The role of lepton flavor symmetries in leptogenesis

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Abstract

The presence of flavor symmetries in the lepton sector may have several consequences for the generation of the baryon asymmetry of the Universe via leptogenesis. We review the mechanism in general type-I, type-II and type-III seesaw models. We then turn to the discussion of the cases when the asymmetry is generated in the context of seesaw models extended with flavor symmetries, before or after flavor symmetry breaking. Finally we explain how the interplay between type-I and type-II seesaws can (or not) lead to viable models for leptogenesis even when there is an exact mixing pattern enforced by the flavor symmetry.

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1 Introduction

Baryogenesis via leptogenesis is a scenario in which the baryon asymmetry of the Universe is first generated in leptons and partially reprocessed—via standard model sphaleron processes—into a baryon asymmetry. From a general point of view three conditions (Sakharov conditions [1]) have to be satisfied in order for leptogenesis to take place at some stage during the evolution of the expanding Universe, namely there must be (*i*) interactions that break lepton number; (*ii*) CP violation; (*iii*) departure from thermodynamical equilibrium. In principle any framework in which these conditions can be satisfied can be regarded as a playground for leptogenesis.

In models for Majorana neutrino masses lepton number is broken, so they provide intrinsic frameworks for leptogenesis. The most well studied scenarios for leptogenesis correspond to the type-I [2, 3, 4, 5, 6, 7], type-II [7, 8, 9, 10] and type-III [11] seesaw models (tree-level realizations of the dimension five effective operator $\mathcal{O}_5 \sim \ell\ell HH$ [12]). In these cases the generation of a $B-L$ asymmetry proceeds via the decay of heavy fermion right-handed electroweak singlets (RH neutrinos for brevity) (see e.g. [13]) or triplets [14, 15] (type-I or type-III) or scalar $SU(2)$ triplets [16, 17, 15]. Due to the different electroweak charges of these states their thermodynamical behavior is different and so is the way in which leptogenesis takes place.

The idea of flavor symmetries dates back to the late 1970's [18]. Initially, due to the lack of experimental data in the lepton sector, flavor symmetries were used to explain quark masses and mixing patterns, but with the advent of neutrino data [19, 20, 21] the idea was increasingly extended to the lepton sector. In particular, in recent years it has been shown that lepton mixing is well described by non-Abelian flavor symmetries (see e.g. [22]). In association with these developments, the issue of leptogenesis in flavor models has attracted a great deal of attention.

In this short review we describe the relationship between flavor symmetries and leptogenesis. In section 2 we provide a brief review of general aspects of leptogenesis, covering leptogenesis in type-I seesaw, and also leptogenesis in type-II and III seesaw models. We take some care in establishing the notation to be used in the other sections. The connection with flavor symmetries has been covered in several works: [23, 24, 25] cover the flavor symmetric phase and are reviewed in section 3. In section 4 we review the results of [26, 27, 28, 29, 30, 31], addressing the case where only type-I seesaw takes place and identify cases where the presence of the symmetries can lead to strong predictions for the viability of leptogenesis. In section 5 we summarize the results from [32], where the scenario considered has both type-I and II seesaws taking place—here too, in certain circumstances conclusions about leptogenesis can be derived due to the presence of the flavor symmetry. Other papers studying leptogenesis in the context of a flavor symmetry include [33, 34, 35, 36, 37, 38, 39] although here we do not review their results explicitly.

For consistency we employ the same notation throughout the review. The notation is mostly based on what was used in [29] and parts of the notation used in [32] (in particular where type-II seesaw is discussed, such as in section 5). In general we consider the basis where the charged lepton mass matrix is diagonal. For type-I seesaw we also take the basis where the RH neutrino mass matrix is diagonal unless otherwise stated. Matrices that appear with a hat are in the basis where that matrix is diagonal (e.g. $\boldsymbol{\hat{m}_D}$) and we denote matrices in boldface.

