Masters Thesis

Fatigue Reliability of joint connections in support structure of offshore wind turbines

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Declaration of Authorship

I, Anh Quang Mai, declare that this thesis titled, 'Fatigue Reliability of joint connections in support structure of offshore wind turbines' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Advanced Master in Naval Construction

Abstract

Fatigue Reliability of joint connections in support structure of offshore wind turbines

by Anh Quang Mai

This study is my initial attempt to investigate the evolution of fatigue reliability of welded joints in offshore wind turbine support structures. This information is crucial to update the inspected data (crack sizes, probability of crack detection) to make the plan for the next inspection and maintenance of existing offshore structures in general. The thesis is limited to tracing back the fatigue reliability of welded joints from their design information. The later work for inspection planning will be continued in the framework of my Ph.D thesis.

This thesis is divided into five chapters. The first chapter is about the motivations and objectives of the research. The fatigue design process is summarized in the second chapter, together with the basis of reliability methods. In the Methodology part (chapter 3), the content is arranged following the procedure to carry out a reliability analysis problem: sources of uncertainties, how those uncertainties appear in the limit state function, transformation of uncertainties, methods to do reliability calculation. Due to the nature of the damage cumulation, the cumulated damage is also a random variable. How do deal with this special random variable is an important section in Methodology chapter. First Order Reliability Method (FORM) is implemented to find reliability index of the joint and then be verified by using the Second Order Reliability Method (SORM). The results of SORM, in turn, is verified by Monte Carlo simulation method. Monte Carlo simulation is also applied to verify the methods of dealing with the special random variable. In the Results part (chapter 4), the methodology is applied to two joints that is designed using two different types of S-N curve. Comparison between FORM/SORM and MCS, assessment of the assumption using for the special random variable are shown at the end of this chapter. In Conclusion chapter (chapter 5), the research objective is restated. The methods to obtain it are summarized. And finally the perspective for further use of the research results is mentioned.

On the basis of the results of this research, it can be concluded that for the fatigue reliability problem, FORM and SORM gives results with high accuracy compared to MCS and that the design point found by FORM/SORM are really the global minimal reliability index. The assumption in dealing with cumulated damage shows a good agreement with MCS. This is the foundation for using a simplified LSF in reliability calculation of accumulative fatigue problem.

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Abbreviations

$\log(\cdot)$	common (10^{th}) logarithm
$\ln(\cdot)$	natural logarithm
CDF	Cumulative Density Function
CoV	Coefficient of Variation

Symbols

Φ	normal cumulative distribution function	$\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}$
$F(x)$	cumulative distribution function	$F(x) = P(X \leq x)$
μ	population mean	mean of population values
$\mathrm{E}(x)$	expectation value	of random variable X
$\mathrm{E}(X\|Y)$	conditional expectation value	of random variable X given Y
$\mathrm{Var}(x)$	variance	of random variable X
σ^2	variance	of population values
σ_X	standard deviation	of random variable X
CoV	coefficient of variation	$\frac{\sigma}{\mu}$

Chapter 1

Introduction

1.1 Motivations

Most offshore structure platforms nowadays are more than 40 years old because they were constructed during the period from 1970 to 1980 - the time period of oil investment growing (El-Reedy, 2012). Although time of duty of those ageing structures would normally be considered for retirement, but with certain processes and criteria, for some of them, lives can be extended for a further period without a reduction in margins below safe operating limits. This is a dual-benefit solution because it reduces ecosystem impacts and the construction costs.

On the other hand, for the in-service offshore structures, particularly offshore wind turbines, the costs for Operation and Maintenance (O&M) are very high. O&M may increases by about 30% the costs of a kWh (Van Bussel and Schöntag, 1997). Risk-Based Inspection (RBI) is the process of developing an optimal inspection plan regarding O&M costs, based on knowledge of the risk of failure of the structures. Thus, to improve the competitiveness of offshore wind energy, it is crucial to apply RBI for offshore wind farms.

The "safe operating limits" of the ageing structures as well as the inspection planning strategy base on one thing in common. It is the critical failure probability designated by specifications. Whenever the failure probability of the structure goes lower than that value, a decision must be made to keep the structure be safe for operation. How to keep track of the failure probability? - fatigue reliability calculation is the starting part of the answer for that question.

1.2 Objectives

As defined by DNV-RP-G101 (2010) standard, "Risk-based Inspection is a decision making technique for inspection planning based on risk - comprising the consequence of failure and probability of failure".

Of all failure modes of offshore structure, the fatigue failure is the most common. For steel structure, welded joints are the potential spots for fatigue fracture. So, in risk-based assessment of offshore structures, it is necessary to know the probability of failure of welded joints under fatigue failure mode. This probability of failure can be done either by using Stress-life approach (i.e. using Miner's rule for accumulative damage) or Fracture mechanics approach (i.e. using Paris law for crack growth model).
In design practice of offshore structures, it is common to use the stress-life method (SN curves) to expect that crack will occur at the end of their service lives. However, due to uncertainties in material, welding quality, loading conditions, cracks may occur during the service life. The question that arises for maintenance work is "how to update reliability of the welded joints once inspection data (possibility of detection, crack dimensions) is known".
The only information contained in SN model is whether the hot spot has failed or survived (or whether the total damage has reached the critical value or not), whereas the FM model gives the crack dimensions after any number of cycle N.
So using FM model in calculating reliability of a welded joint gives the advantage to be able to consider inspection data.

The aim of this master thesis, as an initial step toward inspection planning, is to calculate fatigue reliability of welded joints using SN approach. In later work, this result will be used to find an accurate fracture mechanics (FM) model for updating reliability of existing welded joints in offshore structures.
By "an accurate FM model" I mean the FM model that we use for reliability updating should give good correspondence with life predictions based on the SN approach. Implying that its parameters used in crack growth model must be calibrated so that

the cumulative failure probabilities determined with the FM model are equal to those calculated with the SN model.

1.3 Outline

The thesis is divided into five chapters.

The first chapter is about the motivations and objectives of the research. The fatigue design process is summarized in the second chapter, together with the basis of reliability methods. In Chapter 3, the content is arranged following the procedure to carry out a reliability analysis problem: sources of uncertainties, how those uncertainties appear in the limit state function, transformation of uncertainties, methods to do reliability calculation. In Chapter 4, the methodology is applied to two joints that is designed using two different types of S-N curve. Comparison between FORM/SORM and MCS, assessment of the assumption using for the special random variable are shown at the end of this chapter. In Conclusion chapter - Chapter 5, the research objective is restated. The methods to obtain it are summarized. And finally the perspective for further use of the research results is mentioned.

Chapter 2

Related Works

2.1 Fatigue Limit State

The aim of fatigue design is to ensure that the structure has sufficient resistance against fatigue failure, i.e. that it has an adequate fatigue life.

2.1.1 Fatigue Resistance - SN curves

The resistance against fatigue is normally given in terms of an S-N curve. The S-N curve gives the number of cycles to failure N versus the stress range S. The S-N curve is usually based on fatigue tests in the laboratory. For interpretation of S-N curves from fatigue tests, the fatigue is defined to have occurred when a fatigue crack has grown through the thickness of the structure or structural component. To account for the observed scatter, a regression analysis is carried out to determine the median line and the standard deviation.
The characteristic curve normally shall be taken as the curve that ensures 97.7% probability of survival, i.e. it corresponds to the 2.3% quantile of N for given S (DNV-OS-J101, 2013).

The equation for each curve reads:

$$N = C \cdot S^{-m} \tag{2.1}$$

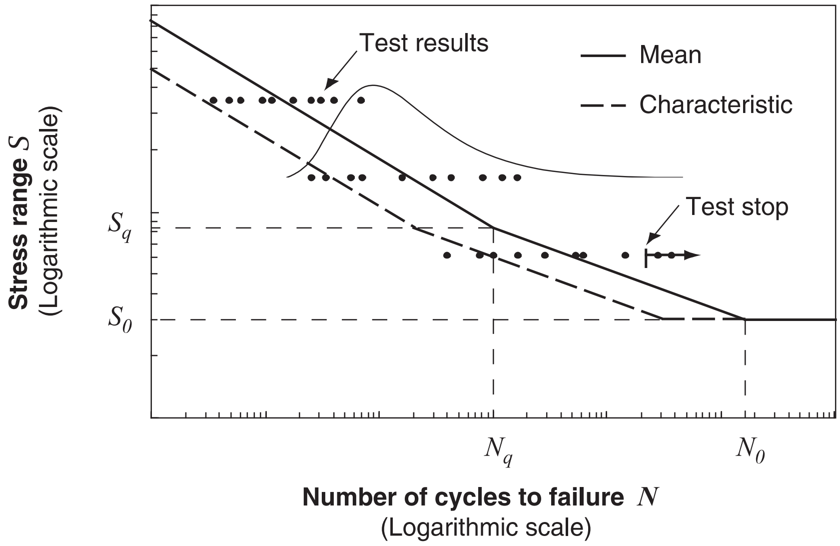

FIGURE 2.1: Illustration of the bi-linear SN curve with cut off ΔS_0, after Straub (2004)

where C and m are constants that are determined by experiments. Alternatively a multi-linear curve, as illustrated in Figure 2.1 and defined in Equation (2.2), is often applied.

$$N = \begin{cases} C \cdot S^{-m_1}, & \text{if } S \geq S_q \\ C \cdot S_q^{(m_2 - m_1)} \cdot S^{-m_2}, & \text{if } S_q \geq S \geq S_0 \\ \infty, & \text{if } S_0 \geq S \end{cases} \tag{2.2}$$

For the SN curve in Figure (2.1) the definition consists additionally of the parameters m_2, ΔS_q and the so-called cut off ΔS_0, below which no failure occurs.

2.1.2 Fatigue Loading

In design procedures fatigue stresses are usually hot spot stresses, which are obtained by combination of the nominal stress with stress concentration factors (SCF) given for specific connection types.

The SN curves are generally derived from test with uniaxial loading while in most real structures, it is multiaxial stresses that occur. It is therefore required to relate these multiaxial stress states to fatigue equivalent stresses. For materials with a ductile

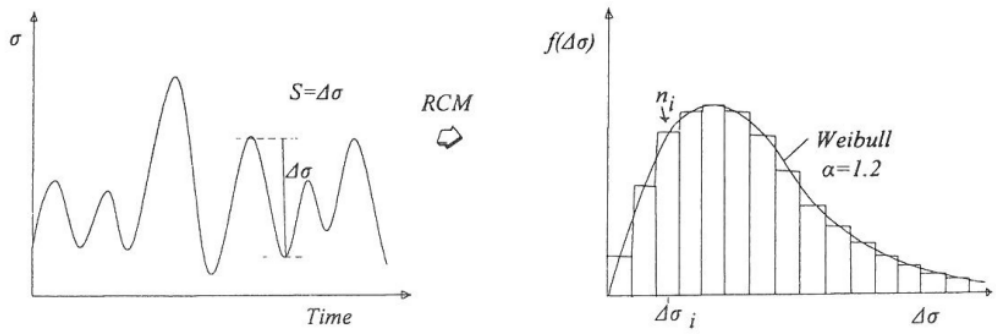

FIGURE 2.2: Variable amplitude loading with fitted Weibull long-term distribution, after Lassen (1997)

behaviour the stress ranges are best described by the von Mises equivalent stresses. For materials with brittle behaviour the use of largest principal stress range is more appropriate (Straub, 2004).

To calculate cumulative fatigue damage, a distribution of stress ranges $f(\Delta\sigma)$ (see Figure 2.2) needs to be established from a given stress process which comes out from measurements or structural analyses. For general loadings it is concluded by Madsen et al. (2006) that the rain-flow counting method generally gives the best results. This is also the method specified in Eurocode-3-1993 (2005) together with the reservoir method (which used for large numbers of cycles asymptotically equivalent).

Apart from using the rainflow-counting for a realisation of the stress process, $f(\Delta\sigma)$ is often approximated by the Raleigh or the Weibull distribution (see an example in Figure 2.2). The Raleigh distribution is generally used when the stress process is both normal and narrow-band, see e.g. Lutes et al. (1984). The Weibull distribution occurs very commonly in natural processes related to dynamic response of elastic systems. For marine structures, the long term stress ranges due to wave loads are often modelled by a Weibull distribution, (Almar-Næss, 1985); for fatigue loads on wind turbines, it is shown in Winterstein and Lange (1995) that the Weibull distribution provides a reasonable fit to observed data.

The fatigue design has to ensure the required fatigue design factor (FDF), which is a ratio between the design fatigue life T_{FL} and the service life T_{SL}:

$$FDF = \frac{T_{FL}}{T_{SL}} \tag{2.3}$$

Because the service life of the structure is fixed, designing for a long fatigue life (or high value of FDF) is an efficient mean to reduce probability of fatigue failure.

In the design loop, to satisfy the requirement of FDF, designers can either change the loading or enhance the geometry of the structure. But the direct consequence is the stress range distribution, which is represented by two parameters of the Weibull distribution (see Equation (3.12)). The scale parameter, k, is representative for the level of stress ranges at the hot spot. The shape parameter, λ, is often representative for a specific geometry and loading type.

In deriving the shape and scale parameters of the Weibull distribution, one can use the curve fitting method for the stress range histogram. In ship structure industry, the shape parameter can be established based on empirical formula for shape parameter which depends on ship length, ship behaviour and the S-N curve; while the scale parameter is derived directly from the shape parameter and the maximum stress range that occurs during the calculated time and the FDF. Reader who interested in can find more details in DNV-RP-C203 (2005) and DNV-CN-30.7 (2003).

2.1.3 Miner's rule

Miner's rule is one of the most widely used method to calculate cumulative fatigue damage for offshore structures. It was popularized by M.A. Miner in 1945. This is an interaction-free theory, i.e. the damage accumulation after n cycles is independent of the order in which these cycles occur. The damage increment for each cycle with stress range S_i is defined as:

$$\Delta D_i = \frac{1}{N_i} \tag{2.4}$$

where N_i is the number of cycles to failure at stress range S_i as given by the associated SN curve.

The total accumulated damage after n cycles is given by:

$$D_{tot} = \sum_{i=1}^{n} \Delta D_i \tag{2.5}$$

2.1.4 Design Criterion

The design criterion is written as:

$$D_D \leq 1.0 \tag{2.6}$$

where D_D is design cumulative fatigue damage.

In practice, the partial safety factor design method is used in design standards. There are two methods to apply the partial safety factor. Readers who interested in can find more detail in DNV-OS-J101 (2013):

- Method (1): Using the "fatigue design factor" FDF as a partial safety factor for the characteristic cumulative damage D_C to calculate the design cumulative damage D_D.

  $$D_D = FDF \cdot D_C \tag{2.7}$$

  where D_C is calculated directly from the characteristic long-term distribution of stress ranges $\Delta\sigma_i$ using Miner's sum.

- Method (2): Using the "material factor" γ_m for fatigue as partial safety factor to calculate the design stress range $\Delta\sigma_{d,i}$ from the characteristic long-term distribution of stress ranges $\Delta\sigma_i$:

  $$\Delta\sigma_{d,i} = \gamma_m \Delta\sigma_i \tag{2.8}$$

  And then the design cumulative fatigue damage D_D is calculated directly from the design stress range $\Delta\sigma_{d,i}$ by Miner's sum.

The relationship between FDF and γ_m can be found in (DNV-OS-J101, 2013) for varieties of joint locations and accessibility.

2.2 Reliability method

2.2.1 Calculation of failure probabilities

In reliability calculation, when both the distribution of the basic variables Z_i and the limit state surface - which divides the z-space into a safe and failure set - are known then

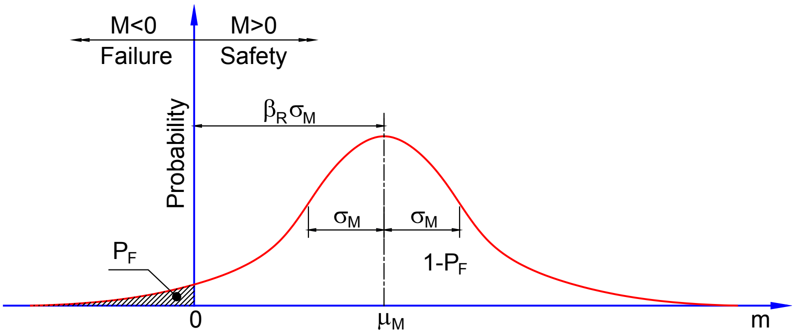

FIGURE 2.3: Illustration of relationship between β_R and P_F for independent, normally distributed variables

level III reliability methods can be applied. The reliability measure is the probability of survival P_R, which is calculated as:

$$P_R = \int_S f_{\mathbf{Z}}(\mathbf{z})d\mathbf{z} \tag{2.9}$$

where $f_{\mathbf{Z}}(\mathbf{z})$ is the joint probability density function of the basic variables, and S is the safe set. In fatigue damage calculation, a sample point of basic variables is said to be on the safe set if its cumulative damage which is calculated using equation (2.5) is smaller than damage criteria Δ.

If we denote $M = \Delta - D_{tot}$ (M is usually called the safety margin) then the probability of survival can be written as:

$$P_R = P(M > 0) \tag{2.10}$$

A reliability index β_R, as an alternative to P_R is defined as:

$$\beta_R = \Phi^{-1}(P_R) = -\Phi^{-1}(P_F) \tag{2.11}$$

where

$$P_F = 1 - P_R = \int_F f_{\mathbf{Z}}(\mathbf{z})d\mathbf{z} \tag{2.12}$$

is the failure probability and F is the failure set. An illustration of the relationship between β_R and P_F is shown in Figure 2.3. In general the integral (2.9) cannot be computed analytically. Alternative evaluation techniques such as numerical integration or simulation are generally very time consuming. In structural reliability, the interested failure probability is usually in the range from 10^{-2} to 10^{-5}, so the analytical methods such as the first order reliability method (FORM) and the second order reliability method (SORM) are more suitable. It is also suggested in (DNV-CN-30.6, 1992) that

FORM and SORM should be applied for failure probabilities less than 0.05 and for larger probabilities the simulation method should be used.

