Wake-Induced Vibration in Power Transmission Line. Parametric study

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ABSTRACT

The paper discusses some aspects of modeling the wake-induced vibrations of the bundle conductors in transmission lines.

The wake coupling between subconductors is modeled with the modified Simpson's approach. In order to apply this model to the analysis of power transmission line (PTL), a modal representation of a span could be used. In present paper we study how application of Dynamic Reduction (Component Mode) technique may improve such analysis under finite-element approach. The present paper, to extent of authors' knowledge, is the first where Component Mode method application is discussed to study the subspan oscillation.

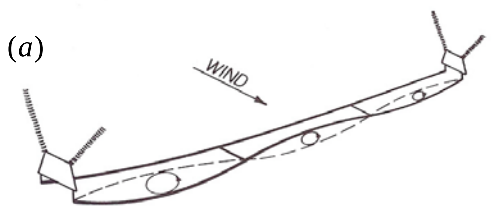
Finally, bundled conductors' susceptibility to wake-induced vibrations is illustrated on an exemplary twin-bundle, three-subspan model. Effects of bundle parameters (initial subconductor spacing, frequency ratio, subconductor mass), and the means to account them when applying the component mode method are discussed.

1. INTRODUCTION

Studies of the wake-induced vibrations in electrical conductors have been actively conducted since mid 60's and across seventies, giving rise to basic understanding of this spectacular phenomenon. Having successfully applied flutter theory to describe instability of leeward conductor in the bundle Simpson (1971/2) has established a range of parameters playing key role in triggering the oscillations. He also illustrated that undamped flutter theory, based on aerodynamics of the flow in the wake, provides necessary conditions of instability.

Effect of the fluid field on the character of instability has been outlined in numerous studies. Wardlaw et al. (1975) measured the aerodynamic loads in the multiple bundles where a leeward conductor is subject to a superposition of wake loads from several conductors. Then the leeward conductor may be found in conditions similar to the tube patterns (e.g. heat exchangers) and studied by Price and Païdoussis (1984) and by Hémon (1999). In these works it was put special attention to accounting the time delay between the motions of neighboring cylinders. However, in bundle conductors accounting the time delay is not as significant, since the frequency of excited eigenmodes is quite low. As shown by Cigada et al. (1995), quasi-stationary hypothesis of the flutter theory is fully applicable to the bundle conductors within the conditions, where wake-induced vibrations are observed:

-wind speed: 7…15 m/s;
-bundle separation 10… 20 conductor diameters;
-conductor mass per unit length: 1.2 … 1.6 kg/m;
- oscillation frequencies: 0.5 … 3 Hz.



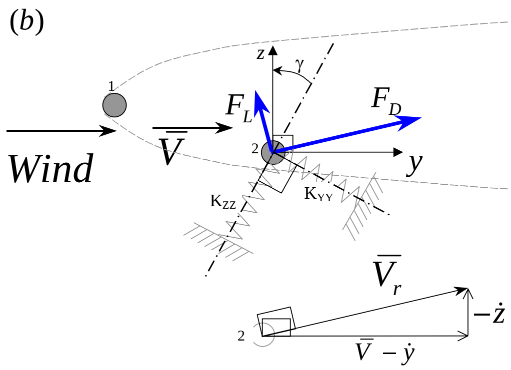


Fig.1. (a) Typical mode of wake-induced oscillation in the conductor bundle;
(b) Cable bundle's model for study of the wake-induced instability

2. MODEL OF WAKE-INDUCED INSTABILITY

To recall the basic ideas, consider a two-degree-of-freedom system shown in Fig.1. Here, the bundle is reduced to the fixed windward conductor and elastically suspended leeward cylinder. Oscillations of the latter around equilibrium position are described by following equation:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \{\ddot{y} \quad \ddot{z}\}^T + q\frac{d}{\overline{V}} \begin{bmatrix} 2C_D & -C_L \\ 2C_L & C_D \end{bmatrix} \{\dot{y} \quad \dot{z}\}^T + \left[\begin{bmatrix} k_{yy} & k_{yz} \\ k_{yz} & k_{zz} \end{bmatrix} - q \begin{bmatrix} C_{D_Y} & C_{D_Z} \\ C_{L_Y} & C_{L_Z} \end{bmatrix} \right] \{y \quad z\}^T = \begin{bmatrix} F_D \\ F_L \end{bmatrix} \quad (1)$$

Here,

q stands for aerodynamic pressure, $kg/(m \cdot s^2)$;

y, z are components of small displacement, m (dots denote time derivatives);

d is the conductor's diameter, m;

$\overline{V}$ is the local wind speed at current position of leeward cylinder in the wake, *m/s*;

k_{yy}, k_{zz}, k_{yz} are the terms of axial and coupled stiffnesses per unit length, *N/m*, - see (3);

m is the mass per unit length, kg/m;

F_D, F_L are the aerodynamic forces, N/m;

$C_D, C_L, C_{Dy}, C_{Ly}, C_{Dz}, C_{Lz}$ are aerodynamic coefficients (dimensionless) and their derivatives vs. displacement components (*1/m*$^{-1}$).