2 Leptogenesis: generalities

We now discuss the general framework of leptogenesis in more detail: in a hot plasma with N lepton number and CP violating states $S_1, \cdots, S_N$, assuming they all have tree-level couplings with the standard model leptons (only with electroweak doublets to simplify the discussion), their out-of-equilibrium tree-level decays will produce a net $B - L$ asymmetry. The determination of the exact amount of $B - L$ asymmetry depends on the dynamics of the S_α states and requires—in general—an analysis based on kinetic equations accounting for the evolution of the S_α densities and the $B - L$ asymmetry density itself. For the evolution of the $B - L$ asymmetry one can write

$$\dot{Y}_{\Delta_{B-L}}(z) = \sum_{\alpha=1}^{N} \dot{Y}^{(S_\alpha)}_{\Delta_{B-L}}(z)\,, \tag{1}$$

where, following ref. [40], we are using the notation $s(z)H(z)z\,dY_X(z)/dz \equiv \dot{Y}_X(z)$. Here $z = M_1/T$ (M_1 being the mass of the lightest state), $Y_{\Delta_X} = n_X - n_{\bar{X}}/s$ with n_X ($n_{\bar{X}}$) the number density of particles (antiparticles), s the entropy density and $H(z)$ the expansion rate of the Universe (the expressions for these functions are given in appendix A). $\dot{Y}^{(S_\alpha)}_{\Delta_{B-L}}(z)$ is the asymmetry generated by each of the states S_α. Note that we have written the dimensionless inverse temperature of the remaining states as $z_\alpha = M_\alpha/M_1\, z$.

The evolution of the asymmetries generated by each S_α is in turn determined by the "competition" between source (S_{S_α}) and washout (W_{S_α}) terms. The size of the source terms is fixed by how much the S_α's deviate from thermodynamical equilibrium when decaying, by the strength of the decays and by the amount of CP violation. The size of the washout terms is, instead, dictated by S_α processes that tend to diminish the lepton asymmetry created via the source terms like e.g. inverse decays and lepton number breaking scatterings.

As discussed in the introduction, models for Majorana neutrino masses are intrinsic scenarios for leptogenesis to take place and indeed from this approach it turns out that there is a link between two in principle unrelated problems: the origin of neutrino masses and the origin of the baryon asymmetry of the Universe. It is well known that Majorana neutrino masses can be generated in a model independent way by adding to the standard model Lagrangian an effective dimension five operator $\mathcal{O}_5 \sim \ell\ell HH$, that generates the corresponding Majorana masses after electroweak symmetry breaking [12]. The tree-level realizations of this operator give rise to type-I, II and III seesaws which constitute the usual frameworks for almost all the studies of leptogenesis.

2.1 Leptogenesis in type-I seesaw

In type-I seesaw the states S_α correspond to RH neutrinos N_α. In a general basis the interactions of these states are given by

$$-\mathcal{L}^{(I)} = \mathrm{i}\bar{N}\,\gamma^\mu\partial_\mu\,N + \bar{\ell}\,\boldsymbol{\lambda}^*\,N\tilde{H} + \frac{1}{2}N^T\,C\,\boldsymbol{M_N}\,N + \text{h.c.}\,, \tag{2}$$

Here $\tilde{H} = \mathrm{i}\tau_2 H^*$, C is the charge conjugation operator, and for 3 N_α, $\boldsymbol{\lambda}^*$ is a 3×3 Yukawa coupling matrix in flavor space and $\boldsymbol{M_N}$ is the 3×3 Majorana mass matrix. At energy scales well below the RH neutrino masses, the light neutrinos masses are determined by the effective matrix

$$\boldsymbol{m_\nu^{\text{eff}}} = \boldsymbol{m_\nu^I} = -\boldsymbol{m_D}\boldsymbol{M_N}^{-1}\boldsymbol{m_D}^T = -\sum_{\alpha=1,2,3} M_{N_\alpha}^{-1}\boldsymbol{m_{D_\alpha}}\otimes\boldsymbol{m_{D_\alpha}}\,, \tag{3}$$

where in order to facilitate the discussion in section 5 we have expressed the matrix in terms of the parameter space vectors $\boldsymbol{m_{D_\alpha}}^T = (m_{D_{\alpha 1}}, m_{D_{\alpha 2}}, m_{D_{\alpha 3}})$, with $\boldsymbol{m_D} = v\,\boldsymbol{\lambda}$ and $v = \langle H\rangle \simeq 174$ GeV. Diagonalization of (3) by means of the PMNS mixing matrix $\boldsymbol{U}$ leads to the light neutrino mass spectrum:

$$\boldsymbol{U}^T\boldsymbol{m_\nu^{\text{eff}}}\boldsymbol{U} = \boldsymbol{\hat{m}_\nu}\,, \tag{4}$$

with $\boldsymbol{U} = \boldsymbol{V}\,\boldsymbol{D}$ (with $\boldsymbol{V}$ the part of the PMNS matrix having a CKM-like form and $\boldsymbol{D} = \text{diag}(e^{\mathrm{i}\phi_1}, e^{\mathrm{i}\phi_2}, 1)$ containing the Majorana CP phases).