2.2.2 Transformation of random variables

The first step in analytical methods consists of a transformation of the physical basic variables into the space of standard normal variables. This transformation implies a transformation of the limit state surface in the space of basic variables to a corresponding limit state surface in the standard normal space. (See Figure 2.4)

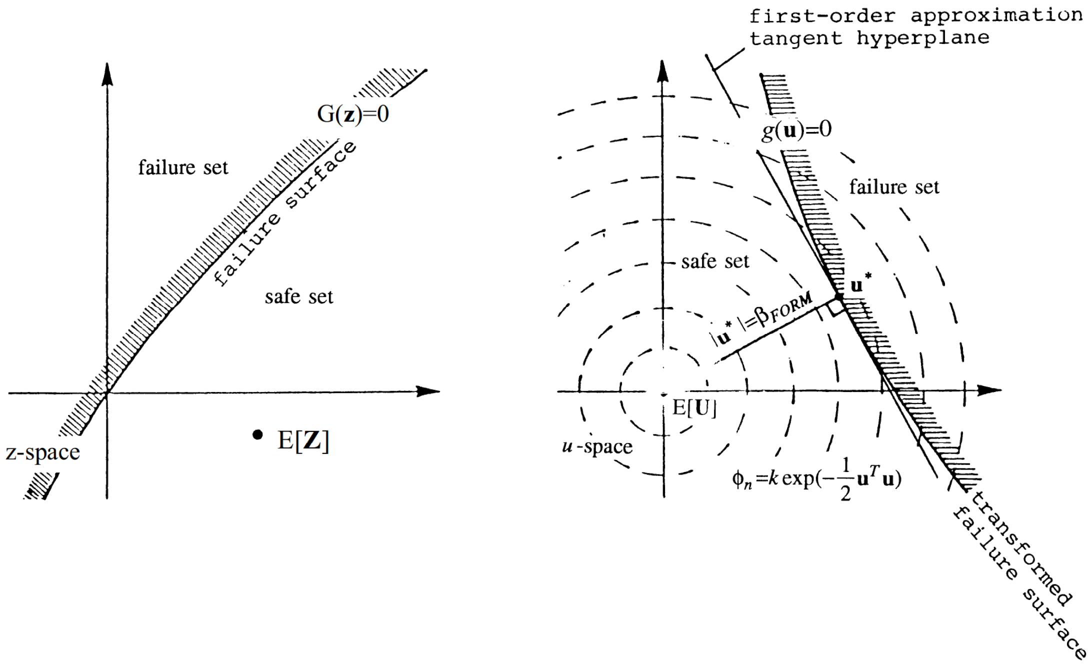

FIGURE 2.4: Transformation from space of basis random variable **z**-space (left) into space of standardized normal variables **u**-space (right)

Depending on the type of basis random variables and their correlations, there are a lot of method to transform them into uncorrelated, normally distributed random variables:

- For correlated, normally distributed variables: Given the covariances between variables, it is possible to write an invertible linear transformation that "uncorrelates" the variables. There are several ways to do this transformation. In Thoft-Christensen and Baker (1982), it is done by determining eigenvalues and eigenvectors of the correlation coefficient matrix. Another way to perform the transformation is using Choleski triangulation given that the correlation coefficient matrix ρ is positive definite. If all the stochastic variables are normally and log-normally distributed then this technique can be well applied because the log-normal variables can easily be transformed to normal variables. The procedure can be shortly described as following:

– The first step is to transform the basis random variables $X_i, i = 1, .., n$ into standardized normally distributed ones $Y_i, i = 1, ..., n$ (with expected value 0 and standard variation 1:

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, i = 1, ..., n \tag{2.13}$$

It is easy to prove that $\mathbf{Y}$ will have a correlation coefficient matrix equal to ρ.

– In the second step, a transformation from $\mathbf{Y}$ to uncorrelated and normalized variables $\mathbf{U}$ with expected value 0 and standard variation 1:

$$\mathbf{Y} = \mathbf{TU} \tag{2.14}$$

where $\mathbf{T}$ is a lower triangular matrix (i.e. $T_{ij} = 0$ for $j > i$). Because $\mathrm{E}[\mathbf{Y}] = \mathrm{E}[\mathbf{Y}^T] = 0$, the covariance matrix $\mathbf{C_Y}$ for $\mathbf{Y}$ can be written as:

$$\mathbf{C_Y} = \mathrm{E}[\mathbf{YY}^T] = \mathrm{E}[\mathbf{TUU}^T\mathbf{T}^T] = \mathbf{T}\mathrm{E}[\mathbf{UU}^T]\mathbf{T}^T = \mathbf{TT}^T = \rho \tag{2.15}$$

The elements in matrix $\mathbf{T}$ are then determined from $\mathbf{TT}^T = \rho$ as:

$$\begin{array}{lll} T_{11} = 1 & & \\ T_{21} = \rho_{12} & T_{22} = \sqrt{1 - T_{21}^2} & \\ T_{31} = \rho_{13} & T_{32} = \dfrac{\rho_{23} - T_{21}T_{31}}{T_{22}} & T_{33} = \sqrt{1 - T_{31}^2 - T_{32}^2} \\ etc. & & \end{array} \tag{2.16}$$

- For independent, non-normally distributed variables: This type of transformation is suitable for fatigue problem in this thesis. More detail of its application is shown in Section 3.4.

- For dependent, non-normally distributed variables: When the basis variables are not mutually independent, the Rosenblatt transformation is a good choice. This method consider the joint PDF of **X** as a product of conditional PDFs:

$$f(\mathbf{x}) = f_1(x_1) \cdot f_2(x_2|x_1) \cdots f_n(x_n|x_1, x_2, ..., x_{n-1}) \tag{2.17}$$

The conditional CDFs are then be written as:

$$\begin{aligned} F(x_1) &= \int_{-\infty}^{x_1} f_1(x_1) dx_1 \\ F(x_2|x_1) &= \int_{-\infty}^{x_2} f_2(x_2|x_1) dx_2 \\ F(x_3|x_1, x_2) &= \int_{-\infty}^{x_3} f_3(x_3|x_1, x_2) dx_3 \\ &\vdots \end{aligned} \tag{2.18}$$

Having these CDFs facilitates the triangular transformation that is referred to as Rosenblatt transformation:

$$\begin{aligned} y_1 &= \Phi^{-1}\left(F_1(x_1)\right) \\ y_2 &= \Phi^{-1}\left(F_2(x_2|x_1)\right) \\ y_3 &= \Phi^{-1}\left(F_3(x_3|x_1, x_2)\right) \\ &\vdots \end{aligned} \tag{2.19}$$

To obtain the inverse transformation it is necessary to solve nonlinear equations for x_i, starting at the top of Equation (2.19). Interested readers can find more in Rosenblatt (1952) for this transformation method.

There is another method named as Nataf transformation. As a first step, this method transform the basis random vector $\mathbf{x}$ to random vector $\mathbf{z}$ which is correlated normally distributed with zero means and unit variances. To facilitate the sought transformation it is assumed that the random variables z_i are jointly normal. This is called the Nataf assumption. Under this assumption it can be shown (Liu and Der Kiureghian, 1986) that the correlation coefficient $\rho_{0,ij}$ between z_i and z_j is related to the correlation coefficient ρ_{ij} between x_i and x_j by the equation:

$$\rho_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{x_i - \mu_i}{\sigma_i}\right)\left(\frac{x_j - \mu_j}{\sigma_j}\right) \varphi_2(z_i, z_j, \rho_{0,ij}) dz_i dz_j \tag{2.20}$$

where φ_2 is the bivariate standard normal PDF:

$$\varphi_2(z_i, z_j, \rho_{0,ij}) = \frac{1}{2\pi\sqrt{1-\rho_{0,ij}^2}} \exp\left[-\frac{z_i^2 + z_j^2 - 2\cdot\rho_{0,ij}\cdot z_1 \cdot z_2}{2\cdot(1-\rho_{0,ij}^2)}\right] \tag{2.21}$$

The Nataf joint distribution model is valid only if the CDFs of x_i be strictly increasing and the correlation matrix of $\mathbf{x}$ and $\mathbf{z}$ are positive definite. Equation (2.20) must be solved for each correlated pair of random variables. Once this is done, the transformation from $\mathbf{z}$ (which is correlated normally distributed with zero mean and a unit covariance matrix) to $\mathbf{y}$ (which is uncorrelated normally distributed with zero mean and a unit covariance matrix) can be obtained using the Choleski decomposition method as mentioned above.

2.2.3 Basis of FORM methods

In principle, random variables are characterized by their first moment (mean), second moment (variance) and higher moments. Different ways of approximating the limit state function form the basis for different reliability analysis algorithms (i.e. FORM, SORM, etc.).

First-order Second Moment (FOSM) Method

This method is not FORM method, but it contains the principles of reliability calculation using linearization methods. This FOSM method also referred as the Mean Value FOSM (MVFOSM), simplifies the functional relationship and alleviates the complexities of the probability-of-failure calculation. The input and output of this method are expressed as the mean and standard deviation. Higher moments such as skew and flatness of the distribution are ignored.

By using the first Taylor expansion, the limit state function $g(X)$ can be approximated as in Equation (2.22) assuming that the variables X are statistically independent.

$$\widetilde{g}(X) = g(\mu_X) + \nabla g(\mu_X)^T (X_i - \mu_{X_i}) \tag{2.22}$$

where, $\mu_X = \{\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_n}\}^T$, and $\nabla g(\mu_X)$ is the gradient of g evaluated at μ_X:

$$\nabla g(\mu_X) = \left\{ \frac{\partial g(\mu_X)}{\partial x_1}, \frac{\partial g(\mu_X)}{\partial x_2}, \cdots, \frac{\partial g(\mu_X)}{\partial x_n} \right\}^T$$

The mean value of the approximate limit state function $\widetilde{g}(X)$ is:

$$\mu_{\widetilde{g}} = \mathrm{E}[g(\mu_X)] = g(\mu_X) \tag{2.23}$$

The variance of $\widetilde{g}(X)$ is calculated as:

$$\begin{aligned} \mathrm{Var}[\widetilde{g}(X)] &= \underbrace{\mathrm{Var}[g(\mu_X)]}_{0} + \mathrm{Var}[\nabla g(\mu_X)^T X] - \mu_X \underbrace{\mathrm{Var}[\nabla g(\mu_X)]}_{0} \\ &= \left[\nabla g(\mu_X)^T\right]^2 \mathrm{Var}(X) \end{aligned} \tag{2.24}$$

So the standard deviation is:

$$\sigma_{\widetilde{g}} = \sqrt{\mathrm{Var}[\widetilde{g}(X)]} = \sqrt{[\nabla g(\mu_X)^T]^2\, \mathrm{Var}(X)} = \left[\sum_{i=1}^{n} \left(\frac{\partial g(\mu_X)}{\partial x_i} \right)^2 \sigma_{x_i}^2 \right]^{\frac{1}{2}} \tag{2.25}$$

Finally, the reliability index β is computed as:

$$\beta = \frac{\mu_{\widetilde{g}}}{\sigma_{\widetilde{g}}} \tag{2.26}$$

There are two drawbacks of this method:

- Large errors in the mean value estimation will happen if the limit state function is highly nonlinear, or if the coefficient of variations of X are large. This can be seen when considering the second order terms in Taylor expansion to assess the mean value of $\widetilde{g}(X)$.
- This method fails to be invariant with different mathematically equivalent formulations of the same problem. This is because the mean point μ_X in the Taylor

expansion is not necessarily on the limit state surface.

Hasofer and Lind Method

In order to avoid the drawbacks of the Mean Value FOSM, the Hasofer and Lind (HL) method aims to search for the design point on the limit state surface. This HL method is usually referred as FORM or advanced FOSM. Original HL method considers only the independent and normally distributed random variables. Those basis random variables in x-space will be transformed into a set of normalized and independent variables u_i in u-space. The limit state function $g(X)$ in x-space is also transformed to $g(U)$ in u-space.
The reliability index β is the solution of a constrained optimization problem in the standard normal space.

$$\text{Minimize:} \qquad \beta_U = (U^T U)^{\frac{1}{2}} \tag{2.27}$$

$$\text{Subject to:} \qquad g(U) = 0 \tag{2.28}$$

Several constrained optimization methods were used to solve this optimization problem, including primal methods (feasible directions, gradient, projection, reduced gradient), penalty methods, dual methods, and Lagrange multiplier methods. Each method has its advantages and disadvantages, depending upon the attributes of the method and the nature of the problem (Choi et al., 2007).

In order to consider non-Gaussian random variables, Rackwitz and Fiessler suggest an algorithm to transfer non-Gaussian variables into equivalent normal variables before using HL method. This algorithm is used to calculate reliability index in the thesis so it will be explained more detail in Section 3.5.

FORM with Adaptive Approximations

In previous section, the limit state function $g(U)$ was approximated by the first-order Taylor expansion at the design point. For nonlinear problems, this approach is only an approximation, and several iterations are usually required. The convergence of the algorithm depends on how well the linearized limit state function approximates the nonlinear function $g(U)$. Another way to approximate the limit state function is by using *Two-point Adaptive Nonlinear Approximations* (TANA), including TANA and TANA2. This new category of approximations is constructed by using the Taylor series expansion in terms of adaptive intervening variables. The nonlinearity of the adaptive approximations is automatically changed by using the known information generated during the iteration process. TANA2 also has a correction term for second-order terms. (Choi et al., 2007)

2.2.4 Basis of SORM methods

Usually FORM works well in case the limit state surface has only one minimal distance point and the function is nearly linear in the neighborhood of the design point. When the failure surface has large curvatures (highly nonlinear), the probability of failure estimated by FORM may give unreasonable and inaccurate results (Melchers, 1999). To handle this problem, the second order Taylor expansion (or other polynomials) is considered. Various nonlinear approximate methods have been proposed in the literature. Breitung (1984), Tvedt (1983), Hohenbichler and Rackwitz (1988), Köylüoğlu and Nielsen (1994), Cai and Elishakoff (1994) have developed SORM using the second order approximation to replace the original surfaces. In this thesis, SORM calculation follows the Breitung (1984) method. Detail of the procedure is explained in Section 3.6.

2.2.5 Principle of the Monte Carlo Simulation method

Monte Carlo Simulation (MCS) is known as a simple random sampling method or statistical trial method that makes realizations based on randomly generated sampling sets for uncertain variables. The computational procedure of MCS for reliability problems is quite simple:

- Select distribution types for the random variables
- Generate sampling sets from the distributions
- Calculate value of the limit state function using the generated sampling sets

- Probability of failure is determined from:

$$P_f = \frac{N_f}{N} \tag{2.29}$$

where N_f is the number of trials that the safety of the structure is violated (i.e. when the limit state function is non-positive) out of N experiments conducted.

Generation of Random Variable: In Monte Carlo Sampling, one of the key features is the generation of a series of values of one or more random variables with specified probability distributions. The most commonly used generation method is the *inverse transform method*. This technique is graphically summarized in Figure 2.5. The random number generator produces uniform random numbers between 0 and 1 based on arbitrarily selected seed values. From the generated uniform random number, the corresponding cumulative density function (CDF) value of the uniform distribution and the target distribution can easily be obtained. The final step is to obtain random number for the target probability density function (PDF) using Equation (2.30).

$$x_i = F_X^{-1}(u_i) \tag{2.30}$$

where u_i is the generated uniformly distributed random number ($0 \leq u_i \leq 1$) and F_X is the CDF of the target random variable x. This method can be applied to variables for which a cumulative distribution function has been obtained from direct observation, or where an analytical expression for the inverse cumulative function, $F_{(.)}^{-1}$ exists.

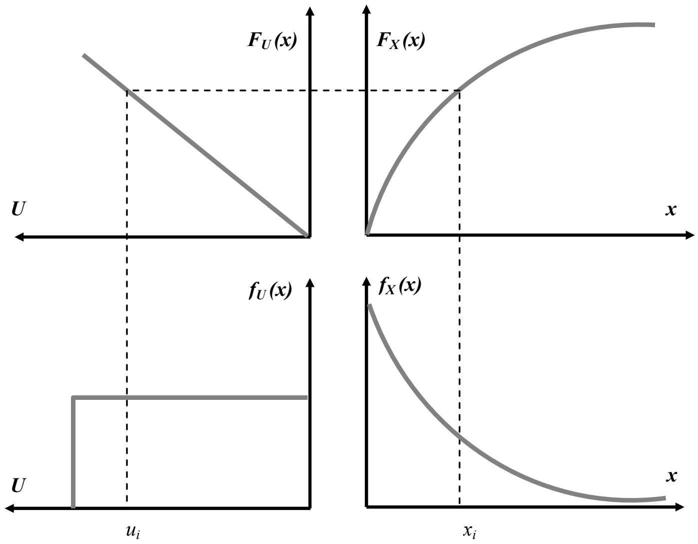

FIGURE 2.5: Transformation from a space of basis random variable **z**-space (left) into a space of standardized normal variables **u**-space (right)

Chapter 3

Methodology

3.1 General

The aim of this thesis is to find the reliability of offshore welded joints once we know the uncertainties in their design stage. In order to do that, this chapter aims to identify the uncertainties, write the causes of failure in a form of Limit State Functions, and finally, describe the suitable methods for reliability analysis.
Structure of this chapter follows the procedure as in Figure 3.1.

3.2 Uncertainties

From the procedure of fatigue design we can see that the uncertainty of the damage result comes from three sources:

1. Uncertainty on the validity of the fatigue modeling (Miner's rule)
2. Uncertainty on the applied SN curve - the fatigue resistance
3. Uncertainty on the loading (natural variability and uncertainty in the environmental modelling and stress calculations)

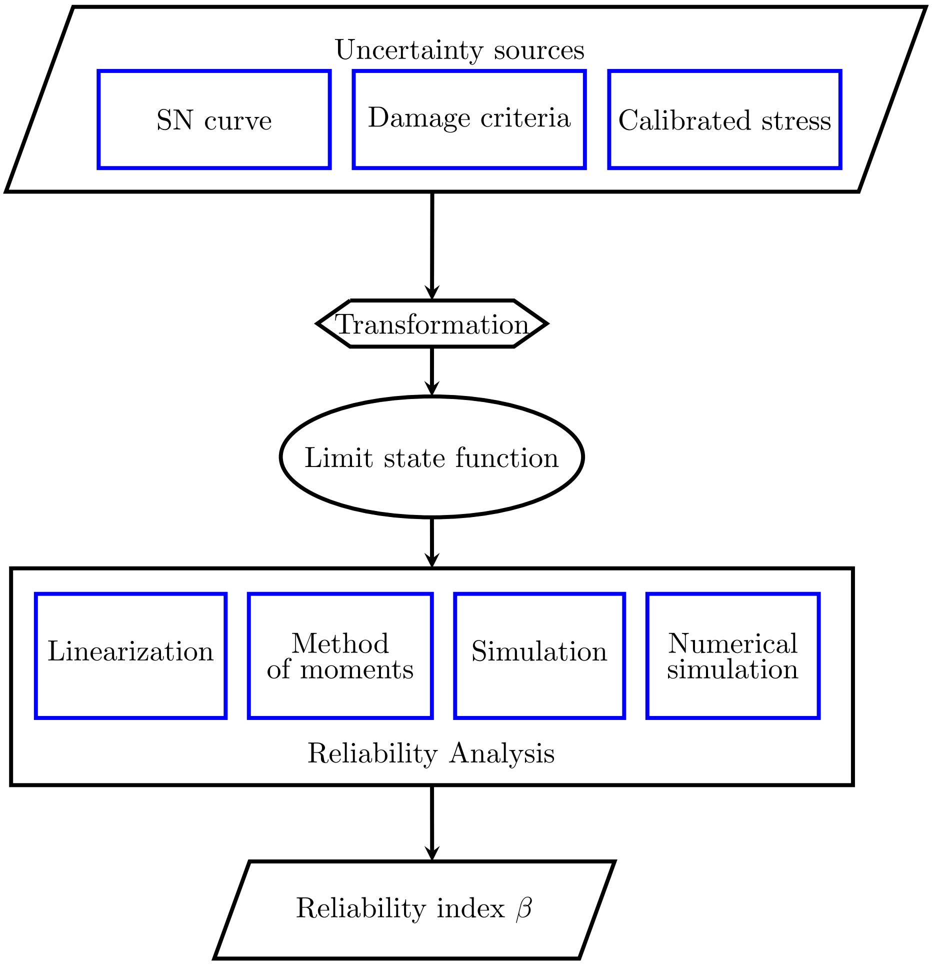

FIGURE 3.1: Procedure for reliability analysis

3.2.1 Uncertainty on fatigue modeling

Due to the inherent deficiencies of the linear damage rules (Miner's Rule), life prediction based on this rule is often unsatisfactory. Experimental evidence under completely reversed loading condition (see Figure 3.2) often indicates that fracture occurs when total damage $\sum D_i > 1$ for a low-to-high loading sequence, and $\sum D_i < 1$ ($\Rightarrow$ Miner's rule is not conservative) for a high-to-low loading sequence (Fatemi and Yang, 1998). This uncertainty is generally modelled by treating the damage at failure, Δ , as a random variable.

