Efficient Computation of the Extrema of Algebraic Quality Measures for Curvilinear Finite Elements

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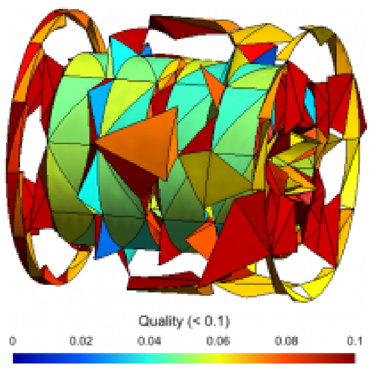
ABSTRACT

The development of high-order computational methods for solving partial differential equations on unstructured grids has been underway for many years. Such methods critically depend on the availability of high-quality curvilinear meshes, as one bad element can degrade the solution in the whole domain [1].

The usual way of generating curved meshes is to first generate a (high-quality) straight-sided mesh. Then, mesh entities that are classified on the boundaries of the domain are curved. This operation introduces a “shape-distortion” that should be controlled. Quality measures allow to quantify to which point an element is well-shaped [2]. They also provide tools to improve the quality of meshes through optimization [5].

In this work we propose an efficient method to compute several quality measures for curved elements, based on the Jacobian of the mapping between the straight-sided elements and the curved ones. Contrary to the approach presented in [4], which relies on an L^2-norm over the elements, we compute the actual minimum and maximum of the local quality measure for each element. The method is an extension of previous works on the validity of those elements [3]. The key feature is that we can adaptively expand functions based on the Jacobian matrix and its determinant in terms of Bézier functions. Bézier functions have both properties of boundedness and positivity, which allow sharp computation of minimum and maximum of the interpolated functions.



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