The 3×3 Dirac mass matrix $\boldsymbol{m_D}$, being a general complex matrix, contains 18 parameters (9 moduli and 9 phases) of which 3 phases can be removed by rotation of the lepton doublets in (2). The number of physical parameters defining $\boldsymbol{m_D}$ is therefore 15. A very useful parametrization in which this is explicitly taken into account is the Casas-Ibarra parametrization [41], in which the Dirac mass matrix is expressed in terms of low-energy neutrino observables and a general complex orthogonal matrix $\boldsymbol{R}$, namely

$$\boldsymbol{m_D} = \boldsymbol{U}^*\,\boldsymbol{\hat{m}_\nu}^{1/2}\boldsymbol{R}\,\boldsymbol{\hat{M}_N}^{1/2}\,. \tag{5}$$

In the conventional thermal leptogenesis scenario the RH neutrino mass spectrum is taken to be hierarchical, $M_{N_\alpha} \ll M_{N_\beta}$ for $\alpha < \beta$ (for a throughout review see [13]). Under this simplification—well justified as far as $T_{\text{Reheat}} < M_{N_{2,3}}$—the effects of $N_{2,3}$ can be neglected and thus the asymmetry is entirely produced by the dynamics of N_1. The kinetic equations that describe the evolution of the asymmetry involve N_1 decays, $\Delta L = 1$ and $\Delta L = 2$ scatterings, and depending on the temperature regimen at which leptogenesis takes place ($T \sim M_{N_1}$) should include the lepton flavor degrees of freedom [42, 43, 44, 45, 46, 47]. At the leading order in the Yukawa couplings, however, the kinetic equations are determined by the decays and the off-shell pieces of the $\Delta L = 2$ scattering processes $\ell_j H \leftrightarrow \ell_i H$ and $\bar{\ell}_j H^\dagger \leftrightarrow \ell_i H$. For $T \gtrsim 10^{12}$ GeV (or otherwise neglecting flavor effects) they read as follows

$$\begin{aligned}
\dot{Y}_{N_1} &= -(y_{N_1} - 1)\gamma_{D_{N_1}} \, , \\
\dot{Y}_{\Delta_{B-L}} &= S_{N_1} + W_{N_1} \\
&= -\left[(y_{N_1} - 1)\epsilon_{N_1} + \frac{y_{\Delta_{B-L}}}{2}\right]\gamma_{D_{N_1}} \, ,
\end{aligned} \tag{6}$$

where we are using the notation $y_X \equiv Y_X/Y_X^{\rm Eq}$ and $y_{\Delta_{B-L}} \equiv Y_{\Delta_{B-L}}/Y_\ell^{\rm Eq}$ (the expressions for the equilibrium densities are given in appendix A). The strength of the decays

$$\tilde{m}_1 = \frac{v^2}{M_{N_1}} \left(\boldsymbol{\lambda}^\dagger \boldsymbol{\lambda}\right)_{11} \tag{7}$$

determines the size of the reaction density $\gamma_{D_{N_1}}$ appearing in the source term S_{N_1} as well as in the washout term W_{N_1}, namely

$$\gamma_{D_{N_1}} = \frac{1}{8\pi^3}\frac{M_{N_1}^5}{v^2}\frac{K_1(z)}{z}\,\tilde{m}_1 \, . \tag{8}$$