Wirsching (1984) proposes a lognormal distribution with median equal to 1.0 and CoV equal to 0.3. This model has become the standard model, e.g. Folsø et al. (2002) and SSC (1996). Lacking alternatives, this model is however recommended if no specific information on the fatigue problem at hand is available.

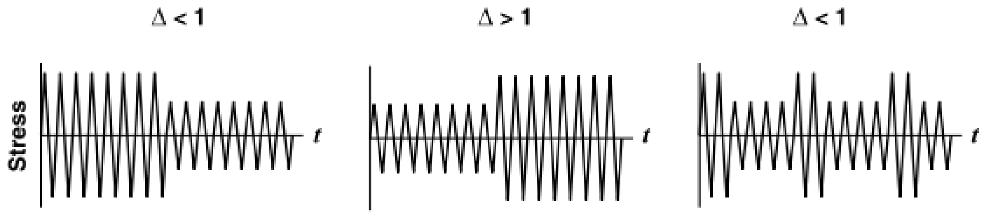

FIGURE 3.2: Sequence effects due to crack initiation, after Dowling (1971)

3.2.2 Uncertainty on the applied SN curve

The nature of SN curves for fatigue design is the relationship between the number of cycle of cyclic loading, N, that a welded joint can sustain before fracture and the amplitude of that cyclic loading. This relationship is represented by a regression line with two parameter C and m:

$$\log N = \log C - m \log S \tag{3.1}$$

These two parameters are evaluated by statistical analysis of the SN (Wöhler) tests. The uncertainty on the constant amplitude fatigue resistance as modelled by the SN curves is commonly accounted for by randomising the parameter C, where $\log C$ is assumed to follow a normal distribution (implying a lognormal distribution for C).
Other parameters in the SN curves, especially m_1 and m_2, are generally modelled as deterministic, mainly due to the limited amount of underlying experimental data.
Although it is easy to find SN curves and their characteristic values for welded joints from design standards, it is rare to find published data on the scatter in C.

There are some pilot research projects funded by Commission des Communautés européennes to study the fatigue behaviour of tubular welded joints such as ARSEM (1985) and Bignonnet et al. (1992). Table 3.1 is the design S-N curve taken from ARSEM (1985) defined by the median number of cycles to reach failure minus two standard deviations. In Table 3.2 we can find an example of the SN curve for tubular joints taken from DET NORSKE VERITAS.

Although information about the standard deviation in S-N curve is required for purpose of probabilistic analysis, it is difficult to present standard deviations that are representative for each of the specific curve (DNV-RP-C203, 2005). This difficulty comes from many reasons such as:

Table 3.1: S-N curves for tubular joints
with reference thickness $t_{ref} = 18$mm (ARSEM, 1985)

Constant amplitude in air $\sigma_{\log N} = 0.275$			
C	m	Range of validity	k
12.29	3	$N < 10^7$	0.29
15.82	5	$N > 10^7$	0.29

Table 3.2: S-N curves for tubular joints
with reference thickness $t_{ref} = 32$mm (DNV-OS-J101, 2013)

In air				In seawater w. cathodic protection				Free corrosion		
$\log C$	m	Validity	k	$\log C$	m	Validity	k	$\log C$	m	k
12.164	3	$N < 10^7$	0.25	11.764	3	$N < 10^6$	0.25	11.687	3	0.25
15.606	5	$N > 10^7$	0.25	15.606	5	$N > 10^6$	0.25			

- Test specimens are in small scale, so they do not include the same amount of residual stresses as a full scale structure.
- The test data may also belong to different R-ratios (where $R = \sigma_{min}/\sigma_{max}$)
- The small scale test specimens are most often more perfect considering tolerances and defects than that of real structures.

However, without having test data established for specific design and fabrication, a typical standard deviation $s_{\log N} = 0.200$ is suggested by DNV-RP-C203 (2005). This value will be used together with the S-N curve in Table 3.2 for reliability index calculation in this thesis.
From the definition of characteristic value for fatigue resistance:

$$\log \bar{C} = \mu_{\log C} - c \cdot \sigma_{\log C} \tag{3.2}$$

With 97.7 % of confidence interval (DNV-RP-C203, 2005, p.126), the number of standard deviations to be subtracted from the mean to derive a S-N is: $c = \Phi^{-1}(97.7\%) = 2$. Given the design value as in Table 3.2, the mean value of $\log C$ random variable, $\mu_{\log C}$, is calculated from:

$$\mu_{\log C} = \log \bar{C} + 2 \cdot \sigma_{\log C} \tag{3.3}$$

where $\log \bar{C}$ is the characteristic value given in Table 3.2 and $\sigma_{\log C} = \sigma_{\log N}$.

Having values of $\mu_{\log C}$ and $\sigma_{\log C}$ in common logarithm (i.e. $\log_{10}(\cdot)$), it is necessary to convert them to natural logarithm (i.e. $\ln(\cdot)$) to be used in the formula of lognormal distribution later:

$$\begin{aligned} \mu_{\ln C} &= \frac{\mu_{\log C}}{\log(e)} \\ \sigma_{\ln C} &= \frac{\sigma_{\log C}}{\log(e)} \end{aligned} \tag{3.4}$$

From information of $\mu_{\ln C}$ and $\sigma_{\ln C}$ of the variable $\ln C$ in normal distribution, one can find back value of μ_C and σ_C in lognormal distribution using the relationship as in Equations (3.5) :

$$\begin{aligned} \sigma_{\ln C} &= \sqrt{\ln\left(\frac{\sigma_C^2}{\mu_C^2} + 1\right)} \\ \mu_{\ln C} &= \ln(\mu_C) - \frac{1}{2}\sigma_{\ln C}^2 \end{aligned} \tag{3.5}$$

This gives back the statistics for the bi-linear S-N curve in seawater with cathodic protection:

$$\begin{aligned} \mu_C &= \exp\left(\mu_{\ln C} + \frac{1}{2}\sigma_{\ln C}^2\right) = 1.6220e12 \\ \text{CoV}_C &= \sqrt{\exp(\sigma_{\ln C}^2) - 1} = 0.486 \end{aligned} \tag{3.6}$$

And the statistics for the linear S-N curve in free corrosion condition:

$$\begin{aligned} \mu_C &= \exp\left(\mu_{\ln C} + \frac{1}{2}\sigma_{\ln C}^2\right) = 1.3585e12 \\ \text{CoV}_C &= \sqrt{\exp(\sigma_{\ln C}^2) - 1} = 0.486 \end{aligned} \tag{3.7}$$

3.2.3 Uncertainty on the fatigue loading

The uncertainty modelling of stress ranges is, by nature, very much depending on the applied stress calculation methods and must thus be considered specifically for the individual cases.

The uncertainties on the calculated stresses are expressed in terms of an error factor B_S which is multiplied on the calculated stress ranges ΔS_{calc}:

$$\Delta S = \Delta S_{calc} \cdot B_S \tag{3.8}$$

The factors contributing to the quantity B_S consists of:

- The uncertainty on the environmental load: seastate discription, wave load predictions
- The uncertainty on the stress concentration factor
- The uncertainty on the nominal stress calculation

For offshore structures, the different contributions of the individual steps in fatigue stress calculations are assessed in Wirsching (1984). These estimations are based on "the experience of several companies" involved in a study of the American Petroleum Institute. He suggested to use median value of B_S is 0.7 and $CoV = 0.5$ for general application. However, nowadays with higher calculation techniques and larger statistic data of environmental load we should have less scattered results of stress calculations. Following the assumed model of (Straub, 2004), the B_S quantity in the calculation of this thesis will be considered as a lognormal distributed random variable with $\mu_{B_S} = 1$ and CoV = 0.25

3.3 Limit State Function

Failure is reached when total damage D_{tot} (calculated from equation (2.5)) reaches Δ, the damage criteria, a random variable which represents the uncertainty in Miner's rule. The limit state function is thus:

$$g_{SN} = \Delta - D_{tot} \tag{3.9}$$

Equation (3.9) can be rewritten as in equation (3.10) for a linear S-N curve [1], and as in equation (3.11) for a bi-linear one [2] (refer to Figure 2.1 for more explanation about types of SN curves).

$$g_{SN} = \Delta - \frac{B_S^m}{C} \sum_{i=1}^{n} S_i^m \tag{3.10}$$

$$g_{SN} = \Delta - \left(\frac{B_S^{m_1}}{C_1} \sum_{i=1}^{n_1} S_{1i}^{m_1} + \frac{B_S^{m_2}}{C_2} \sum_{j=1}^{n_2} S_{2j}^{m_2} \right) \tag{3.11}$$

[1] See section 3.3.1.1 for the calculation of the mean and the variance of the sum random variable
[2] See section 3.3.1.2 for the calculation of the mean and the variance of the two sum random variables

where $n_1 + n_2 = n$ and the relationship between C_1 and C_2 is based on the knee point (N_q, S_q) on the S-N curve:

$$N_q = C_1 S_q^{-m_1} = C_2 S_q^{-m2}$$
$$\rightarrow C_2 = C_1 S_q^{m_2 - m_1}$$

There are two methods to simplify the limit state function as in equation (3.9). One method is to follow the instruction of Madsen et al. (2006), the other is to simplify the damage expectation - proposed by Straub (2004) .

3.3.1 Madsen's method

To compute the reliability, the distribution of $\sum S_i^m$ must be computed. However the distribution type of this random variable is generally difficult to obtain. If the number of stress cycles is large and the dependence between stress ranges is sufficiently weak, the distribution is well approximated by the normal distribution (Madsen et al., 2006).

3.3.1.1 For linear S-N curve

Given that the stress range random variable, S, is Weibull distributed, the probability density function of S is written as:

$$f(S) = \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} \cdot e^{-\left(\frac{S}{k}\right)^{\lambda}}; \qquad S \geq 0 \tag{3.12}$$

Assuming a deterministic number of stress cycles $n = \nu \cdot T$ in the time period $[0, T]$; where ν is the stress cycle rate per year and T is the number of year in service, then for a linear S-N curve:

$$\begin{aligned} \mathrm{E}\left[\sum_{i=1}^{n} S^m\right] &= n\mathrm{E}\left[S^m\right] \\ &= \nu \cdot T \int_0^{\infty} S^m f(S) dS \\ &= \nu \cdot T \int_0^{\infty} S^m \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} e^{-\left(\frac{S}{k}\right)^{\lambda}} dS \end{aligned} \tag{3.13}$$

Set $t = \left(\frac{S}{k}\right)^{\lambda}$

$\rightarrow dt = \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} dS$

$\rightarrow S = k \cdot t^{\frac{1}{\lambda}} \rightarrow S^{2m} = k^{2m} \cdot t^{2m/\lambda}$

Substitute into equation (3.13):

$$\begin{aligned} \mathrm{E}\left[\sum_{i=1}^{n} S^m\right] &= \nu \cdot T \int_{0}^{\infty} k^m \cdot t^{\frac{m}{\lambda} \cdot e^{-t} \cdot dt} \\ &= \nu \cdot T \cdot k^m \Gamma\left(\frac{m}{\lambda} + 1\right) \end{aligned} \tag{3.14}$$

The variance of the random sum can be written as:

$$\begin{aligned} \mathrm{Var}\left[\sum_{i=1}^{n} S^m\right] &= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}\left[S_i^m, S_j^m\right] \\ &= n\mathrm{Var}\left[S_i^m\right] + 2\sum_{k=1}^{n-1}(n-k)\mathrm{Cov}\left[S_i^m, S_{i+k}^m\right] \end{aligned} \tag{3.15}$$

The variance of S_i^m is computed as:

$$\begin{aligned} \mathrm{Var}\left[S_i^m\right] &= \mathrm{E}\left[S_i^{2m}\right] - \mathrm{E}\left[S_i^m\right]^2 \\ &= k^{2m}\left\{\Gamma\left(\frac{2m}{\lambda} + 1\right) - \Gamma^2\left(\frac{m}{\lambda} + 1\right)\right\} \end{aligned} \tag{3.16}$$

For the covariance Cov $\left[S_i^m, S_{i+k}^m\right]$, it is possible to find the method to calculate in Madsen et al. (2006) from the envelope process of stress cycle. However, since all information about the sequence of stress cycles is ignored during stress range counting, it makes little sense to consider the covariance Cov $\left[S_i^m, S_{i+k}^m\right]$.

To check the effect of this covariance Cov $\left[S_i^m, S_{i+k}^m\right]$, this Madsen's method will be used in the calculation latter with the assumption that Cov $\left[S_i^m, S_{i+k}^m\right] \approx 0$.

3.3.1.2 For bi-linear S-N curve

For a bi-linear SN curve with the limit state function in (3.11), we separate the sum random variable into two. The first one, $\sum_{i=1}^{n_1} S_{1i}^{m_1}$, consists of all the stress ranges that below the stress range value S_q, the second one is the rest. The two quantities n_1 and

n_2 in equation (3.11) are the number of cycles whose stress ranges are below and above the stress range value S_q, respectively.

- Above stress range value S_q, the proportion of the number of cycle in this range is calculated from:

$$\frac{n_2}{n} = P(S > S_q) = \int_{S_q}^{\infty} f(S)dS = \int_{S_q}^{\infty} \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} e^{-(\frac{S}{k})^{\lambda}} dS$$

Set $\left(\frac{S}{k}\right)^{\lambda} = t$ then $dt = \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} dS$

So

$$\frac{n_2}{n} = \int_{\left(\frac{S_q}{k}\right)^{\lambda}}^{\infty} e^{-t}dt = \exp\left(-\left(\frac{S_q}{k}\right)^{\lambda}\right)$$

- Below stress range value S_q:

$$\frac{n_1}{n} = [P(0 < S < S_q) = \int_{0}^{S_q} f(S)dS = \int_{0}^{\infty} f(S)dS - \int_{S_q}^{\infty} f(S)dS$$
$$= 1 - \exp\left(-\left(\frac{S_q}{k}\right)^{\lambda}\right)$$

- Mean and Variance of the sum $\sum_{i=1}^{n_1} S_{1i}^{m_1}$:

$$\begin{aligned} \mathrm{E}\left[\sum_{i=1}^{n_1} S_{1i}^{m_1}\right] &= n\mathrm{E}[S_1^{m_1}] \\ &= n\int_{S_q}^{\infty} S^{m_1} f(S)dS \\ &= n\int_{S_q}^{\infty} S^{m_1}\frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} e^{-(\frac{S}{k})^{\lambda}} dS \end{aligned} \tag{3.17}$$

Set $t = \left(\frac{S}{k}\right)^{\lambda}$

$\to dt = \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} dS$

$\to S = k \cdot t^{\frac{1}{\lambda}} \to S^{m_1} = k^{m_1} \cdot t^{m_1/\lambda}$

Substitute into equation (3.17) we have:

$$\begin{aligned} \mathrm{E}\left[\sum_{i=1}^{n_1} S_{1i}^{m_1}\right] &= n \int\limits_{\left(\frac{S_q}{k}\right)^{\lambda}}^{\infty} k^{m_1} \cdot t^{\frac{m_1}{\lambda}} \cdot e^{-t} \cdot dt \\ &= nk^{m_1}\Gamma\left(\frac{m_1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right) \end{aligned} \tag{3.18}$$

The variance of $\sum\limits_{i=1}^{n_1} S_{1i}^{m_1}$ is derived similarly to (3.15) and (3.16), where:

$$\begin{aligned} \mathrm{Var}\left[S_1^{m_1}\right] &= \mathrm{E}[S_1^{2m_1}] - \mathrm{E}[S_1^{m_1}]^2 \\ &= k^{2m_1}\left\{\Gamma\left(\frac{2m_1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right) - \Gamma^2\left(\frac{m_1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right\} \end{aligned} \tag{3.19}$$

So:

$$\begin{aligned} \mathrm{Var}\left[\sum_{i=1}^{n_1} S_{1i}^{m_1}\right] &= n\mathrm{Var}\left[S_1^{m_1}\right] \\ &= nk^{2m_1}\left\{\Gamma\left(\frac{2m_1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right) - \Gamma^2\left(\frac{m_1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right\} \end{aligned} \tag{3.20}$$

- Similarly, Mean and Variance of the sum $\sum\limits_{j=1}^{n_2} S_{2j}^{m_2}$:

$$\begin{aligned} \mathrm{E}\left[\sum_{j=1}^{n_2} S_{2j}^{m_2}\right] &= n\int\limits_{0}^{\infty} S^{m_2} f(S)dS - n\int\limits_{S_q}^{\infty} S^{m_2} f(S)dS \\ &= nk^{m_2}\left\{\Gamma\left(\frac{m_2}{\lambda}+1\right) - \Gamma\left(\frac{m_2}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right\} \end{aligned} \tag{3.21}$$

$$\begin{aligned}
\text{Var}\left[\sum_{j=1}^{n_2} S_{2j}^{m_2}\right] &= n\text{Var}[S_{2j}^{m_2}] \\
&= n\left(E[S_{2j}^{2m_2}] - E[S_{2j}^{m_2}]^2\right) \\
&= nk^{2m_2}\left\{\Gamma\left(\frac{2m_2}{\lambda}+1\right) - \Gamma\left(\frac{2m_2}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right\} - \\
& nk^{2m_2}\left\{\Gamma\left(\frac{m_2}{\lambda}+1\right) - \Gamma\left(\frac{m_2}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right\}^2
\end{aligned} \tag{3.22}$$

Similar to the above derivation we can find the results for the mean and the variance of the sum random variables in case of bi-linear S-N curve with a cut-off. However, it will not be shown here due to the fact that for offshore wind turbine we don't consider the cut-off.

3.3.2 Approximate damage expectation

Straub (2004) proposed to replace the sum random variable by its expectation. By doing that, he avoids dealing with the above problem but still keep the value of damage expectation doesn't change.

The total damage D_{tot} in the equation (3.9) can be approximated as following:

$$D_{tot} = \sum_{i=1}^{n} D_i \approx n \cdot \text{E}[D_i] \tag{3.23}$$

The total damage D_{tot} is a function of the error factor B_S of stress calculation, C and $\sum_i S_i^m$ random variables. For given values of C and B_S, the approximation of the expectation $\text{E}[D_i]$ can be calculated for each type of S-N curve as following, given that stress ranges are Weibull distributed with pdf as in equation (3.12).