By equating right-hand part of (1) to zero, providing aerodynamic terms from wind tunnel test data and passing to dimensionless displacements, we may find numerically (via system's roots) or semi-analytically (via Routh test functions T_3 (Simpson, 1971) or T_2 (Kern, 1995)) the system's eigenvalues whose real part vanishes to zero at the boundary of instability.

On basis of (1) an important condition of undamped flutter theory may be obtained, that wake-induced instability of system is possible as the secondary diagonal terms in aerodynamic stiffness matrix become of opposite signs:

$$C_{L_Y} C_{D_Z} < 0 \quad (2)$$

Being an indicator of unstable zones in the wake, this condition does not explain yet, why these zones are asymmetrical with respect to wind axis. As is known (EPRI, 1979), electrical conductor is prone to wake-induced vibrations only in the lower half-wake. It was found after wind-tunnel tests and shown with the aid of (1), that asymmetry is imparted by static coupling between suspension's modes, which in reality belongs to a blowback of conductor under the wind (Tsui, 1977):

$$\begin{bmatrix} k_{YY} & k_{YZ} \\ k_{YZ} & k_{ZZ} \end{bmatrix} = R \begin{bmatrix} K_{YY} & 0 \\ 0 & K_{ZZ} \end{bmatrix} R^T \quad (3)$$

Another important condition related to modal properties of structure is that initial (without wind) eigenfrequencies of two interacting eigenmodes should be separated (of the order $\omega_Z / \omega_Y \sim 1.06 \ldots 1.1$). As shown in Fig. 2, this condition, fully appropriate to flutter instability, presumes that eigenfrequencies at zero wind must be distant enough so that, upon contribution of aerodynamic stiffness and damping terms, the unstable mode might arise.

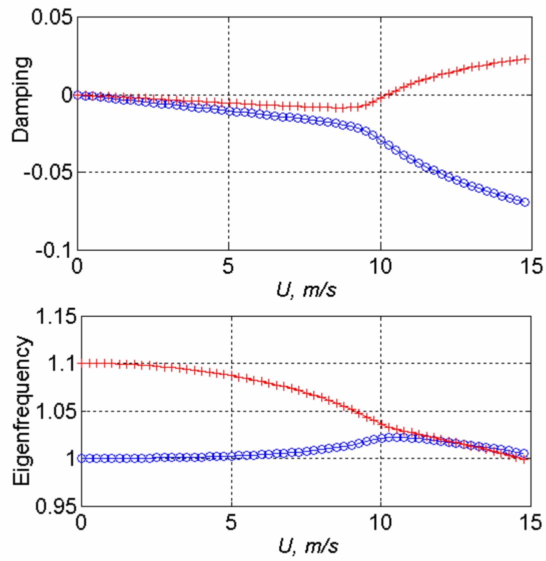


Fig. 2. Variation of 2-dof system eigenvalues on the wind speed. '+' marker corresponds to the in-plane, or "vertical", mode; 'o' marker – to out-of-plane, or "horizontal" mode.

3. APPLICATION TO THE SUBSPAN INSTABILITY

Above approach is, in general, easy to apply throughout various techniques, including finite-element method. Its direct implementation to the model of the line span already makes it possible to obtain its complex modes whose real part can characterize if the span is stable or not. However, such an approach implies certain numerical difficulties, e.g. at the stage of extracting the modeshapes. Also, the span system is treated in a global manner, and results may hardly indicate what are stability conditions in a particular subconductor of a particular subspan.

To give finite element model a more clear meaning in the stability study, we turn to the representation of structure on basis of Component Mode (or Craig-Bampton) method. This widespread technique for reduction of the order of finite-element model in both linear and nonlinear problems is especially attractive in the scope of our study, due to possibility of compact representation of structure through its modal content.