Here $K_1(z)$ is the first-order modified Bessel function of the second-type. The amount of CP violation in N_1 decays is given by the CP violating asymmetry $\epsilon_{N_1} = \sum_{i=e,\mu,\tau}\epsilon_{N_1}^{\ell_i}$. At the leading order this quantity arises through the interference between the N_1 tree-level decay and the one-loop vertex and wave function corrections [48]. The flavored CP violating asymmetries arising from the diagrams depicted in figure 1 read

$$\begin{aligned}
\epsilon_{N_1}^{\ell_i({\rm V})} &= \frac{1}{8\pi}\sum_{\beta\neq 1}\frac{\mathbb{I}{\rm m}[\sqrt{\omega_\beta}(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{\beta 1}\lambda_{i\beta}^*\lambda_{i1}]}{(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{11}} f(\omega_\beta) \, , \\
\epsilon_{N_1}^{\ell_i({\rm W})} &= -\frac{1}{8\pi}\sum_{\beta\neq 1}\frac{\mathbb{I}{\rm m}\{[(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{1\beta} + \sqrt{\omega_\beta}(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{\beta 1}]\lambda_{i\beta}^*\lambda_{i1}\}}{(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{11}} g(\omega_\beta) \, ,
\end{aligned} \tag{9}$$

with obvious generalization if the decaying state is N_α, where $\omega_\beta = M_{N_\beta}^2/M_{N_\alpha}^2$ and the loop functions are

$$\begin{aligned}
f(\omega_\beta) &= (1+\omega_\beta)\ln\left(\frac{\omega_\beta + 1}{\omega_\beta}\right) - 1 \, , \\
g(\omega_\beta) &= \frac{1}{\omega_\beta - 1} \, .
\end{aligned} \tag{10}$$

Since $\omega_\beta^{-1} \ll 1$ the loop functions can be expanded in powers of ω_β^{-1} dropping the subleading terms. At leading order in these parameters the flavored CP violating

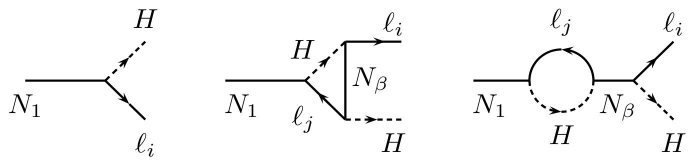

Figure 1: Feynman diagrams generating the CP violating asymmetry $\epsilon_{N_1}^{\ell_i}$ in type-I seesaw.

asymmetries can be expressed as

$$\epsilon_{N_1}^{\ell_i} = -\frac{1}{8\pi(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{11}} \sum_{\beta\neq 1} \mathbb{I}\mathrm{m}\left\{ \left[\frac{(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{1\beta}}{\omega_\beta} + \frac{3(\boldsymbol{\lambda}^\dagger\boldsymbol{\lambda})_{\beta 1}}{2\sqrt{\omega_\beta}} \right] \lambda_{i\beta}^* \lambda_{i1} \right\} . \tag{11}$$

The total CP asymmetry then is obtained from

$$\epsilon_{N_1} = \sum_{i=e,\mu,\tau} \epsilon_{N_1}^{\ell_i} = -\frac{3}{16\pi v^2} \sum_\beta \frac{1}{\sqrt{\omega_\beta}} \frac{\mathbb{I}\mathrm{m}[(\boldsymbol{m_D}^\dagger \boldsymbol{m_D})_{\beta 1}^2]}{(\boldsymbol{m_D}^\dagger \boldsymbol{m_D})_{11}} , \tag{12}$$

where anticipating the discussions of sections 4 and 5 we have rewritten the CP violating asymmetry in terms of the Dirac mass matrix (in the basis where the RH neutrino mass matrix is diagonal).

Formal integration of eqs. (6), using the integrating factor technique and assuming a vanishing primordial $B-L$ asymmetry, gives

$$Y_{\Delta_{B-L}}(z) = -\epsilon_{N_1} Y_{N_1}^{\mathrm{Eq}}(z \to 0)\, \eta(z) . \tag{13}$$

Here $\eta(z)$ is the efficiency function that determines the evolution of $Y_{\Delta_{B-L}}(z)$ and its final value at $z \to \infty$ (see appendix A for details). At $\mathcal{O}(\boldsymbol{\lambda}^2)$ (leading order) the problem of determining the final $Y_{\Delta_{B-L}}$ is a two parameters problem, $\tilde{m}_1$ and ϵ_{N_1}, and requires numerical solutions of eqs. (6) [1]. Figure 2 shows the efficiency factor $\eta \equiv \eta(z \to \infty)$ as a function of the parameter $\tilde{m}_1$.