3.3.2.1 For linear SN curve

Similar to the derivation of equation (3.14) we get:

$$\begin{aligned}
\text{E}[D_i|(B_S, C)] &= \int_0^{\infty} \frac{{B_S}^m}{C} S^m f(S) dS \\
&= \frac{{B_S}^m}{C} k^m \Gamma\left(\frac{m}{\lambda}+1\right)
\end{aligned} \tag{3.24}$$

Approximate the term $\mathrm{E}[D_i]$ in (3.23) by $\mathrm{E}[D_i|(B_S,C)]$ in (3.24), the Limit State Function for linear SN curve can be written as:

$$g_{SN} = \Delta - n\frac{{B_S}^m}{C}k^m\Gamma\left(\frac{m}{\lambda}+1\right) \tag{3.25}$$

3.3.2.2 For bi-linear SN curve

$$\begin{aligned}\mathrm{E}[D_i|(B_S,C)] &= \int_0^{S_q}\frac{{B_S}^{m_2}}{C_2}S^{m_2}f(S)dS + \int_{S_q}^{\infty}\frac{{B_S}^{m_1}}{C_1}S^{m_1}f(S)dS \\ &= E2 + E1\end{aligned} \tag{3.26}$$

- Component $E1$ can be calculated as:

$$\begin{aligned}E1 &= \int_{S_q}^{\infty}\frac{{B_S}^{m_1}}{C1}\Delta S^{m_1}f(S)dS \\ &= \frac{{B_S}^{m_1}}{C1}\int_{S_q}^{\infty}S^{m1}\frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1}\cdot e^{-(\frac{S}{k})^{\lambda}}dS\end{aligned} \tag{3.27}$$

Set $t = \left(\frac{S}{k}\right)^{\lambda}$:

$\rightarrow dt = \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} dS$

$\rightarrow S = k\cdot t^{1/\lambda} \rightarrow S^{m1} = k^{m1}\cdot t^{m1/\lambda}$

Substitute into equation (3.27):

$$\begin{aligned}E1 &= \frac{{B_S}^{m_1}}{C1}\int_{\left(\frac{S_q}{k}\right)^{\lambda}}^{\infty} k^{m1}\cdot t^{m1/\lambda}\cdot e^{-t}\cdot dt \\ &= \frac{{B_S}^{m_1}k^{m1}}{C1}\Gamma\left(\frac{m1}{\lambda}+1,\left(\frac{S_q}{k}\right)^{\lambda}\right)\end{aligned} \tag{3.28}$$

- Similarly, $E2$ can be calculated as:

$$\begin{aligned}E2 &= \int_0^{S_q} \frac{{B_S}^{m_2}}{C_1 \cdot S_q^{m_2-m_1}} \cdot S^{m_2} f(S) dS \\ &= \frac{{B_S}^{m_2} S_q^{m_1-m_2}}{C_1} \int_0^{S_q} S^{m_2} \frac{\lambda}{k}\left(\frac{S}{k}\right)^{\lambda-1} \cdot e^{-(\frac{S}{k})^{\lambda}} dS \\ &= \frac{{B_S}^{m_2} S_q^{m_1-m_2}}{C_1} \cdot k^{m_2}\left[\Gamma\left(\frac{m_2}{\lambda}+1\right) - \Gamma\left(\frac{m_2}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right]\end{aligned} \tag{3.29}$$

- In conclusion, expectation of damage per cycle can be calculated as:

$$\begin{aligned}E[D_i|(B_S, C)] = &\frac{{B_S}^{m_1}}{C1} k^{m1} \Gamma\left(\frac{m1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right) \dots \\ &+ \frac{{B_S}^{m_2} S_q^{m1-m2}}{C1} \cdot k^{m2}\left[\Gamma\left(\frac{m2}{\lambda}+1\right) - \Gamma\left(\frac{m2}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right]\end{aligned} \tag{3.30}$$

Approximate the term E$[D_i]$ in (3.23) by E$[D_i|(B_S, C)]$ in (3.30), the Limit State Function for bi-linear SN curve can be written as:

$$\begin{aligned}g_{SN} = \Delta - n &\frac{{B_S}^{m_1}}{C1} k^{m1} \Gamma\left(\frac{m1}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right) \dots \\ - n &\frac{{B_S}^{m_2} S_q^{m1-m2}}{C1} \cdot k^{m2}\left[\Gamma\left(\frac{m2}{\lambda}+1\right) - \Gamma\left(\frac{m2}{\lambda}+1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right]\end{aligned} \tag{3.31}$$

3.4 Transformation

In this section we propose a method to transform random variables in the current fatigue reliability problem into independent, standardized normally distributed ones. Because damage criteria Δ, S-N curve parameter C, stress calculation error B_S are independent random variables, the transformation procedure is quite simple. For other reliability problems where basis random variables are correlated, methods to transform are mentioned in Section 2.2.2.

To transform a non-normally distributed random variable X_i to a standardized normally distributed random variable U_i we use the relationship:

$$\Phi(U_i) = F_{X_i}(X_i) \tag{3.32}$$

where F_{X_i} is the distribution function (or CDF) of X_i. Given a realization $\mathbf{u}$ of $\mathbf{U}$, a realization $\mathbf{x}$ of $\mathbf{X}$ can be determined from:

$$\begin{aligned} x_1 &= F_{X_i}^{-1}\left(\Phi(u_i)\right) \\ &\vdots \\ x_n &= F_{X_n}^{-1}\left(\Phi(u_n)\right) \end{aligned} \tag{3.33}$$

3.4.1 Transformation of Damage criteria Δ random variable

As mentioned earlier, random variable Δ is lognormal distributed with mean μ_Δ and coefficient of variation CoV_Δ.

Note that these values of the mean and the standard variation are those of the variable's natural logarithm (implies that the variable's logarithm is normally distributed).

The distribution function of Δ is:

$$F_\Delta(\Delta) = \Phi\left(\frac{ln(\Delta) - \mu_{L\Delta}}{\sigma_{L\Delta}}\right) \tag{3.34}$$

where $\sigma_{L\Delta} = \sqrt{ln\left(\frac{\sigma_\Delta^2}{\mu_\Delta^2} + 1\right)}$ and $\mu_{L\Delta} = ln(\mu_\Delta) - \frac{1}{2}\sigma_{L\Delta}^2$

Let U_1 be a transformed random variable of Δ which is standardized normally distributed, we have:

$$F_\Delta(\Delta) = \Phi(U_1) \tag{3.35}$$

From equations (3.34) and (3.35) we have:

$$\Delta = \exp\left(U_1\sigma_{L\Delta} + \mu_{L\Delta}\right) \tag{3.36}$$

3.4.2 Transformation of SN curve uncertainty - C random variable

Random variable C is lognormal distributed with mean μ_C and coefficient of variation CoV_C.

Let U_2, which is standardized normally distributed, be a transformed random variable of C. Similarly to random variable Δ, we have:

$$C = \exp(U_2\sigma_{LC} + \mu_{LC}) \tag{3.37}$$

3.4.3 Transformation of stress calculation error - B_S random variable

Similar to the transformations of Δ and C, for the lognormal distributed random variable B_S if we know the mean μ_{B_S} and CoV_{B_S} then the transformation of B_S into a standardized normally distributed random variable U_3 can be written as:

$$B_S = \exp(U_3\sigma_{LB_S} + \mu_{LB_S}) \tag{3.38}$$

3.4.4 Transformation of $\sum_i S_i^m$- the sum random variable

Since the random variable $\sum_i S_i^m$ is normally distributed. The transformation of this sum random variable into a standardized normally distributed random variable U_3 can done by:

$$U_3 = \frac{\sum_i S_i^m - \mu_{\sum}}{\sigma_{\sum}} \tag{3.39}$$

Then:

$$\sum_i S_i^m = U_3\sigma_{\sum} + \mu_{\sum} \tag{3.40}$$

where $\mu_{\sum}$ and $\sigma_{\sum}$ are mean and standard deviation of the sum random variable $\sum_i S_i^m$.

3.4.5 Limit State Functions after transformation

From the transformations of Δ, B_S, C and $\sum_i S_i^m$ random variables in equations (3.36), (3.38), (3.37) and (3.40):

- Substitute into equation (3.10), we have the transformation of the LSF following Madsen's method for linear SN curve:

$$\begin{aligned} g_{SN} &= \Delta - \frac{{B_S}^m}{C} \sum_{i=1}^{n} S_i^m \\ &= \exp\left(U_1\sigma_{L\Delta} + \mu_{L\Delta}\right) - \frac{[\exp(U_3\sigma_{LB_S} + \mu_{LB_S})]^m}{\exp(U_2\sigma_{LC} + \mu_{LC})}\left(U_4\sigma_{\Sigma} + \mu_{\Sigma}\right) \end{aligned} \tag{3.41}$$

- Substitute into equation (3.11), we have the transformation of the LSF following Madsen's method for bi-linear SN curve:

$$\begin{aligned} g_{SN} &= \Delta - \left(\frac{B_S{}^m}{C} \sum_{i=1}^{n_1} S_i^m + \frac{B_S{}^m}{S_q^{m_2-m_1} C} \sum_{j=1}^{n_2} S_j^m \right) \\ &= \exp\left(U_1 \sigma_{L\Delta} + \mu_{L\Delta}\right) - \frac{[\exp(U_3 \sigma_{LB_S} + \mu_{LB_S})]^{m_1}}{\exp(U_2 \sigma_{LC} + \mu_{LC})} \left(U_4 \sigma_{\Sigma 1} + \mu_{\Sigma 1}\right) \\ &\quad - \frac{[\exp(U_3 \sigma_{LB_S} + \mu_{LB_S})]^{m_2}}{S_q^{m_2-m_1} \exp(U_2 \sigma_{LC} + \mu_{LC})} \left(U_5 \sigma_{\Sigma 2} + \mu_{\Sigma 2}\right) \end{aligned} \tag{3.42}$$

- Substitute into equation (3.25), we have the transformation of the LSF using Approximate damage expectation method for linear SN curve:

$$\begin{aligned} g_{SN} &= \Delta - n \frac{B_S{}^m}{C} k^m \Gamma\left(\frac{m}{\lambda} + 1\right) \\ &= \exp\left(U_1 \sigma_{L\Delta} + \mu_{L\Delta}\right) - n \frac{[\exp(U_3 \sigma_{LB_S} + \mu_{LB_S})]^m}{\exp(U_2 \sigma_{LC} + \mu_{LC})} k^m \Gamma\left(\frac{m}{\lambda} + 1\right) \end{aligned} \tag{3.43}$$

- Substitute into equation (3.31), we have the transformation of the LSF using Approximate damage expectation method for bi-linear SN curve:

$$\begin{aligned} g_{SN} &= \Delta - n \frac{B_S{}^{m_1}}{C1} k^{m1} \Gamma\left(\frac{m1}{\lambda} + 1, \left(\frac{S_q}{k}\right)^{\lambda}\right) \ldots \\ &\quad + n \frac{B_S{}^{m_2} S_q^{m1-m2}}{C1} \cdot k^{m2} \left[\Gamma\left(\frac{m2}{\lambda} + 1\right) - \Gamma\left(\frac{m2}{\lambda} + 1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right] \\ &= \exp\left(U_1 \sigma_{L\Delta} + \mu_{L\Delta}\right) \ldots \\ &\quad - n \frac{[\exp(U_3 \sigma_{LB_S} + \mu_{LB_S})]^{m_1}}{\exp(U_2 \sigma_{LC} + \mu_{LC})} k^{m1} \Gamma\left(\frac{m1}{\lambda} + 1, \left(\frac{S_q}{k}\right)^{\lambda}\right) \ldots \\ &\quad - n \frac{[\exp(U_3 \sigma_{LB_S} + \mu_{LB_S})]^{m_2}}{S_q^{m_2-m_1} \exp(U_2 \sigma_{LC} + \mu_{LC})} k^{m2} \left[\Gamma\left(\frac{m2}{\lambda} + 1\right) - \Gamma\left(\frac{m2}{\lambda} + 1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right] \end{aligned} \tag{3.44}$$

3.5 First Order Reliability Method

3.5.1 Procedure of FORM

As a continuation of Section 2.2.3, this section explains the procedure of FORM.

Let's consider the limit state function $g(U)$ is the transformation of the limit state function $g(X)$ from basis x-space into the normalized u-space.

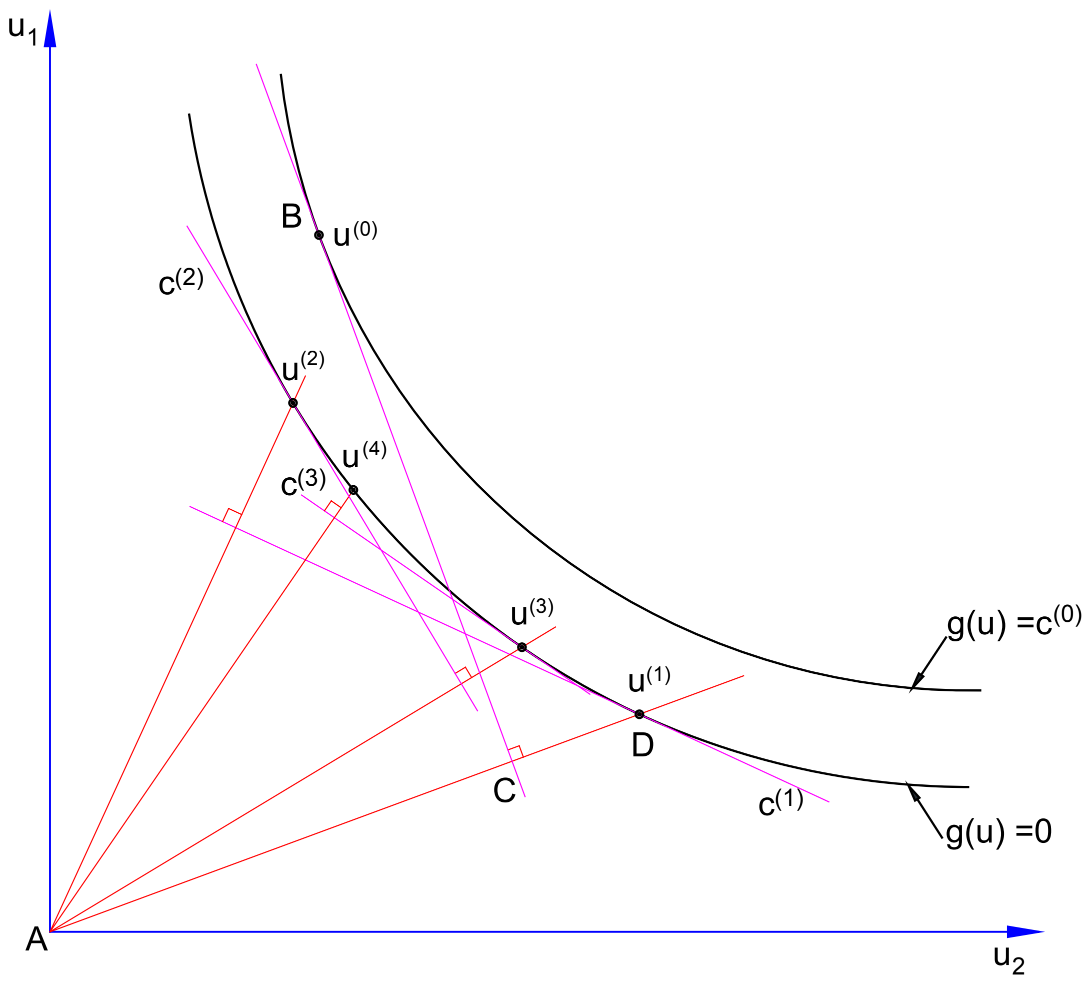

FIGURE 3.3: Algorithm to determine the reliability index β_{HL}

By using the first-order Taylor expansion, the limit state surface $g(U)$ is replaced by its tangent hyperplane $\widetilde{g}(U)$ at each point on the trajectory (i.e. hyperplanes $c^{(1)}, c^{(2)}, c^{(3)}, \cdots$ on figure 3.3). Reliability index β is the shortest distance from the origin to the limit state surface in u-space. So by iteration, this algorithm aims to find the point on the trajectory that has the shortest distance to the tangent hyperplane at that point.

At step m, the point is $\mathbf{u}^{(m)}$ and the surface $g(U)$ is replaced by its tangent hyperplane at $\mathbf{u}^{(m)}$. The point $\mathbf{u}^{(m)}$ on the limit state function should ensure $g(\mathbf{u}^{(m)}) = 0$, so we have:

$$g(\mathbf{u}^{(m)}) + \sum_{i=1}^{n} \frac{\partial g(\mathbf{u}^{(m)})}{\partial u_i}(u_i - u_i^{(m)}) = 0 \tag{3.45}$$

The next iterated point $\mathbf{u}^{(m+1)}$ is obtained as the sum of two terms. First $\mathbf{u}^{(m)}$ is projected on the normal $\alpha^{(m)}$ to the trajectory and then an additional term is introduced to account for the fact that $g(\mathbf{u}^{(m)})$ may be different from zero:

$$\mathbf{u}^{(m+1)} = (\mathbf{u}^{(m)\,T}\boldsymbol{\alpha}^{(m)})\boldsymbol{\alpha}^{(m)} + \frac{g(\mathbf{u}^{(m)})}{|\nabla g(\mathbf{u}^{(m)})|}\boldsymbol{\alpha}^{(m)} \tag{3.46}$$

And when the sequence converges toward a point $\mathbf{u}^*$ then:

$$\mathbf{u}^* = \beta\boldsymbol{\alpha}^*, \qquad g(\mathbf{u}^*) = 0 \tag{3.47}$$

where the unit normal vector $\boldsymbol{\alpha}^{(m)}$ to the trajectory is calculated from:

$$\boldsymbol{\alpha}^{(m)} = -\frac{\nabla g(\mathbf{u}^{(m)})}{|\nabla g(\mathbf{u}^{(m)})|} \tag{3.48}$$

here $\nabla g(\mathbf{u}^{(m)})$ is the gradient vector which is assumed to exist:

$$\nabla g(\mathbf{u}) = \left(\frac{\partial g}{\partial u_1}(\mathbf{u}), \cdots \frac{\partial g}{\partial u_n}(\mathbf{u})\right) \tag{3.49}$$

The procedure of applying the Rackwitz and Fiessler algorithm for reliability calculation can be listed as following:

- Step 1: guess the initial value of the coordinate vector: $\mathbf{u}^{(0)}$
- Step 2: calculate $g(\mathbf{u}^{(i)})$
- Step 3: calculate value of the gradient vector at the current point $\nabla g(\mathbf{u}^{(i)})$
- Step 4: calculate an improved guess of the β point using Equation (3.46)
- Step 5: calculate the corresponding reliability index

$$\beta_{i+1} = \sqrt{(\mathbf{u}^{(i+1)})^T\mathbf{u}^{(i+1)}} \tag{3.50}$$

- Step 6: The loop can stop when β converges, if not, continue by $i = i + 1$ and go to Step 2.

3.5.2 Sensitivity Factors

The sensitivity factor shows the relative importance of each random variable to the failure probability. From the definition of $\beta(\mathbf{u})$ as the length of the vector $\mathbf{u}$ it follows that:

$$\frac{\partial\beta(\mathbf{u})}{\partial u_i} = \frac{\partial}{\partial u_i}\sqrt{u_1^2 + u_2^2 + \cdots + u_n^2} = \frac{u_i}{\beta(\mathbf{u})} = \alpha_i, \qquad (i = 1, 2, \cdots, n) \tag{3.51}$$

From the definition of vector $\boldsymbol{\alpha}$ as in (3.48) we have:

$$\alpha_1^2 + \alpha_2^2 + \cdots + \alpha_n^2 = 1 \tag{3.52}$$

That means the larger the α_i value is, the higher the contribution towards the failure probability.

