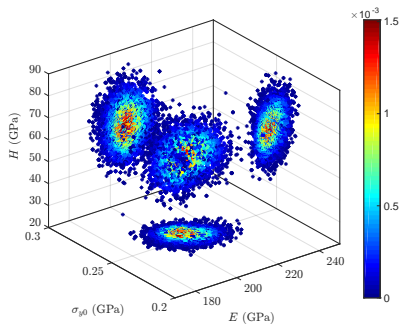
Bayesian inference: Young's modulus identification



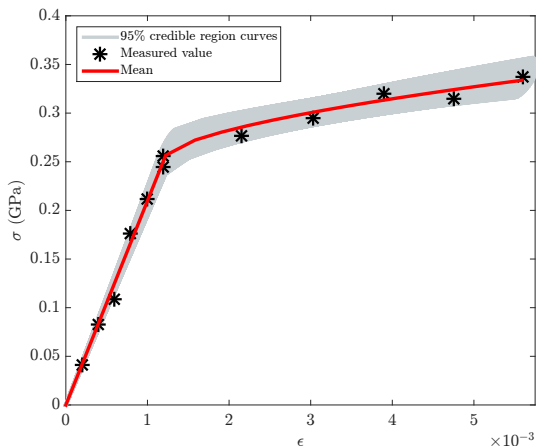
Bayesian inference: linear elastic-perfectly plastic model identification



Bayesian inference: linear elastic-linear hardening model identification



Bayesian inference: linear elastic-nonlinear hardening model identification

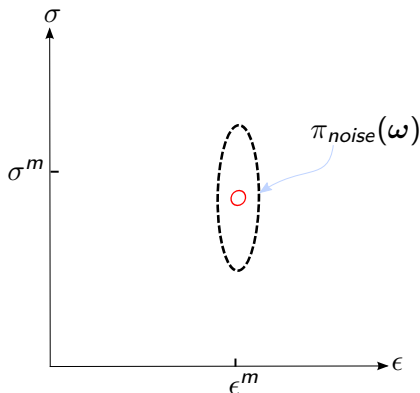


Bayesian inference: Young's modulus identification with double uncertainty

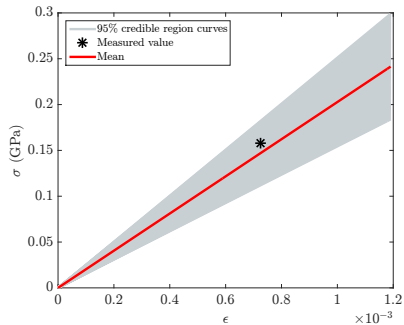
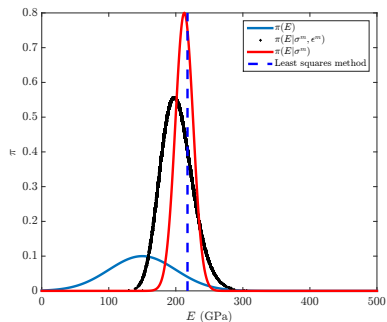
$$\sigma^m = E\epsilon + \Omega_\sigma$$

$$\epsilon^m = \epsilon + \Omega_\epsilon$$

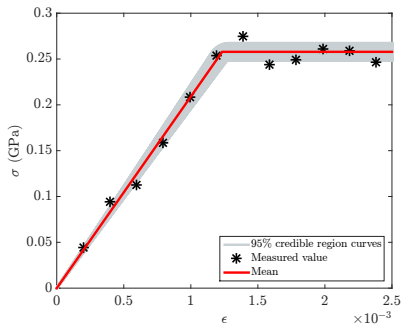
$$\Omega_\sigma \text{ and } \Omega_\epsilon \sim \pi_{noise}(\omega)$$



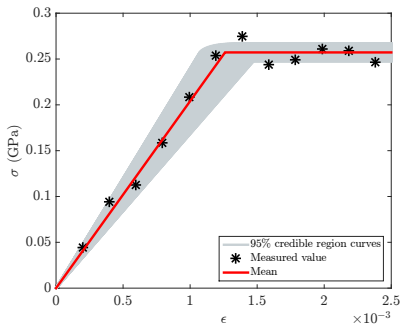
Bayesian inference: Young's modulus identification with double uncertainty