2.2 Leptogenesis in type-II seesaw

Consistent models for leptogenesis involving scalar electroweak triplets require going beyond a single scalar triplet. There are several ways in which this can be done, namely adding at least another triplet [51], adding RH neutrinos [16] or adding fermionic triplets. Here we will discuss scenarios that include RH neutrinos, as these schemes will be further analyzed in the context of flavor symmetries in section 5. In this case, depending on the triplet and RH mass spectrum, the states S_α can be identified with the RH neutrinos, the triplet or both.

[1] Analytically the problem has been addressed yielding quite accurate expressions for the efficiency [49, 50].

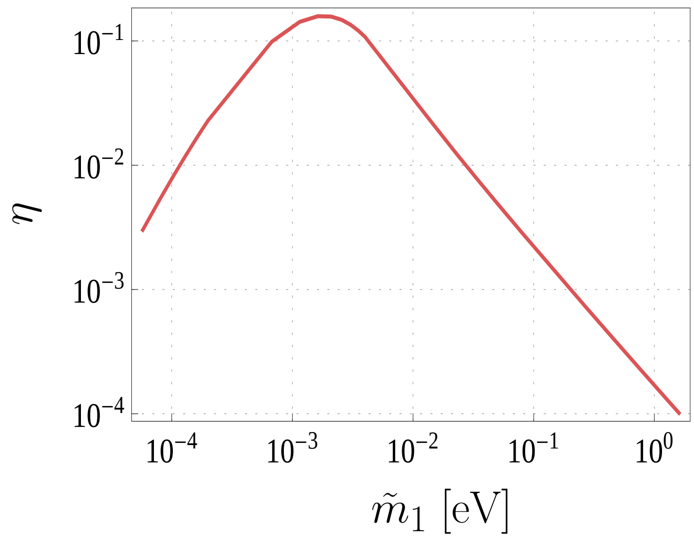

Figure 2: Efficiency factor as a function of the parameter $\tilde{m}_1$ in type-I seesaw. The efficiency has been calculated from leading order Boltzmann equations in the one-flavor approximation.

The interactions of the RH neutrinos are given by the Lagrangian in (2) whereas the interaction of the triplet are determined by

$$-\mathcal{L}^{(II)} = \ell^T C \boldsymbol{Y} \mathrm{i}\tau_2 \boldsymbol{\tau} \ell \boldsymbol{\Delta} + M_\Delta^2 \mathrm{Tr} \boldsymbol{\Delta}^\dagger \boldsymbol{\Delta} - \mu H^T \mathrm{i}\tau_2 \boldsymbol{\tau} H \boldsymbol{\Delta} + \text{h.c.}\,. \tag{14}$$

Here $\boldsymbol{Y}$ is a 3×3 matrix in flavor space and $\boldsymbol{\Delta}$, the $SU(2)$ scalar electroweak triplet has hypercharge $+1$ (to the lepton doublets -1/2) and is given by

$$\boldsymbol{\Delta} = \begin{pmatrix} \Delta^{++} & \Delta^{+}/\sqrt{2} \\ \Delta^{+}/\sqrt{2} & \Delta^{0} \end{pmatrix}\,. \tag{15}$$

After electroweak symmetry breaking the light neutrino mass matrix receives the contributions from the dimension five effective operators of the RH neutrino and triplet

$$\boldsymbol{m}_{\boldsymbol{\nu}}^{\textbf{eff}} = \boldsymbol{m}_{\boldsymbol{\nu}}^{\boldsymbol{I}} + \boldsymbol{m}_{\boldsymbol{\nu}}^{\boldsymbol{II}} \quad \text{with} \quad \boldsymbol{m}_{\boldsymbol{\nu}}^{\boldsymbol{II}} = 2\, v_\Delta \boldsymbol{Y}\,. \tag{16}$$

The first term is the contribution from the RH neutrinos (eq. (3)) whereas the second one is the contribution from the triplet, with the triplet vacuum expectation value fixed by $\langle \Delta^0 \rangle = v_\Delta = \mu^*\, v^2/M_\Delta^2$.