3.6 Second Order Reliability Method

3.6.1 Breitung Algorithm

To verify the result of FORM method, reliability index is recalculated using SORM method. In the work of Breitung (1984) he proved that by using second order Taylor expansion to represent the limit state function, the probability of failure can be modified as:

$$P_f \approx \Phi(-\beta)\prod_{j=1}^{n-1}(1 + k_j\beta)^{-1/2} \tag{3.53}$$

where k_j is the main curvature of the surface at the design point. The procedure to apply the algorithm of Breitung can be listed as follow:

- Conduct the safety-index search and locate the design point U^* by using the algorithm in FORM method.
- Compute the second order derivatives of the limit state surface at U^* and form the B matrix:

$$B = \frac{\nabla^2 g(U^*)}{|\nabla g(U^*)|} \tag{3.54}$$

- Calculate the orthogonal matrix H for the rotated transformation (see Section 3.6.2).
- Compute the main curvatures of the failure surface at the design point (which is the elements on diagonal matrix in (3.58)).
- Compute the failure probability P_f using equation (3.53).

3.6.2 Orthogonal Transformation

In order to follow the Breitung algorithm, it is necessary to use an orthogonal transformation to transform random variables in standardized normally distributed u-space into a rotated y-space as in Figure 3.4.

To conduct the rotated transformation, an orthogonal matrix H needs to be generated

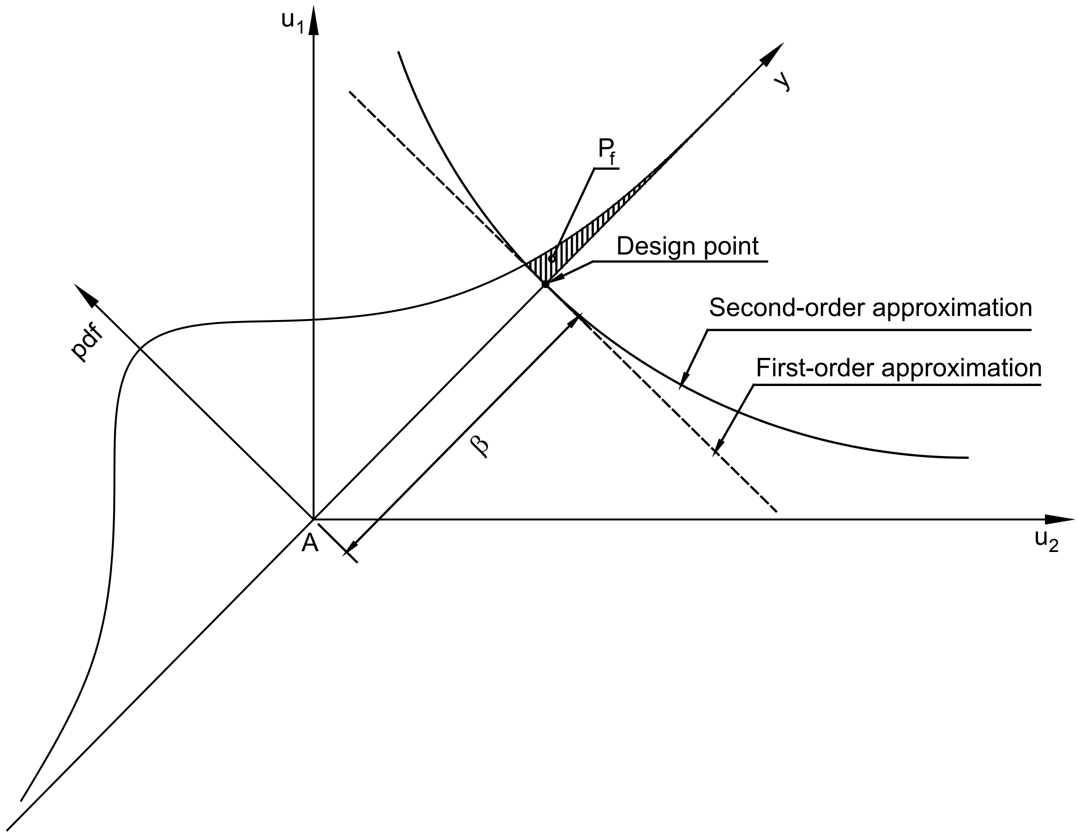

FIGURE 3.4: Second-order approximation of the LSF in rotated y-space

in which the n^{th} row of H is the unit normal of the limit state function at the design point (i.e. vector $\boldsymbol{\alpha}$ in equation (3.48)).

To generate H, first, an initial matrix is selected as follows:

$$\begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{3.55}$$

where the last $n-1$ rows consist of zeros and unity on the diagonal. The Gram-Schmidt algorithm is used to orthogonalize the above matrix to obtain an orthogonal matrix. First, let $f_1, f_2, \cdots, f_n$ denote the first, second, $\cdots$,n^{th} row vector of the above matrix, respectively:

$$\begin{aligned} f_1 &= \boldsymbol{\alpha} \\ f_2 &= \{0, 1, 0, \cdots, 0\} \\ &\vdots \\ f_n &= \{0, 0, 0, \cdots, 1\} \end{aligned}$$

Set $D_1 = (f_1 \cdot f_1^T)$, $\quad e_{11} = \frac{1}{D_1}$, $\quad \gamma_1 = e_{11} f_1$

$D_2 = [(f_2 \cdot f_2^T) - (f_2 \cdot \gamma_1^T)]^{\frac{1}{2}}$, $\quad e_{12} = -\frac{(f_2 \cdot \gamma_1)}{D_2}$, $\quad e_{22} = \frac{1}{D_2}$, $\quad \gamma_2 = e_{12}\gamma_1 + e_{22} f_2$

and in general,

$D_k = [(f_k \cdot f_k^T) - (f_k \cdot \gamma_1^T)^2 - (f_k \cdot \gamma_2^T)^2 - \cdots - (f_k \cdot \gamma_{k-1}^T)^2]^{\frac{1}{2}}$

$e_{1k} = -\frac{(f_k \cdot \gamma_1^T)}{D_k}$, $\quad e_{2k} = -\frac{(f_k \cdot \gamma_2^T)}{D_k}$, $\quad e_{k-1,k} = -\frac{(f_k \cdot \gamma_{k-1}^T)}{D_k}$

$e_{kk} = \frac{1}{D_k}$, $\quad \gamma_k = e_{1k}\gamma_1 + e_{2k}\gamma_2 + \cdots + e_{k-1,k}\gamma_{k-1} + e_{kk} f_k$

The generated orthogonal matrix H_0 is:

$$H_0 = \{\gamma_1; \gamma_2; \cdots; \gamma_n\} \tag{3.56}$$

To satisfy that the n^{th} row of H is $\boldsymbol{\alpha}$, the first row of the orthogonal matrix is moved to the last row. So the H matrix is written as:

$$H = \{\gamma_2; \gamma_3; \cdots; \gamma_n; \gamma_1\} \tag{3.57}$$

The curvatures of the response surface at the design point is the elements on the diagonal matrix in equation (3.58).

$$\bar{H}\bar{B}\bar{H}^T = \begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & k_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & k_{n-1} \end{bmatrix} \tag{3.58}$$

where quantities associated with $n-1$ variables are denoted with a bar.
It is worth to mention here that the orthogonal matrix H can also be generated using the eigenvectors and eigenvalues of the initial matrix. In matlab, one can use the orthogonal-triangular decomposition command for the initial matrix, which is selected as follows:

$$\begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ \alpha_2 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_n & 0 & \cdots & 1 \end{bmatrix} \tag{3.59}$$

3.7 Monte Carlo Simulation Method

Monte Carlo Simulation method will be used in the calculation of this thesis to:

- verify the accuracy of FORM and SORM method in finding the global minimal reliability index.
- determine the error caused by simplified limit state functions.

3.7.1 Verifying the global minimal reliability index

Monte Carlo simulation will be applied for each limit state function in order to compare with the results of FORM and SORM. More specifically, there are four simulation problems:

- Fatigue reliability of Joint A with the simplified limit state function following Madsen's method:

  - The limit state function is written as in Equation (3.10) with four random variables to consider.
  - The summation $\sum_{i=1}^{n} S_i^m$ is considered as a normal random variable with its two moments determined from equations (3.14) and (3.15)

- Fatigue reliability of Joint A with the simplified limit state function using Approximate damage expectation method:
  - The limit state function is written as in Equation (3.25)
  - There are three random variables to consider.

- Fatigue reliability of Joint B with the simplified limit state function following Madsen's method:
  - The limit state function is written as in Equation (3.11) with five random variables to consider.
  - The two summations $\sum_{i=1}^{n_1} S_i^{m_1}$ and $\sum_{j=1}^{n_2} S_j^{m_2}$ are considered as two normal random variables with their two moments determined from equations (3.17) to (3.22)

- Fatigue reliability of Joint B with the simplified limit state function using Approximate damage expectation method:
  - The limit state function is written as in Equation (3.31)
  - There are three random variables to consider.

3.7.2 Verifying the assumptions used for the simplified limit state functions

Instead of considering the summations $\sum_{i=1}^{n_1} S_i^{m_1}$ and $\sum_{j=1}^{n_2} S_j^{m_2}$ as random variables, this simulation will calculate them from the basis random variable S, which is the stress ranges and weibull-distributed random variable.

The simulation will be conducted for the year $T = 20$, which requires a smaller number of sample (i.e. cheaper in terms of calculation time and computer resources). The probability of failure in addition to the variance of the two summations $\sum_{i=1}^{n_1} S_i^{m_1}$ and $\sum_{j=1}^{n_2} S_j^{m_2}$ will be compared with the those from the Madsen's method for bi-linear S-N curve.

Chapter 4

Results

4.1 Information about welded joints

We will consider two existing tubular welded joints as in Figure 4.1. From the design stage we have:

4.1.1 Joint A

We consider Joint A with the following design information:

- Working in free corrosion condition
- Thickness $t = 16$ mm
- Linear S-N curve as shown in Table 3.2: C=4.8641e11 and m=3.
- The stress range follows Weibull distribution with shape parameter $\lambda = 1.2$ and scale parameter $k = 7.152$ MPa. (see Figure 4.2)
- Mean value of stress calculation error $\mu_{B_S} = 1$.
- Stress cycles rate per year: $\nu = 10^7$
- Service life $T = 20$ years, with FDF=2
- FDF value:

$$FDF = \frac{1}{D_{tot}} = \frac{1}{\nu \cdot T \cdot \frac{B_S^m}{C} k^m \Gamma\left(\frac{m}{\lambda} + 1\right)} = 2 \tag{4.1}$$

FIGURE 4.1: Position of Joint A and Joint B in a floating offshore wind turbine (source: Wikipedia)

4.1.2 Joint B

We consider Joint B with following design information:

- Working in seawater with cathodic protection condition
- Thickness $t = 16$ mm
- Bi-linear S-N curve as shown in Table 3.2:

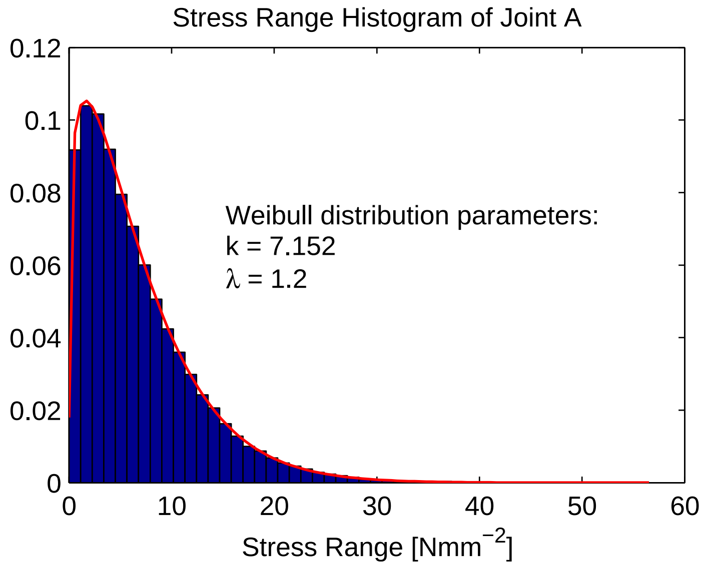

Figure 4.2: Joint A - Stress Range Histogram

  - C_1=5.8076e11 and m_1=3 when $N < 10^6$
  - C_2=4.0365e15 and m_2=5 when $N > 10^6$

- Stress range that follows Weibull distribution with shape parameter $\lambda = 1.2$ and scale parameter $k = 12.6890$ MPa. (see Figure 4.3)

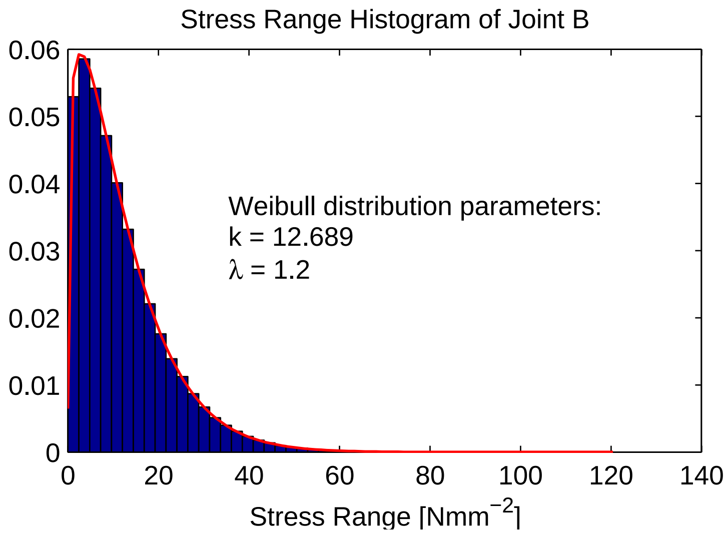

Figure 4.3: Joint B - Stress Range Histogram

- Mean value of stress calculation error $\mu_{B_S} = 1$.
- Stress cycles rate per year: $\nu = 10^7$
- Service life $T = 20$ years, with FDF=2

- FDF value:

$$FDF = \frac{1}{D_{tot}} = 2 \tag{4.2}$$

where

$$D_{tot} = \nu \cdot T \cdot \left\{ \frac{{B_S}^{m_1}}{C_1} k^{m_1} \Gamma\left(\frac{m_1}{\lambda} + 1, \left(\frac{S_q}{k}\right)^{\lambda}\right) \ldots \right.$$
$$\left. + \frac{{B_S}^{m_2} S_q^{m_1 - m_2}}{C_1} \cdot k^{m_2} \left[\Gamma\left(\frac{m_2}{\lambda} + 1\right) - \Gamma\left(\frac{m_2}{\lambda} + 1, \left(\frac{S_q}{k}\right)^{\lambda}\right)\right]\right\} \tag{4.3}$$

4.2 Input data for reliability calculation

The statistics information of relevant uncertainties (i.e. distribution types, their two moments) is discussed in Chapter 3. In this section they are summarized in Table 4.1 for Joint A and in Table 4.2 for Joint B.

TABLE 4.1: Input data for reliability calculation of Joint A

Parameter	Dimension	Distribution	Mean	CoV
Δ	-	Lognormal	1	0.3
ν	yr^{-1}	Deterministic	10^7	
T_{SL}	yr	Deterministic	20	
$k(FDF = 2)$	Nmm^{-2}	Deterministic	7.152	
B_S	-	Lognormal	1	0.25
λ	-	Deterministic	1.2	
C	$(Nmm^{-2})^{m_1}$	Lognormal	1.3585e12	0.486
m	-	Deterministic	3	

4.3 Reliability Calculation on Joint A

4.3.1 The Limit State Function using the Madsen's Method

Using the method mentioned in Madsen et al. (2006), the limit state function for a linear S-N curve fatigue-reliability problem become as in Equation (3.10), where the variance of the sum random variable is simplified by considering that the quantity $\mathrm{Cov}[S_i^m, S_{i+k}^m]$ is not significant. Results of the calculation for Joint A using this simplified limit state function is shown in Table 4.3 and Figure 4.4.

TABLE 4.2: Input data for reliability calculation of Joint B

Parameter	Dimension	Distribution	Mean	CoV
Δ	-	Lognormal	1	0.3
ν	yr^{-1}	Deterministic	10^7	
T_{SL}	yr	Deterministic	20	
$k(FDF = 2)$	Nmm^{-2}	Deterministic	12.6890	
B_S	-	Lognormal	1	0.25
λ	-	Deterministic	1.2	
C_1	$(Nmm^{-2})^{m_1}$	Lognormal	1.6220e12	0.486
m_1	-	Deterministic	3	
m_2	-	Deterministic	5	
N_q	-	Deterministic	10^6	

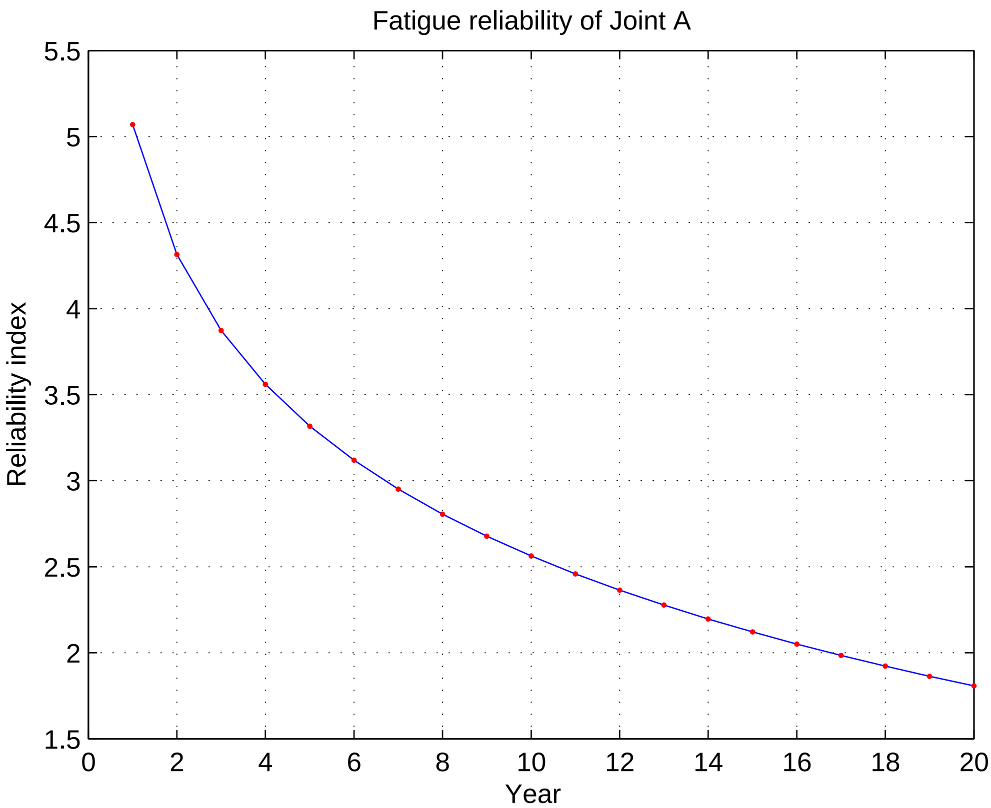

FIGURE 4.4: Result of fatigue reliability calculation
Madsen's method - Joint A

Table 4.3: Fatigue reliability results of Joint A using Madsen's method

Service life	Reliability index β			Probability of failure P_f		
(year)	FORM	SORM	MCS	FORM	SORM	MCS
1	5.069244	5.069212	5.123447	0.000000	0.000000	0.000000
2	4.314690	4.314658	4.298962	0.000008	0.000008	0.000009
3	3.873331	3.873299	3.875926	0.000054	0.000054	0.000053
4	3.560165	3.560132	3.561103	0.000185	0.000185	0.000185
5	3.317178	3.317238	3.316275	0.000455	0.000455	0.000456
6	3.118728	3.118693	3.120866	0.000908	0.000908	0.000902
7	2.950946	2.950911	2.952226	0.001584	0.001584	0.001577
8	2.805610	2.805575	2.807234	0.002511	0.002511	0.002498
9	2.677324	2.677382	2.679109	0.003711	0.003710	0.003691
10	2.562654	2.562618	2.564785	0.005194	0.005194	0.005162
11	2.458925	2.458889	2.460786	0.006968	0.006968	0.006932
12	2.364135	2.364192	2.365781	0.009036	0.009035	0.008996
13	2.277025	2.277081	2.278520	0.011392	0.011391	0.011348
14	2.196375	2.196337	2.197585	0.014033	0.014034	0.013989
15	2.121293	2.121255	2.122660	0.016949	0.016950	0.016891
16	2.050966	2.051021	2.052567	0.020135	0.020132	0.020057
17	1.984994	1.985048	1.986490	0.023573	0.023570	0.023489
18	1.922794	1.922754	1.923945	0.027253	0.027255	0.027181
19	1.863959	1.863919	1.865465	0.031164	0.031167	0.031058
20	1.808050	1.808104	1.809103	0.035300	0.035295	0.035217

4.3.2 The Limit State Function Using Approximate Damage Expectation

Another simplifying method to consider the limit state of the fatigue reliability problem is to neglect the uncertainty of the quantity $\sum_i S_i^m$. The simplified limit state function is written as in Equation (3.25). Results of the calculation using this simplified limit state function is shown in Table 4.4 and Figure 4.5.