Bayesian inference: linear elastic-perfectly plastic with double uncertainty

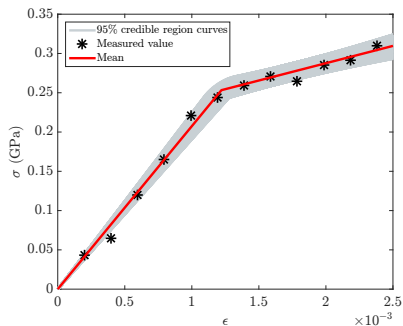


single uncertainty

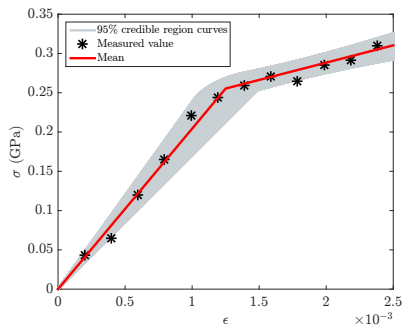


double uncertainty

Bayesian inference: linear elastic-linear hardening with double uncertainty

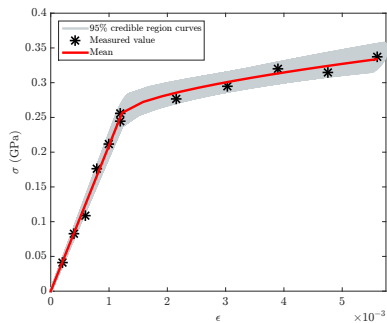


single uncertainty

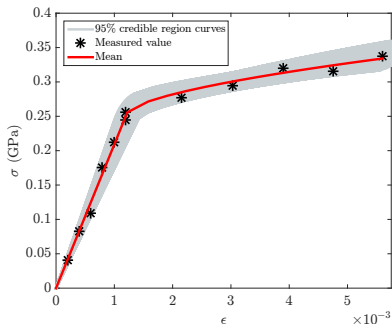


double uncertainty

Bayesian inference: linear elastic-nonlinear hardening with double uncertainty



single uncertainty



double uncertainty

Conclusion

- ‘Closed form expression’ of the posterior for:
 - linear elasticity,
 - elastoplasticity with perfect plasticity,
 - elastoplasticity with linear hardening, and
 - elastoplasticity with nonlinear hardening.

Conclousion

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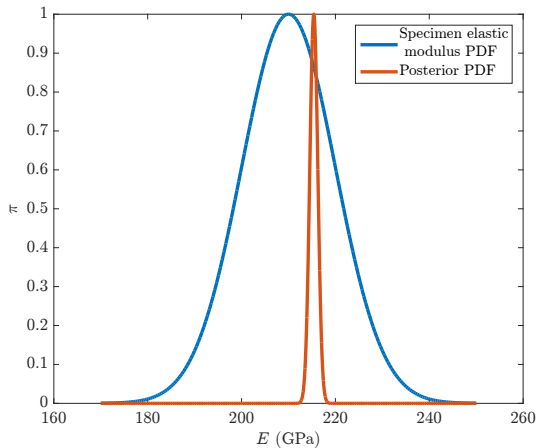
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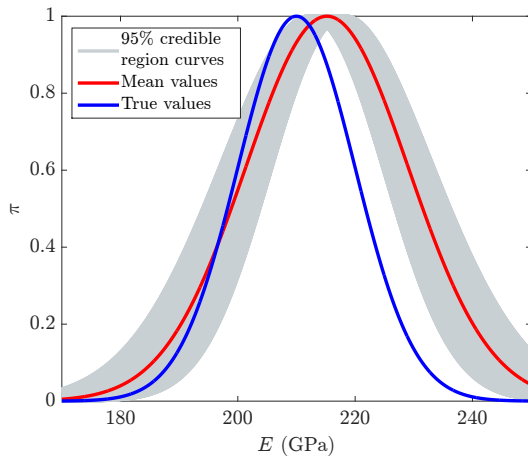
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- The results of BI cannot directly be compared to those of the least squares method.
- The selected prior distribution has a significant effect on the results.
- BI leads to a distribution for the considered parameters, however the resulting distribution do *not* reflect the heterogeneity of the material parameters.

Future work



Future work



A special acknowledgement

Computational Science Priority, University of Luxembourg

The End