As already mentioned the generation of $B-L$ asymmetry depends on the heavy mass spectrum. One can define three possible scenarios:

- $M_{N_1} \ll M_\Delta$: the effects of $\boldsymbol{\Delta}$ are decoupled and the lepton asymmetry is generated via N_1 dynamics. This case resembles leptogenesis in type-I seesaw.

- $M_{N_1} \gg M_\Delta$: the lepton asymmetry is entirely produced by the dynamics of $\boldsymbol{\Delta}$ [15].

- $M_\Delta \sim M_{N_1}$: both the triplet and the lightest RH neutrino generate the asymmetry [32].

Here we will discuss the third possibility in the regimen $M_{N_1,\Delta} > 10^{12}$ GeV. We will closely follow the presentation in [32].

Since in this case the scalar triplet, carrying non-trivial $SU(2)$ quantum numbers, couples to the standard model electroweak gauge bosons, and the number of degrees of freedom participating in the generation of the lepton asymmetry is larger, the Boltzmann equations are more involved. At leading order in the Yukawa couplings $\boldsymbol{\lambda}$ and $\boldsymbol{Y}$ the kinetic equation for the lepton asymmetry, Y_{Δ_L}[2], involve the RH neutrino and triplet decays and inverse decays $N_1 \leftrightarrow \ell \tilde{H}^\dagger$ and $\boldsymbol{\Delta} \leftrightarrow \bar{\ell}\bar{\ell}$ and the off-shell Yukawa generated scattering reactions $\ell \tilde{H}^\dagger \leftrightarrow \ell \tilde{H}^\dagger$ and $H^\dagger H^\dagger \leftrightarrow \ell\ell$. In addition to the evolution of the Y_{Δ_L} asymmetry the full network of Boltzmann equations should include the equations accounting for the evolution of the RH neutrino and triplet number densities and the triplet and Higgs asymmetries[3]. The resulting system of five coupled differential equations can be reduced to four by using the constraint imposed by hypercharge neutrality [15]:

$$2\,Y_{\Delta_\Delta} + Y_{\Delta H} - Y_{\Delta_L} = 0\,. \tag{17}$$

The resulting kinetic equations can thus be written as

$$\begin{aligned}
\dot{Y}_{N_1} &= -(y_{N_1} - 1)\,\gamma_{D_{N_1}}\,,\\
\dot{Y}_{\Sigma} &= -(y_\Sigma - 1)\,\gamma_{D_\Delta} - 2(y_\Sigma^2 - 1)\,\gamma_A\,,\\
\dot{Y}_{\Delta_L} &= \left[(y_{N_1} - 1)\,\epsilon_{N_1}^{\rm tot} - \left(y_{\Delta_L} + y^H_{\Delta_\Delta}\right)\right]\,\gamma_{D_{N_1}} + \left[(y_\Sigma - 1)\,\epsilon_\Delta - 2K_\ell\,(y_{\Delta_L} + y_{\Delta_\Delta})\right]\,\gamma_{D_\Delta}\,,\\
\dot{Y}_{\Delta_\Delta} &= -\left[y_{\Delta_\Delta} + (K_\ell - K_H)\,y_{\Delta_L} + 2K_H\,y^H_{\Delta_\Delta}\right]\,,
\end{aligned} \tag{18}$$

where $\Sigma \equiv \Delta + \Delta^\dagger$ and $y^H_{\Delta_\Delta} \equiv Y_{\Delta_\Delta}/Y_H^{\rm Eq}$ and the rest of the variables in the equations follow the conventions introduced in the previous section when writing the eqs. in (6). The reaction densities involving the triplet are given by

$$\gamma_{D_\Delta} = \frac{1}{8\,\pi^3}\frac{M_\Delta^5}{v^2}\frac{K_1(z)}{z}\,\left(\tilde{m}_\Delta^\ell + \frac{\tilde{m}_\Delta^2}{4\tilde{m}_\Delta^\ell}\right)\,,\quad \gamma_A(z) = \frac{M_\Delta^4}{64\,\pi^4}\int_4^\infty dx\sqrt{x}\frac{K_1(zx)}{z}\,\widehat{\sigma}_A(x)\,, \tag{19}$$