4.3.3 Comparison of the two simplified limit state functions

Figure 4.6 displays the variation of the difference between FORM results taken from Table 4.4 with the one taken from Table 4.3.

From this figure we can see that:

- Reliability indexes taken from the LSF that ignores the uncertainty in $\sum_i S_i^m$ (i.e. using Approximate damage expectation method) are larger than those taken from

Table 4.4: Fatigue reliability results of Joint A using Approximate damage expectation method

Service life (year)	Reliability index β FORM	SORM	MCS	Probability of failure P_f FORM	SORM	MCS
1	5.069247	5.069215	5.088977	0.000000	0.000000	0.000000
2	4.314691	4.314658	4.322602	0.000008	0.000008	0.000008
3	3.873332	3.873300	3.873826	0.000054	0.000054	0.000054
4	3.560165	3.560133	3.564666	0.000185	0.000185	0.000182
5	3.317178	3.317238	3.319403	0.000455	0.000455	0.000451
6	3.118728	3.118694	3.121674	0.000908	0.000908	0.000899
7	2.950946	2.950912	2.953617	0.001584	0.001584	0.001570
8	2.805610	2.805576	2.807501	0.002511	0.002511	0.002496
9	2.677324	2.677382	2.678186	0.003711	0.003710	0.003701
10	2.562655	2.562618	2.564391	0.005194	0.005194	0.005168
11	2.458925	2.458889	2.460952	0.006968	0.006968	0.006928
12	2.364135	2.364192	2.366460	0.009036	0.009035	0.008980
13	2.277025	2.277081	2.278530	0.011392	0.011391	0.011348
14	2.196375	2.196337	2.198363	0.014033	0.014034	0.013962
15	2.121293	2.121255	2.122695	0.016949	0.016950	0.016890
16	2.050966	2.051021	2.052185	0.020135	0.020132	0.020076
17	1.984994	1.985048	1.986019	0.023573	0.023570	0.023516
18	1.922794	1.922754	1.924303	0.027253	0.027256	0.027158
19	1.863959	1.863919	1.865277	0.031164	0.031167	0.031071
20	1.808050	1.808104	1.809451	0.035300	0.035300	0.035191

the LSF which considers that uncertainty (i.e. using Madsen's method). This result is intuitively easy to understand because the more uncertain input random variables are, the lower reliability index is.

- However, the results show that the differences in reliability indexes are negligible (the maximum difference is about 3×10^{-6} after 1 year). And the difference gets smaller when then the number of cycles gets higher.

- By checking the sensitivity factor of the quantity $\sum_i S_i^m$ in Table 4.5 we can see that it is very small compare to those of others random variables and it is reduced with the increment of number of cycles of fatigue loading. That is to say, for the case of linear S-N curve as Joint A, if the assumption about $\text{Cov}[S_i^m, S_{i+k}^m] \approx 0$ is acceptable then the simplified LSF that ignores the uncertainty of $\sum_i S_i^m$ is justified.

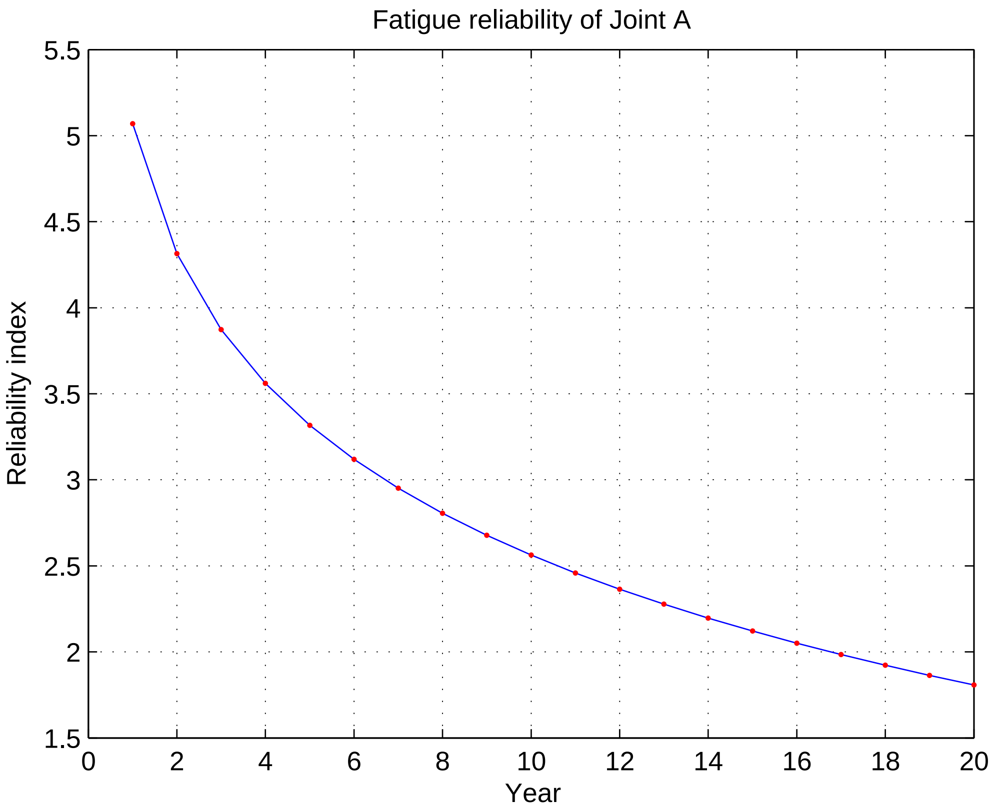

FIGURE 4.5: Result of fatigue reliability calculation
Approximate Damage Expectation method - Joint A

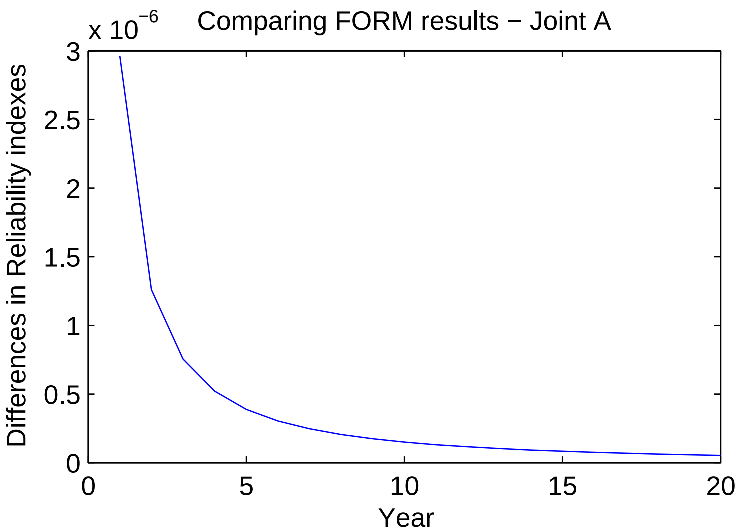

FIGURE 4.6: Variation of the difference
between the FORM results in Table 4.4 with those in Table 4.3 for Joint A

TABLE 4.5: Sensitivity factors - Joint A

Service life (year)	Sensitivity factor of random variable Δ	C	$\sum_i S_i^m$	B_s
1	-0.3200	-0.5012	0.0011	0.8040
2	-0.3200	-0.5012	0.0008	0.8040
3	-0.3200	-0.5012	0.0006	0.8040
4	-0.3200	-0.5012	0.0005	0.8040
5	-0.3200	-0.5012	0.0005	0.8040
6	-0.3200	-0.5012	0.0004	0.8040
7	-0.3200	-0.5012	0.0004	0.8040
8	-0.3200	-0.5012	0.0004	0.8040
9	-0.3200	-0.5012	0.0004	0.8040
10	-0.3200	-0.5012	0.0003	0.8040
11	-0.3200	-0.5012	0.0003	0.8040
12	-0.3200	-0.5012	0.0003	0.8040
13	-0.3200	-0.5012	0.0003	0.8040
14	-0.3200	-0.5012	0.0003	0.8040
15	-0.3200	-0.5012	0.0003	0.8040
16	-0.3200	-0.5012	0.0003	0.8040
17	-0.3200	-0.5012	0.0003	0.8040
18	-0.3200	-0.5012	0.0003	0.8040
19	-0.3200	-0.5012	0.0002	0.8040
20	-0.3200	-0.5012	0.0002	0.8040

4.4 Reliability Calculation on Joint B

4.4.1 The LSF Following Madsen's Method

Similar to Joint A, the simplified limit state function following Madsen's method (with the assumption about the variance $\mathrm{Cov}[S_i^m, S_{i+k}^m]$) is applied to Joint B which is designed using bi-linear S-N curve. The limit state function is shown in Equation (3.11) for basis variables and in Equation (3.42) after transforming into standardized normally distributed u-space. Results of the calculation for Joint B using this limit state function is shown in Table 4.6 and Figure 4.7.

4.4.2 The LSF Using Approximate Damage Expectation

The Approximate damage expectation method is also applied to calculate fatigue reliability of Joint B. The simplified limit state function is written in Equation (3.31) for basis random variables and in Equation (3.44) after transforming into standardized

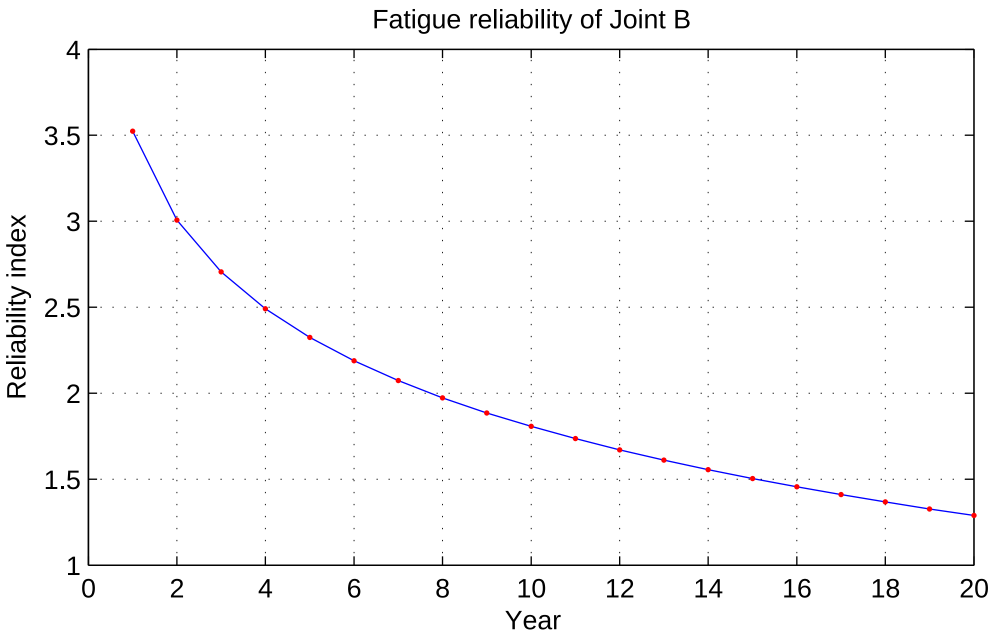

FIGURE 4.7: Result of fatigue reliability calculation using Madsen's method - Joint B

normally distributed u-space. Results of the calculation using this simplified limit state function is shown in Table 4.7 and Figure 4.8.

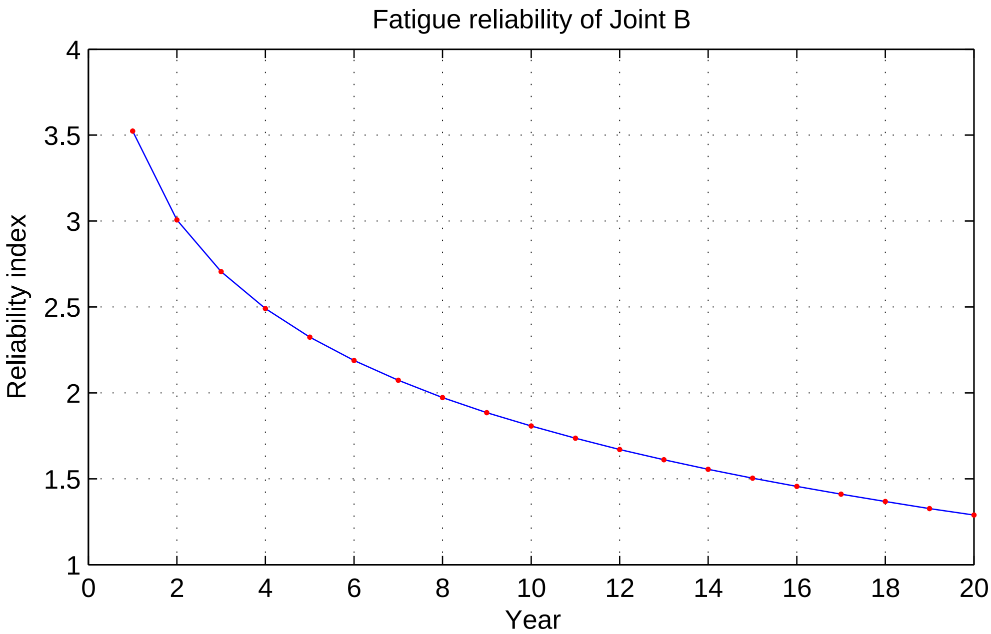

FIGURE 4.8: Result of fatigue reliability calculation using Approximate damage expectation method - Joint B

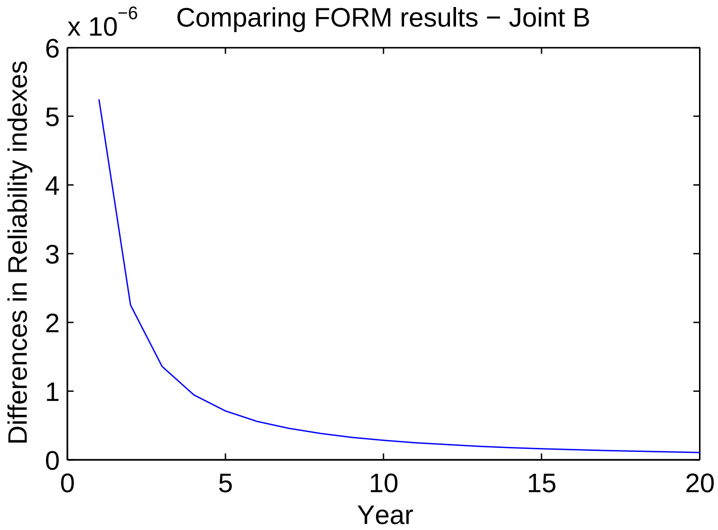

FIGURE 4.9: Variation of the difference between the FORM results in Table 4.7 with those in Table 4.6 for Joint B

4.4.3 Comparison of the two simplified limit state functions

Figure 4.9 shows the difference between the FORM result taken from Table 4.6 with the one taken from Table 4.7. From this figure we can see that:

- Although the maximum difference in case of bi-linear S-N curve (in Figure 4.9 it is about 6×10^{-6}) is higher than the one in the linear case (in Figure 4.6 it is about 3×10^{-6}), they are both negligible when the number of cycles gets higher.
- As a factor to consolidate the disregard for the uncertainty of the quantity $\sum_i S_i^m$, Table 4.8 shows that the sensitivity factors of $\sum_{i=1}^{n1} S_i^{m_1}$ and $\sum_{j=1}^{n2} S_i^{m_2}$ are very small compare to other random variables. And they get smaller when the number of cycles gets larger.

4.5 Verifying numbers of design point

FORM method give only a single design point. It is possible that this given design point is a local optimal solution. To check if that local design point is sufficient as an estimate of global reliability, the simulation method can be used.

Table 4.9 and Figure 4.10 show that the MCS converges when number of simulation reaches $N = 10^8$.