3.1 Component Mode method

After extracting the normal modes of structure and expressing the internal degrees of freedom via modal coordinates, and boundary DOF via static modes, one obtains the super element stiffness and mass matrices as:

$$K_R = \begin{bmatrix} \bar{K}_{II} & \bar{K}_{IF} \\ \bar{K}_{IF} & \bar{K}_{FF} \end{bmatrix}; \quad M_R = \begin{bmatrix} \bar{M}_{II} & \bar{M}_{IF} \\ \bar{M}_{IF} & \bar{M}_{FF} \end{bmatrix} \qquad (4)$$

The portions $\bar{K}_{FF}$ and $\bar{M}_{FF}$ (call it modal matrices) are especially important to us, as they result from left-right product of original matrices of subconductor (normal degrees of freedom) and of normal eigenmodes:

$$\bar{K}_{FF} = \phi^T K_{FF} \phi \qquad (5)$$

Known that only two normal modes contribute into the wake-induced flutter, we aim at obtaining the stiffness and mass modal matrices of each subconductor of the order 2x2. Furthermore, we need from another subconductor only the information about its relative position, and only to the extent of obtaining aerodynamic stiffness and damping terms. At the stage of super element construction, the remaining part of line span may be omitted. However, we keep in parallel with the subconductor's component modes, all necessary aerodynamic terms assembled in 2x2 matrices, C_{aer} and K_{aer} for each subconductor and for each subspan.

The dynamic reduction flowchart applied to a subconductor is shown in Fig. 3. The retained boundary nodes of subconductor are its span connections, as well as the points of fixation to the spacers. We look for the pairs of basic in-plane and out-of-plane subspan eigenmodes. Bringing them together, we come to the following modal stiffness matrix of subconductor :

$$\bar{K}_{FF} = \begin{bmatrix} \begin{bmatrix} \bar{K}_{11} & 0 \\ 0 & \bar{K}_{21} \end{bmatrix} & 0 & 0 \\ & \cdots & 0 \\ sym & & \begin{bmatrix} \bar{K}_{1k} & 0 \\ 0 & \bar{K}_{2k} \end{bmatrix} \end{bmatrix}$$

Here, each 2x2 block represents the in-plane and out-of-plane stiffnesses of orthogonal modes in subspans.

By analogy to the Simpson's approach, we should take into account the stiffnesses' cross-coupling via the blowback angle (3). However, instead of applying to diagonal stiffness matrix, the back-transformation can be done to the aerodynamic stiffness and damping terms:

$$\begin{aligned} \bar{C}_{aer} &= R^T C_{aer} R \\ \bar{K}_{aer} &= R^T K_{aer} R \end{aligned} \qquad (6)$$

Here,

$$R = \begin{bmatrix} \begin{bmatrix} \cos\gamma_1 & -\sin\gamma_1 \\ \sin\gamma_1 & \cos\gamma_1 \end{bmatrix} & 0 & 0 \\ & \cdots & 0 \\ sym & & \begin{bmatrix} \cos\gamma_k & -\sin\gamma_k \\ \sin\gamma_k & \cos\gamma_k \end{bmatrix} \end{bmatrix}$$

is a rotation matrix made with blowback angles in each subspan.

Thus, we keep the diagonal shape of structural stiffness matrix at this stage. Resulting system of equations for studying dynamic stability will now read (*r* denoting the vector of small displacements, in the sense of normal, or modal, coordinates for summary of subspans):

$$\bar{M}_{FF}\ddot{r} + \bar{C}_{aer}\dot{r} + \left[\bar{K}_{aer} + \bar{K}_{FF}\right] r = 0 \qquad (7)$$

3.1.1 Account of spacer flexibility

Matrix of subconductor's component modes, obtained via the super element provides necessary information for stability study but is always diagonal. From the modeling point of view, diagonal matrix indicates perfectly rigid spacers. If a flexible spacer or spacer-damper must be considered in the models, the approach is straightforward. The stiffness properties enter the matrix in order to couple respective degrees of freedom. Thus, we can proceed just in the same manner as in transfer matrix method forwarded by

Rawlins (1977) and Claren et al. (1971).