with $x = s/M_\Delta^2$. The reduced cross section $\widehat{\sigma}_A(x) = 2\,x\,\lambda(1, x^{-1}, 0)$ (where we have $\lambda(a,b,c) = (a-b-c)^2 - 4bc$) can be found in appendix A. The factors $K_{\ell,H}$ resemble the flavor projectors defined in standard flavored leptogenesis [45, 47] as they project triplet decays into either the Higgs or the lepton doublet directions. They are defined as follows

$$K_\ell = \frac{\tilde{m}_\Delta^\ell}{\tilde{m}_\Delta^\ell + \frac{\tilde{m}_\Delta^2}{4\,\tilde{m}_\Delta^\ell}} \qquad \text{and} \qquad K_H = \frac{\tilde{m}_\Delta^2}{4\,\tilde{m}_\Delta^\ell\left(\tilde{m}_\Delta^\ell + \frac{\tilde{m}_\Delta^2}{4\,\tilde{m}_\Delta^\ell}\right)}\,, \tag{20}$$

[2]In contrast to the previous section, in this case we do not include the change in the lepton densities due to sphaleron processes, and thus study only the evolution of the L asymmetry.

[3]These asymmetries are a consequence of these fields not being self-conjugate.

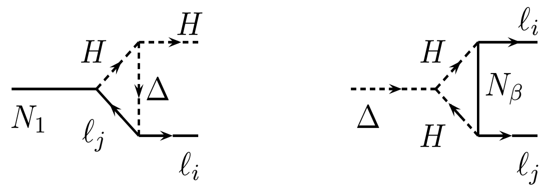

Figure 3: Left-hand side: Vertex loop correction involving the triplet and contributing to $\epsilon^{\Delta}_{N_1}$. Right-hand side: Vertex loop correction for triplet decays [16].

where the parameters $\tilde{m}^{\ell}_{\Delta}$ and $\tilde{m}^2_{\Delta}$ are given by

$$\tilde{m}^{\ell}_{\Delta} = \frac{v^2\,|\boldsymbol{Y}|^2}{M_\Delta} \qquad \text{and} \qquad \tilde{m}^2_{\Delta} = \mathrm{Tr}[\boldsymbol{m_\nu^{II}} \boldsymbol{m_\nu^{II}}^\dagger]\,, \tag{21}$$

with $|\boldsymbol{Y}|^2 = \mathrm{Tr}[\boldsymbol{Y}\,\boldsymbol{Y}^\dagger]$. In these definitions we have replaced the trilinear coupling μ by the contribution of the type-II sector to the effective light neutrino mass matrix, encoded in $\tilde{m}^2_{\Delta}$. In principle this is just a matter of choice, but it proves to be quite convenient given that in contrast to μ the parameter $\tilde{m}_\Delta$ is (partially) constrained by experimental neutrino data.

The CP asymmetry for the RH neutrino arises as in type-I but, due to the trilinear scalar coupling in (14), there is an additional contribution coming from a vertex correction involving the triplet, as shown in fig. 3 (left-hand side). The interference between the tree-level decay $N_\alpha \to \ell\,\tilde{H}^\dagger$ and this 1-loop vertex diagram yields [16, 17][4]

$$\epsilon^{\Delta}_{N_1} = -\frac{3}{2\,\pi\,M_\Delta}\frac{1}{(\boldsymbol{m_D}\,\boldsymbol{m_D}^\dagger)_{11}}\mathfrak{Im}\left[\left(\boldsymbol{m_D}\,\boldsymbol{Y}^*\boldsymbol{m_D}^T\right)_{11}\,\mu\right]\,h(\sigma_1)\,. \tag{22}$$

The function $h(\sigma_1)$, with $\sigma_\alpha = M^2_\Delta/M^2_{N_\alpha}$, is given by

$$h(\sigma_1) = \sqrt{\sigma_1}\left[1 - \sigma_1 \log\left(\frac{1+\sigma_1}{\sigma_1}\right)\right]\,. \tag{23}$$

The total CP violating asymmetry in N_1 decays therefore reads

$$\epsilon^{\mathrm{tot}}_{N_1} = \epsilon_{N_1} + \epsilon^{\Delta}_{N_1}\,, \tag{24}$$

where, for the scenario considered, ϵ_{N_1} is determined by eqs. (11) and (12).