Table 4.6: Fatigue reliability results of Joint B using Madsen's method

Service life	Reliability index β			Probability of failure P_f		
(year)	FORM	SORM	MCS	FORM	SORM	MCS
1	3.523334	3.523415	3.526935	0.000213	0.000213	0.000210
2	3.007079	3.007186	3.007786	0.001319	0.001318	0.001316
3	2.704875	2.705092	2.705742	0.003417	0.003414	0.003408
4	2.490455	2.490590	2.491858	0.006379	0.006377	0.006354
5	2.324026	2.324171	2.324722	0.010062	0.010058	0.010043
6	2.188011	2.188164	2.189478	0.014334	0.014329	0.014281
7	2.072986	2.073146	2.073611	0.019087	0.019079	0.019058
8	1.973233	1.973492	1.974749	0.024235	0.024220	0.024148
9	1.885310	1.885573	1.886564	0.029694	0.029676	0.029609
10	1.806646	1.806819	1.807728	0.035409	0.035395	0.035324
11	1.735474	1.735651	1.736441	0.041328	0.041313	0.041243
12	1.670489	1.670670	1.671651	0.047411	0.047393	0.047297
13	1.610699	1.610883	1.611813	0.053623	0.053603	0.053501
14	1.555242	1.555521	1.556313	0.059944	0.059911	0.059817
15	1.503693	1.503974	1.504727	0.066330	0.066294	0.066197
16	1.455466	1.455655	1.456439	0.072770	0.072744	0.072636
17	1.410158	1.410349	1.411103	0.079246	0.079218	0.079107
18	1.367436	1.367629	1.368269	0.085744	0.085714	0.085614
19	1.326927	1.327214	1.328110	0.092266	0.092219	0.092071
20	1.288581	1.288869	1.289741	0.098772	0.098722	0.098570

Table 4.7: Fatigue reliability results of Joint B
using Approximate damage expectation method

Service life	Reliability index β			Probability of failure P_f		
(year)	FORM	SORM	MCS	FORM	SORM	MCS
1	3.523339	3.523422	3.526935	0.000213	0.000213	0.000210
2	3.007082	3.007189	3.009222	0.001319	0.001318	0.001310
3	2.704876	2.705094	2.706084	0.003416	0.003414	0.003404
4	2.490456	2.490592	2.491671	0.006379	0.006377	0.006357
5	2.324027	2.324172	2.324895	0.010062	0.010058	0.010039
6	2.188012	2.188165	2.189209	0.014334	0.014329	0.014291
7	2.072987	2.073147	2.074075	0.019087	0.019079	0.019036
8	1.973233	1.973492	1.973953	0.024235	0.024220	0.024194
9	1.885310	1.885574	1.886801	0.029694	0.029676	0.029594
10	1.806647	1.806820	1.807744	0.035409	0.035395	0.035323
11	1.735474	1.735652	1.736202	0.041328	0.041313	0.041264
12	1.670489	1.670670	1.671170	0.047411	0.047393	0.047344
13	1.610699	1.610883	1.611869	0.053623	0.053603	0.053495
14	1.555242	1.555522	1.556424	0.059944	0.059911	0.059804
15	1.503693	1.503974	1.504798	0.066330	0.066294	0.066188
16	1.455466	1.455655	1.456969	0.072770	0.072744	0.072563
17	1.410159	1.410349	1.411142	0.079246	0.079218	0.079101
18	1.367436	1.367629	1.368184	0.085744	0.085714	0.085627
19	1.326927	1.327214	1.328032	0.092266	0.092219	0.092084
20	1.288581	1.288869	1.289559	0.098772	0.098722	0.098602

Table 4.8: Sensitivity factors - Joint B

Service life (year)	Sensitivity factor of random variable Δ	C	$\sum_{i=1}^{n1} S_i^{m_1}$	$\sum_{j=1}^{n2} S_i^{m_2}$	B_s
1	-0.218991	-0.342803	0.000237	0.001707	0.913524
2	-0.219150	-0.343044	0.000211	0.001205	0.913396
3	-0.219271	-0.343212	0.000197	0.000983	0.913304
4	-0.219338	-0.343348	0.000188	0.000851	0.913237
5	-0.219411	-0.343461	0.000181	0.000760	0.913177
6	-0.219472	-0.343561	0.000176	0.000694	0.913125
7	-0.219524	-0.343650	0.000171	0.000642	0.913079
8	-0.219600	-0.343728	0.000168	0.000600	0.913031
9	-0.219642	-0.343803	0.000164	0.000566	0.912993
10	-0.219681	-0.343872	0.000161	0.000536	0.912958
11	-0.219717	-0.343937	0.000159	0.000511	0.912925
12	-0.219750	-0.343997	0.000157	0.000489	0.912894
13	-0.219782	-0.344055	0.000154	0.000470	0.912865
14	-0.219840	-0.344107	0.000153	0.000453	0.912831
15	-0.219867	-0.344159	0.000151	0.000437	0.912805
16	-0.219894	-0.344209	0.000149	0.000423	0.912780
17	-0.219919	-0.344257	0.000148	0.000410	0.912756
18	-0.219943	-0.344303	0.000146	0.000399	0.912732
19	-0.219994	-0.344345	0.000145	0.000388	0.912704
20	-0.220015	-0.344388	0.000144	0.000378	0.912683

The probability of failure when MCS converges is $P_f = 0.0352$ which is almost the same as the asymptotic solution in Table 4.3, row $T = 20$ years: $P_f = 0.035217$. This result implies that the FORM method is capable of getting the global optimal design point for the current fatigue problem. The result of SORM stays in between, showing the efficiency of considering the higher order in Taylor expansion for the simplified limit state function.

Use the number of sample $N = 10^8$ to verify reliability index and failure probability from FORM for all the value of service life T. Results are shown in Tables 4.3, 4.4, 4.6 and 4.7.

Table 4.9: Convergence of the Monte Carlo Simulation - Joint A, at T=20 years

N	10^3	10^4	$5{\times}10^4$	10^5	5×10^5	10^6	5×10^6	10^7	5×10^7	10^8
P_f	0.0340	0.0347	0.0355	0.0346	0.0349	0.0352	0.0353	0.0352	0.0352	0.0352

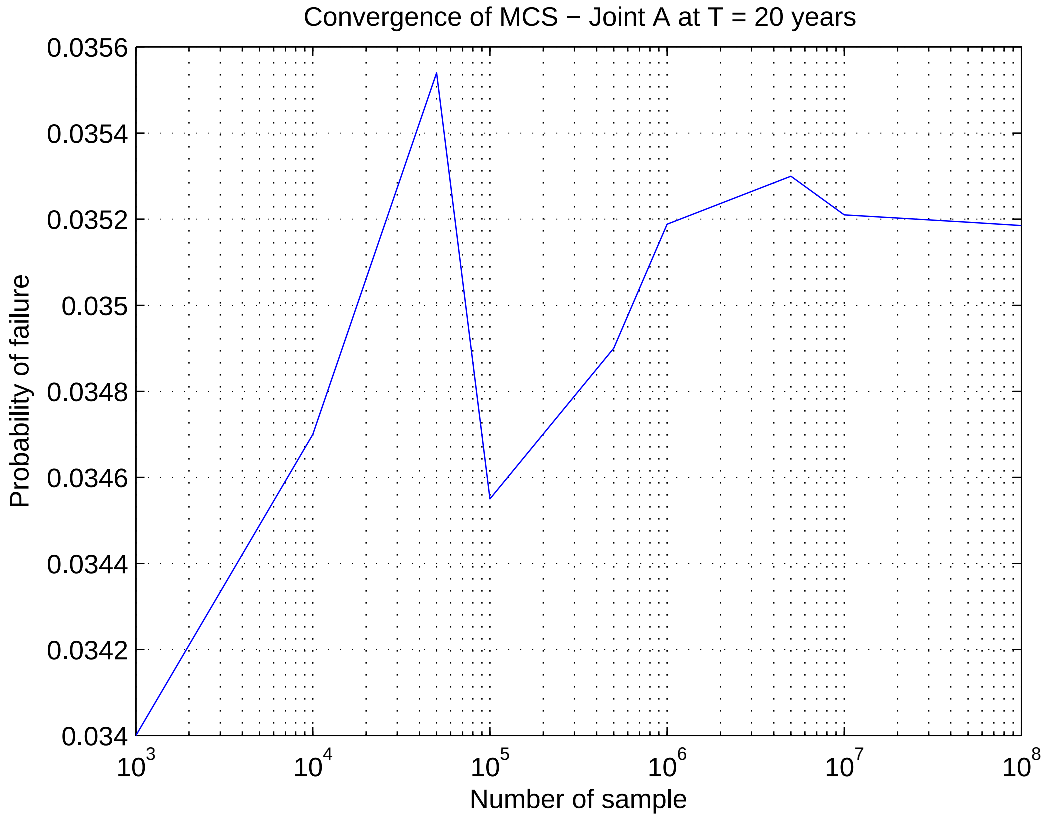

Figure 4.10: Convergence of MCS

4.6 Accuracy of the assumptions concerning the LSFs

4.6.1 Input data for verification

Data for the Monte Carlo simulation in this Section is different from those in sections 4.3 and 4.4. The mean value of C is taken from the statistics of joints working in air condition, while the knee point in the SN curve is taken from the statistics of the joints working in seawater with cathodic protection (see Table 3.2). This weird input data is just because of a mistake in data selection for the knee point in MCS (and because the simulation is very time consuming, restating it with the correct knee point is not a wise choice). However this should not effect the legitimacy of the verification work provided that this input is kept consistent for all the methods in comparison.

The stress-range value at the knee point (of the design S-N curve) is calculated as in equation (4.4).

$$\Delta S_q = \left(\frac{\bar{C}_2}{\bar{C}_1}\right)^{\frac{1}{m_2 - m_1}} = \left(\frac{10^{15.606}}{10^{11.764}}\right)^{\frac{1}{5-3}} = 83.3681 \text{ (MPa)} \tag{4.4}$$

Table 4.10: Input data for verifying the assumptions in the LSFs

Parameter	Dimension	Distribution	Mean	CoV
Δ	-	Lognormal	1	0.3
ν	yr^{-1}	Deterministic	10^7	
T_{SL}	yr	Deterministic	20	
$k(FDF = 2)$	Nmm^{-2}	Deterministic	12.6928	
B_S	-	Lognormal	1	0.25
λ	-	Deterministic	1.2	
C_1	$(\text{Nmm}^{-2})^{m_1}$	Lognormal	4.0743e12	0.486
m_1	-	Deterministic	3	
m_2	-	Deterministic	5	
N_q	-	Deterministic	10^6	

4.6.2 Results of the verification

The Asymptotic solution (FORM) gives the result of fatigue reliability as in Figure 4.11. At $T = 20$ years, the two moments values of the summation random variables $\sum_{i=1}^{n1} S_i^{m_1}$ and $\sum_{j=1}^{n2} S_i^{m_2}$ are shown in Table 4.11.

The results of MCS are also shown in Table 4.11. From Figure 4.14 it is shown that the MCS is converged at $N = 10^5$.

Comparing the values of mean and standard deviation from the two methods we can see that they are almost the same. In addition, Figures 4.12 and 4.13 show that the values of $\sum_{i=1}^{n1} S_i^{m_1}$ and $\sum_{j=1}^{n2} S_i^{m_2}$ from MCS are definitely fitted to normal distribution [1]. That means the assumption $\text{Cov}\left[S_i^m, S_{i+k}^m\right] \approx 0$, which is used to simplify the variance in Equation (3.15) is acceptable.

[1] Using order statistics to draw the relationship between the sorted value of the summation random variable with the corresponding Standard normal quantile. If the points approximately follow a straight line then the normal distribution is a reasonable model for the sample.

TABLE 4.11: Results of the FORM and Monte Carlo method solutions

		FORM	MCS
	Number of cycles	2.0e+08	2.0e+08
N	Number of sample	(no sample)	108032
$\sum_{i=1}^{n1} S_i^{m_1}$	Distribution	Assumed: normal	Verified in Figure 4.12
	Mean	1.0543e+10	1.0543e+10
	Std	9.2459e+07	9.2202e+07
	CoV	0.0088	0.0087
$\sum_{j=1}^{n2} S_i^{m_2}$	Distribution	Assumed: normal	Verified in Figure 4.13
	Mean	1.9479e+15	1.9479e+15
	Std	1.0064e+12	1.0044e+12
	CoV	5.1664e-04	5.1564e-04

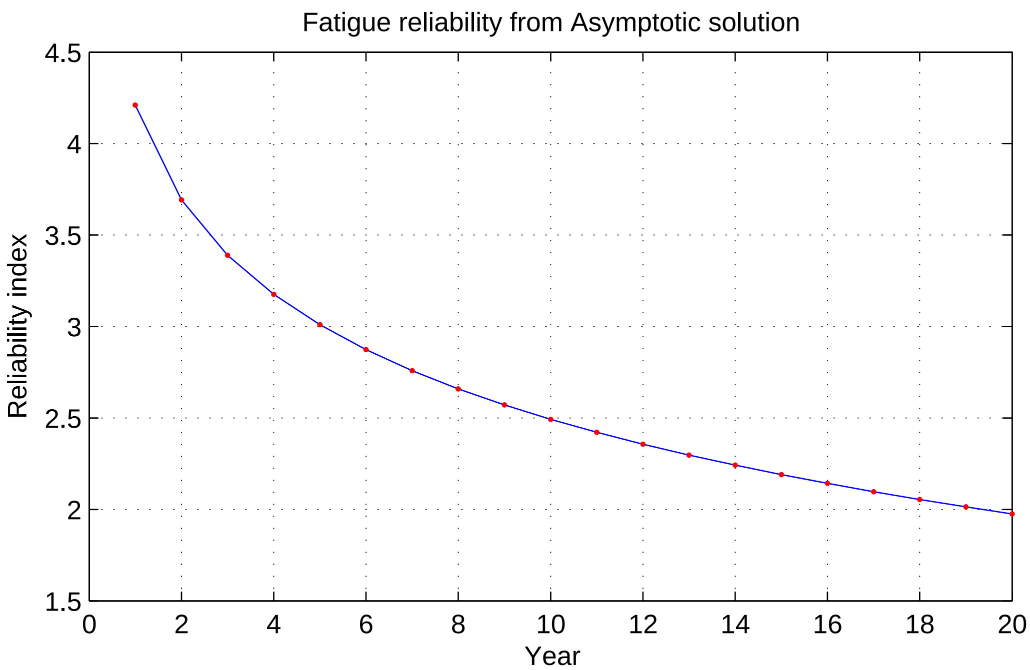

FIGURE 4.11: Result of Asymptotic solution

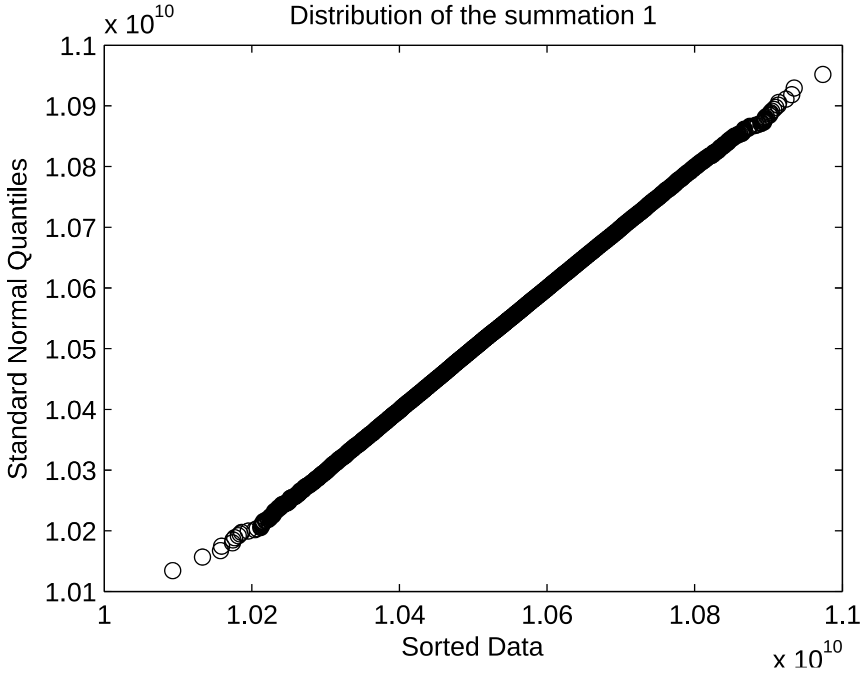

FIGURE 4.12: Verifying if the random variable $\sum_{i=1}^{n1} S_i^{m_1}$ is normally distributed

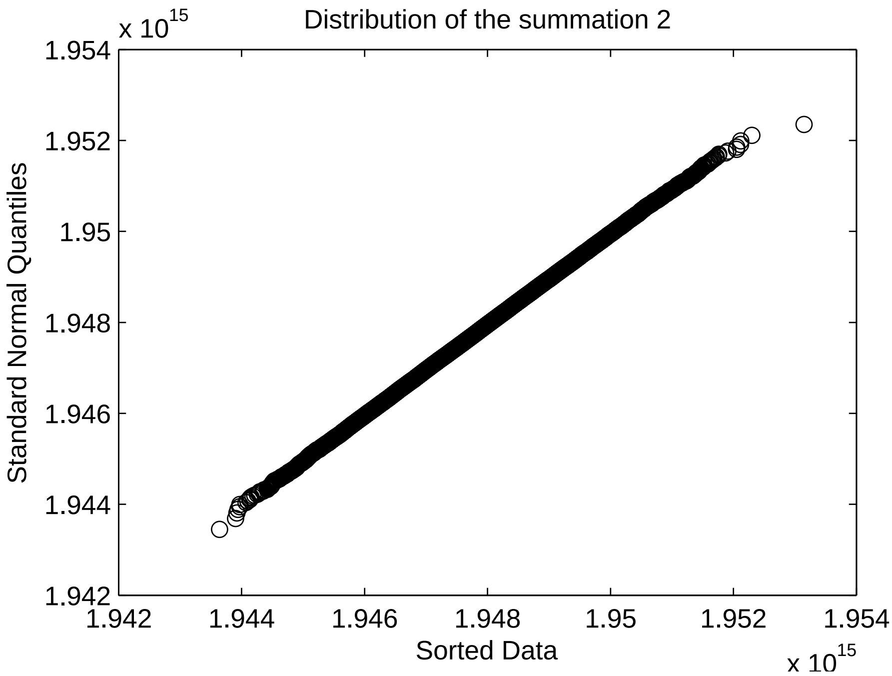

FIGURE 4.13: Verifying if the random variable $\sum_{j=1}^{n2} S_i^{m_2}$ is normally distributed

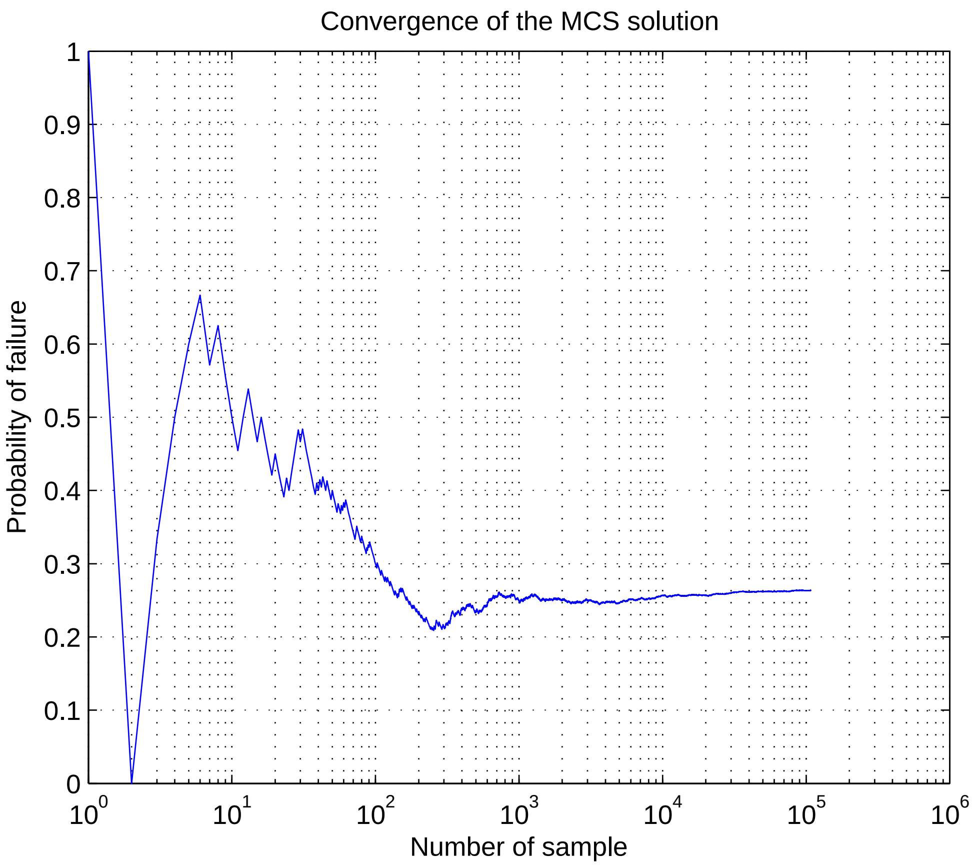

FIGURE 4.14: Convergence of the Monte Carlo Simulation

Chapter 5

Conclusion

Information of fatigue reliability at the design stage is very important for inspection planning and maintenance of existing offshore structures, especially for large offshore wind farms. Due to the empirical nature of fatigue, reliability calculation for welded joints is complicated, and need to be simplified in some steps to be suitable with practical use.