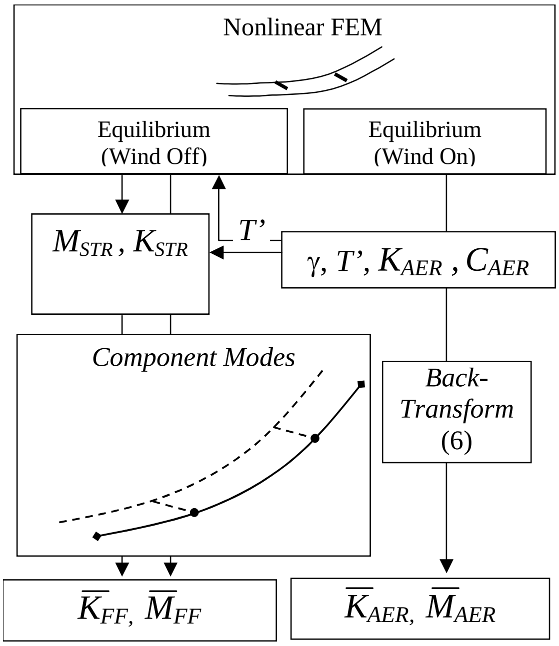


Fig.3. Flowchart for extraction of aerodynamic and modal matrices to equation (7).

4. ANALYSIS EXAMPLE

Consider a span made of a bundle of two conductors. The span length is 120 m; conductor is ASTER-570 (diameter 30.5 mm) Initial tension in subconductors is 7.55 kN.

Initial separation of subconductors in the bundle was 10 diameters (in horizontal) and –2.2 diameters in vertical direction, so that leeward conductor is found in unstable position. However, the wind velocity was selected not too high (10 m/s).

The bundle is composed of three subspans. In basic case all subspans are 40 m long. We shall vary the position of spacers in a way that the length of a middle subspan will remain constant.

Following above chart, we compute in the beginning of each case the bundle's equilibrium position to obtain the generalized 2x2 aerodynamic matrices. We then re-compute the bundle's static position without the wind, however, we retain the tension in subconductors obtained in previous stage. This involves a certain simplification to the strained state (and, thus, modeshape) of subconductor which does have appropriate tension, *T'*, but not the appropriate sag. In the future work we plan to calculate it more precisely.

Finally, the tangent stiffness and mass matrices are extracted for eigenvalue computation over the subconductor. In fact, as we look for unstable modes of leeward subconductor only, the rest of structure is omitted from super element. After generation of super element, we keep its modal part while fixing all attachment points to the spacers and retaining necessary number of modes. In this case, for example, we proceed with an eigenvalue problem of sixth order for stability study.

Summary of analysis cases is presented in the Table below.

	Subspan Length, m		
No. case	no.1	no.2	no.3
1	40	40	40
2	35	40	45
3	30	40	50
4	25	40	55
5	20	40	60

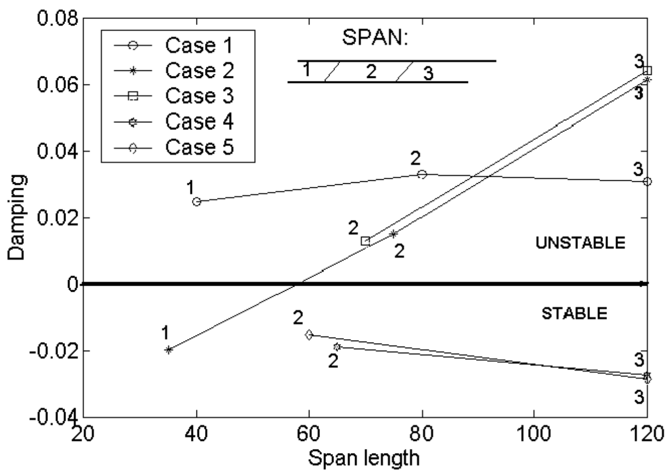


Fig.4. Variation of real part of in-plane mode in subspans.

Shown in Fig. 4 are the damping values corresponding to in-plane modes (in fact, those real parts uniquely provide instability to the bundle just as shown in Fig. 2). Each line corresponds to an appropriate analysis case (see the Table above). Each point refers to the subspan. Abscissas indicate the spacers' coordinate (except '3").

The case of equal subspans is always avoided in practice, however, it is quite illustrative. The slight difference in span lengths gives rise to disbalance in values of damping. This image also indicates one drawback of method: because of higher modes, the eigenfrequencies for a shorter subspan were left apart and, instead of them, higher modes in adjacent spans appeared (they were excluded from study). The direct way to identify the lacking modes is to keep some more eigenmodes in the subconductor super-element.

5. CONCLUSIONS

Further development shall include the inter-subspan coupling via transfer matrix technique; studies of the connection to the spacer-damper; improved account of wind loads at the stage of line span stiffness matrix generation prior to extraction of subconductor's super element; and study of the reference case presented by Hearnshaw (1974). At present, the basic approaches for handling the cable model by Component Mode technique are established and first successful testings have been done.

6. ACKNOWLEDGMENTS

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