5.1 Simplification of fatigue limit state function

Calculation of fatigue damage needs to deal with the quantity $\sum_{i=1}^{n} S^m$. In fatigue reliability analysis, calculate variance of this quantity is hopeless since the information about the covariance between two certain stress point on the stress process is ignored in rain-flow counting. Fortunately, for offshore structure, the stress cycle rate is quite large $\nu > 10^7$ (year^{-1}), and then the uncertainty of this quantity can be ignored. This conclusion is proved in this thesis (see Chapter 4) through two steps. Firstly, it is shown in Section 4.6 that the variance of the quantity $\sum_{i=1}^{n} S^m$ can be simplified based on the Monte Carlo simulation. Secondly, since the importance factor of this quantity is very small compare to others uncertainty, the "Approximate Damage Expectation method" considers that quantity as deterministic, and as a consequence, the result in comparison with the Madsen's method is almost the same.

5.2 FORM/SORM for the accumulated fatigue problems

Two types of asymptotic solution are applied in fatigue reliability calculation of welded joints in this thesis. They are FORM and SORM, the first order reliability method and the second order reliability method. The results of calculation shows that they work well for the cumulative fatigue problem (i.e. fatigue damage calculation using Miner rule). The results from the asymptotic solutions are verified by comparing with the Monte Carlo simulation method. It shows that the design point found by FORM/SORM are really the global minimal reliability index.

5.3 Perspective

The aim of this master thesis, as an initial step toward inspection planning, is to calculate fatigue reliability of welded joints using SN approach. In later work, this result will be used to find an accurate fracture mechanics (FM) model for updating reliability of existing welded joints in offshore structures. That FM model will be able to update the current probability of failure of a welded joint base on the information about crack size from the inspection data.

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Appendix A

FORM solution for Joint A

```
clc
clear all;
t = cputime;
%-----------------------------------------------
% Random variable \Delta:
mu_Delta = 1;
CoV_Delta = 0.3;
sig_Delta = mu_Delta * CoV_Delta;

%=====
vtem = (sig_Delta/mu_Delta)^2;
sig_L_Delta = sqrt(log(vtem + 1));
mu_L_Delta = log(mu_Delta) - 0.5*sig_L_Delta^2;

%-----------------------------------------------
% Random variable \BS:
mu_BS = 1;
CoV_BS = 0.25;
sig_BS = mu_BS * CoV_BS;

%=====
vtem = (sig_BS/mu_BS)^2;
sig_L_BS = sqrt(log(vtem + 1));
mu_L_BS = log(mu_BS) - 0.5*sig_L_BS^2;
%-----------------------------------------------
% Random variable C:
mu_C = 1.3585e12;
CoV_C = 0.486;
sig_C = mu_C * CoV_C;

%=====
```

```
vtem1 = (sig_C/mu_C)^2;
sig_L_C = sqrt(log(vtem1 + 1));
mu_L_C = log(mu_C) - 0.5*sig_L_C^2;

%----------------------------------------------
 nu = 1e7;
 %----
 B = 1.2;

 A = 7.152;
 %----
 %C1 = 10^11.764;
 m = 3;
 Mean_sum = A^(m)* gamma(1+m/B);

%----------------------------------------------
%----------------------------------------------

syms x y u
f=@(x,y,u) exp(x*sig_L_Delta + mu_L_Delta)-...
    nu_T_sum*(exp(u*sig_L_BS + mu_L_BS))^m/exp(y*sig_L_C + mu_L_C);
kq=[diff(f,x) diff(f,y) diff(f,u)];
for j=1:1:20
T(j) = j;
nu_T_sum=nu*T(j)*Mean_sum;

xyz=[0 0 0];

ss=1;
i=1;
beta(i) = norm(xyz);
while abs(ss)>0.0001
    i=i+1
    grad=subs(kq,[x y u],xyz);
    seVar = subs(f,[x y u],xyz);
    seVar = subs(seVar);
    alpha =-grad./ norm(grad);
    xyz   = (xyz*alpha')*alpha + (seVar/norm(grad)/16)*alpha;
    beta(i) = norm(xyz);
    ss = beta(i)-beta(i-1);
    kq_xyz(i,:) = xyz;
end
kq_beta(j)=beta(i);

end

solving_time = cputime-t
figure;
```

```
plot(T,kq_beta,T,kq_beta,'r.')
xlabel('Year')
ylabel('Reliability index')
```

Appendix B

SORM solution for Joint B

```
clc
clear all;
t = cputime;
%-----------------------------------------------
% Random variable \Delta:
mu_Delta = 1;
CoV_Delta = 0.3;
sig_Delta = mu_Delta * CoV_Delta;

%=====
vtem = (sig_Delta/mu_Delta)^2;
sig_L_Delta = sqrt(log(vtem + 1));
mu_L_Delta = log(mu_Delta) - 0.5*sig_L_Delta^2;

%-----------------------------------------------
% Random variable \BS:
mu_BS = 1;
CoV_BS = 0.25;
sig_BS = mu_BS * CoV_BS;

%=====
vtem = (sig_BS/mu_BS)^2;
sig_L_BS = sqrt(log(vtem + 1));
mu_L_BS = log(mu_BS) - 0.5*sig_L_BS^2;




%-----------------------------------------------
% Random variable C:
mu_C = 1.6220e12;
CoV_C = 0.486;
```

```
sig_C = mu_C * CoV_C;

%=====
vtem1 = (sig_C/mu_C)^2;
sig_L_C = sqrt(log(vtem1 + 1));
mu_L_C = log(mu_C) - 0.5*sig_L_C^2;

%-----------------------------------------------
% Random variable \sum:

 nu = 1e7;
 %----
 B = 1.2;
 A = 12.6890;
 %----
 C1 = 10^11.764;
 C2 = 10^15.606;
 m1 = 3;  m2 = 5;

 Sq = (C2/C1)^(1/(m2-m1));



 %----
 % The two sum random variables:

 Mean_sum_Sm1 = A^(m1)* gamma(1+m1/B)*...
     gammainc((Sq/A)^B, 1+m1/B,'upper') ;

 Mean_sum_Sm2 = A^(m2)* gamma(1+m2/B)*(1 -  ...
     gammainc((Sq/A)^B, 1+m2/B,'upper'));

 Var_sum_Sm1 = A^(2*m1)* (gamma(1+2*m1/B)*...
     gammainc((Sq/A)^B, 1+2*m1/B,'upper') - ...
    (gamma(1+m1/B)*gammainc((Sq/A)^B, 1+m1/B,'upper'))^2);

 Var_sum_Sm2 = A^(2*m2)* gamma(1+2*m2/B)*...
     (1 -gammainc((Sq/A)^B, 1+2*m2/B,'upper'))...
    -A^(2*m2)*(gamma(1+m2/B))^2 *(1- gammainc((Sq/A)^B, 1+m2/B,'upper'))^2;




%-----------------------------------------------
%-----------------------------------------------

syms x y z1 z2 u
f=@(x,y,z1,z2,u) exp(x*sig_L_Delta + mu_L_Delta)-...
    (exp(u*sig_L_BS + mu_L_BS))^m1*...
    (z1*sig_sum1 + mu_sum1)/exp(y*sig_L_C + mu_L_C) - ...
```

```
    (exp(u*sig_L_BS + mu_L_BS))^m2*...
    (z2*sig_sum2 + mu_sum2)/exp(y*sig_L_C + mu_L_C)/Sq^(m2-m1);
kq=[diff(f,x) diff(f,y) diff(f,z1) diff(f,z2) diff(f,u)];
Hess=[diff(kq(1),x) diff(kq(1),y)  diff(kq(1),z1) ...
    diff(kq(1),z2) diff(kq(1),u);...
      diff(kq(2),x) diff(kq(2),y)  diff(kq(2),z1) ...
      diff(kq(2),z2) diff(kq(2),u);...
      diff(kq(3),x) diff(kq(3),y)  diff(kq(3),z1) ...
      diff(kq(3),z2) diff(kq(3),u);...
      diff(kq(4),x) diff(kq(4),y)  diff(kq(4),z1) ...
      diff(kq(4),z2) diff(kq(4),u);...
      diff(kq(5),x) diff(kq(5),y)  diff(kq(5),z1) ...
      diff(kq(5),z2) diff(kq(5),u)];
for j=1:1:20
T(j) = j;
mu_sum1 = r1*nu*T(j)* Mean_sum_Sm1;
sig_sum1 = (r1*nu*T(j)* Var_sum_Sm1)^0.5;

mu_sum2 = r2*nu*T(j)* Mean_sum_Sm2;
sig_sum2 = (r2*nu*T(j)* Var_sum_Sm2)^0.5;

xyz=[0 0 0 0 0];
ss=1;
i=1;
beta(i) = norm(xyz);
while abs(ss)>0.0001
    i=i+1
    grad=subs(kq,[x y z1 z2 u],xyz);
    seVar = subs(f,[x y z1 z2 u],xyz);
    seVar = subs(seVar);
    alpha =-grad./ norm(grad);
    xyz   = (xyz*alpha')*alpha + (seVar/norm(grad)/16)*alpha;
    beta(i) = norm(xyz);



    val_Hess=subs(Hess,[x y z1 z2 u],xyz);
    val_Hess=-(norm(grad))^(-1) .* subs(val_Hess);



    f1 = alpha;
    f2 = [0 1 0 0 0];
    f3 = [0 0 1 0 0];
    f4 = [0 0 0 1 0];
    f5 = [0 0 0 0 1];

    D1 = (f1*f1')^0.5;
```

```
    e11 = 1/D1;
    gama1 = e11*f1

    D2 = ((f2*f2')-(f2*gama1')^2)^0.5;
    e12 = -(f2*gama1')/D2;
    e22 = 1/D2;
    gama2 = e12*gama1 + e22*f2;

    D3 = ((f3*f3')-(f3*gama1')^2 - (f3*gama2')^2)^0.5;
    e13 = -(f3*gama1')/D3;
    e23 = -(f3*gama2')/D3;
    e33 = 1/D3;
    gama3 = e13*gama1 + e23*gama2 + e33*f3;

    D4 = ((f4*f4')-(f4*gama1')^2 -(f4*gama2')^2 - (f4*gama3')^2)^0.5;
    e14 = -(f4*gama1')/D4;
    e24 = -(f4*gama2')/D4;
    e34 = -(f4*gama3')/D4;
    e44 = 1/D4;
    gama4 = e14*gama1 + e24*gama2 + e34*gama3 + e44*f4;

    D5 = ((f5*f5')-(f5*gama1')^2 -(f5*gama2')^2 -...
        (f5*gama3')^2 - (f5*gama4')^2)^0.5;
    e15 = -(f5*gama1')/D5;
    e25 = -(f5*gama2')/D5;
    e35 = -(f5*gama3')/D5;
    e45 = -(f5*gama4')/D5;
    e55 = 1/D5;
    gama5 = e15*gama1 + e25*gama2 + e35*gama3 + e45*gama4 + e55*f5;

    H = [gama2; gama3; gama4; gama5; gama1];

    HBH = H*val_Hess*H'


    %kata=eig(HBH);


%     J_temp = 1-kata(1);
    J_temp1 = (1+beta(i)*HBH(1,1));
    J_temp2 = (1+beta(i)*HBH(2,2));
    J_temp3 = (1+beta(i)*HBH(3,3));
    J_temp4 = (1+beta(i)*HBH(4,4));
    J = J_temp1*J_temp2*J_temp3*J_temp4;
    J = J^(-0.5);
%     for k=2:length(kata)-1
%         J_temp=J_temp*(1-kata(k));
%     end
```

```
%      J=J_temp^(-0.5);
    % update beta:

    P=normcdf(-beta(i))*J;
    beta(i)=-norminv(P);

    ss = beta(i)-beta(i-1);
    %kq_xyz(i,:) = xyz;
end
kq_beta(j)=beta(i);
sen_fact(j,:)=alpha;
end
save('SORM_Mad_JointB.mat','kq_beta');
save('senstivity_Mad_SORM.mat','sen_fact');
% plot3(kq_xyz(:,1),kq_xyz(:,2),kq_xyz(:,3))
solving_time = cputime-t
figure;
plot(T,kq_beta,T,kq_beta,'r.')
title('Fatigue reliability of Joint B')
xlabel('Year')
ylabel('Reliability index')
```

Appendix C

MCS for verifying global minimal reliability index

```
clc
clear all;
t = cputime;

%-----------------------------------------------
% Random variable \Delta:
mu_Delta = 1;
CoV_Delta = 0.3;
sig_Delta = mu_Delta * CoV_Delta;

%=====
vtem = (sig_Delta/mu_Delta)^2;
sig_L_Delta = sqrt(log(vtem + 1));
mu_L_Delta = log(mu_Delta) - 0.5*sig_L_Delta^2;

%-----------------------------------------------
% Random variable \BS:
mu_BS = 1;
CoV_BS = 0.25;
sig_BS = mu_BS * CoV_BS;

%=====
vtem = (sig_BS/mu_BS)^2;
sig_L_BS = sqrt(log(vtem + 1));
mu_L_BS = log(mu_BS) - 0.5*sig_L_BS^2;
```

```
%------------------------------------------------
% Random variable C:
mu_C = 1.6220e12;
CoV_C = 0.486;
sig_C = mu_C * CoV_C;

%=====
vtem1 = (sig_C/mu_C)^2;
sig_L_C = sqrt(log(vtem1 + 1));
mu_L_C = log(mu_C) - 0.5*sig_L_C^2;

%------------------------------------------------



 nu = 1e7;
 %----
 B = 1.2;

 A = 17.7662;
 %----
C1 = 10^11.764;
 C2 = 10^15.606;
 m1 = 3;  m2 = 5;

 Sq = (C2/C1)^(1/(m2-m1));



 r2 = exp(-(Sq/A)^B);
 r1 = 1 - r2;
 %----



Mean_sum1 = A^(m1)* gamma(1+m1/B)* gammainc((Sq/A)^B, 1+m1/B,'upper');
Mean_sum2 = A^(m2)* gamma(1+m2/B)*(1-gammainc((Sq/A)^B,1+m2/B,'upper')) ;

TT=20;
%------------------------------------------------
ten_file1 = strcat('All_',num2str(TT),'years_result_MCS_Mad_bi.txt')
ten_file2 = strcat('All_',num2str(TT),'years_rand_state.mat')
ten_file3 = strcat('All_',num2str(TT),'years_randn_state.mat')
fileID = fopen(ten_file1,'w');
for j=1:1
    T=j;
    nu_T_sum1=r1*nu*T*Mean_sum1;
    nu_T_sum2=r2*nu*T*Mean_sum2;


m=1e8;
```

```
nn=10000;
qq = m/nn;
C=zeros(qq,nn);
Delta=zeros(qq,nn);
BS=zeros(qq,nn);
Sum_Sm1=zeros(qq,nn);
Sum_Sm2=zeros(qq,nn);
parfor k=1:qq
C_tam = lognrnd(mu_L_C,sig_L_C,1,nn);
C(k,:)=C_tam;
C_tam=[];
Delta_tam = lognrnd(mu_L_Delta,sig_L_Delta,1,nn);
Delta(k,:)=Delta_tam;
Delta_tam=[];
BS_tam = lognrnd(mu_L_BS,sig_L_BS,1,nn);
BS(k,:)=BS_tam;
BS_tam=[];

end

% Calculating damage:
    D_tot = nu_T_sum1 * (BS.^m1)./C + nu_T_sum2 * (BS.^m2)./C/Sq^(m2-m1);

    BS=[];
    C=[];

    g = Delta - D_tot;
    Delta=[];
    D_tot=[];

dem = sum(sum(logical(g<0)));
g=[];
Pf(j) = dem/m
dem=[];
bt(j) = -norminv(Pf(j));
fprintf(fileID,'%8.6f%12.6f\r\n',bt(j),Pf(j));
end
fclose(fileID);


solving_time = cputime-t
```

Appendix D

MCS for verifying assumptions in LSF

```
clc;
close all;
clear all;
t = cputime;

load state_20year_rand % St1
load state_20year_randn % St2
rand('state',St1);
randn('state',St2);


 C1 = 10^11.764;
 C2 = 10^15.606;

 m1 = 3; m2 = 5;

 DSq = (C2/C1)^(1/(m2-m1));
 DSo =0;

 %DSq = (C1/Nq)^(1/m1);
 %DSo = (DSq^m2 * Nq/No)^(1/m2);

% % % DSq=0;
% % % DSo=0;

 B = 1.2;

 A = 12.6928;
```

```
 T=20; % service life = 10 years
 nu = 1e7; % cycle rate per year - IS IT A FIXED VALUE?


 n=nu*T;
 m = 100000;

% C random variable:
    Cov_C = 0.486;
    m_C = 4.0743e12;
    v_C = m_C^2 * Cov_C^2;

    mu_C = log((m_C^2)/sqrt(v_C+m_C^2));
    sig_C = sqrt(log(v_C/(m_C^2)+1));

% Delta random varialbe:
    Cov_Del = 0.3;
    m_Del = 1;
    v_Del = m_Del^2 * Cov_Del^2;

    mu_Del = log((m_Del^2)/sqrt(v_Del+m_Del^2));
    sig_Del = sqrt(log(v_Del/(m_Del^2)+1));

% Bs random varialbe:
    Cov_BS = 0.25;
    m_BS = 1;
    v_BS = m_BS^2 * Cov_BS^2;

    mu_BS = log((m_BS^2)/sqrt(v_BS+m_BS^2));
    sig_BS = sqrt(log(v_BS/(m_BS^2)+1));


sd = T*nu/100000;

D1 = zeros(1,sd);
D2 = zeros(1,sd);
ten_file1 = strcat(num2str(T),'years_kqua2.txt')
ten_file2 = strcat(num2str(T),'years_rand_state.mat')
ten_file3 = strcat(num2str(T),'years_randn_state.mat')
fileID = fopen(ten_file1,'w');

for i=1:m
    St1=rand('state');
    St2=randn('state');

    save(ten_file2,'St1');
    save(ten_file3,'St2');
```

```
    i
    %rand('state',i);

% Generating stress-ranges:
    % DS = wblrnd(A,B,1,n);

% Generating BS:
    % BS = lognrnd(mu_BS,sig_BS);
    % BS = lognrnd(mu_BS,sig_BS,1,n);
% Generating C:
    C = lognrnd(mu_C,sig_C);
% Generating Delta:
    Delta = lognrnd(mu_Del,sig_Del);
    BS = lognrnd(mu_BS,sig_BS);

% Calculating damage:

parfor j=1:sd
     % rand('state',(j-1)*100000);
     DS = wblrnd(A,B,1,100000); % the best value is 1e6 for this generator

     ind_lower = logical(DS < DSq);
     ind_upper = logical(DS > DSq);

     D1(j) = sum((DS(ind_upper).^m1));
     D2(j) = sum((DS(ind_lower).^m2));
     D(j) = D1(j)*BS^m1/C1 + D2(j)*BS^m2/C1/DSq^(m2-m1);
end


% Calculating values of the Limit State Function:

    g = Delta - sum(D);

    q1=sum(D1);
    q2=sum(D2);
    q3=sum(D);


     fprintf(fileID,'%12.8f %12.8f %12.8f %12.8f\r\n',g, q1, q2,q3);

end

fclose(fileID);

solving_time = cputime-t
```