The CP violating asymmetry in triplet decays arises from the interference between the tree-level $\boldsymbol{\Delta} \to \ell\ell$ process and the interference with the 1-loop vertex diagram shown in figure 3 (right-hand side). The result reads [16]

$$\epsilon_\Delta = -\frac{1}{8\,\pi\,v^2}\frac{1}{M_\Delta}\frac{\sum_\beta \mathfrak{Im}\left[\left(\boldsymbol{m_D}\,\boldsymbol{Y}^*\,\boldsymbol{m_D}^T\right)_{\beta\beta}\,\mu\right]}{\mathrm{Tr}\left[\boldsymbol{Y}\,\boldsymbol{Y}^\dagger\right] + \mu^2/M^2_\Delta}\,H(\sigma_\beta)\,, \tag{25}$$

[4] This equation follows from [17] which differs from [16] by a factor of 3/2.

where the loop function in this case is given by

$$H(\sigma_\beta) = \frac{1}{\sqrt{\sigma_\beta}} \log\left(1+\sigma_\beta\right) \,. \tag{26}$$

As in the type-I case the kinetic equation for the lepton asymmetry can be formally integrated. The resulting asymmetry, assuming a zero primordial asymmetry, can be expressed in two different ways [32]

$$Y_{\Delta_L}(z) = -\epsilon_{N_1}^{\text{tot}}\, Y_{\text{tot}}^{\text{Eq}}\, \eta^{I}(z) \quad \text{or} \quad Y_{\Delta_L}(z) = -\epsilon_\Delta\, Y_{\text{tot}}^{\text{Eq}}\, \eta^{II}(z) \,. \tag{27}$$

The functions $\eta^{I,II}(z)$ are defined in such a way that in the limit in which the triplet (RH neutrino) interactions are absent η^I (η^{II}) corresponds to the efficiency function of standard leptogenesis (pure triplet leptogenesis), see appendix A for details. As in the type-I case the final L asymmetry is obtained from these functions in the limit $z \to \infty$.

A precise determination of the lepton asymmetry generated in N_1 and $\boldsymbol{\Delta}$ decays requires solving the network of equations in (18). Taking $z = M_\Delta/T$ and $z_N = rz$, with $r = \sigma_1^{-1/2} = M_{N_1}/M_\Delta$, and once the CP asymmetries $\epsilon_{N_1}^{\text{tot}}$ and ϵ_Δ are fixed, the problem of studying the evolution of the lepton asymmetry is entirely determined by five parameters: $\tilde{m}_1$, $\tilde{m}_\Delta$, $\tilde{m}_\Delta^\ell$, M_Δ and r [5].

As pointed out in [32], in models featuring a mild hierarchy between M_Δ and M_{N_1} three scenarios can be defined:

I. Purely triplet scalar leptogenesis models:
The relevant parameters follow the hierarchy $\tilde{m}_1 \ll \tilde{m}_\Delta^\ell, \tilde{m}_\Delta$. The L asymmetry is generated through the processes $\boldsymbol{\Delta} \to \bar{\ell}\bar{\ell}$ or $\boldsymbol{\Delta} \to HH$ and the details strongly depend on whether $\tilde{m}_\Delta^\ell \gg \tilde{m}_\Delta$, $\tilde{m}_\Delta^\ell \ll \tilde{m}_\Delta$ or $\tilde{m}_\Delta^\ell \sim \tilde{m}_\Delta$. Interestingly, when $\tilde{m}_\Delta^\ell \gg \tilde{m}_\Delta$ the Higgs asymmetry—being weakly washed out—turns out to be large and implies a large lepton asymmetry.

II. Singlet dominated leptogenesis models:
These scenarios are defined according to $\tilde{m}_1 \gg \tilde{m}_\Delta^\ell, \tilde{m}_\Delta$ thus leptogenesis is mainly determined by N_1 dynamics. The relative difference between the parameters $\tilde{m}_\Delta^\ell$ and $\tilde{m}_\Delta$ determines whether either the Higgs asymmetry or the L asymmetry are strongly or weakly washed out, thus three cases can be distinguished: $\tilde{m}_\Delta^\ell \gg \tilde{m}_\Delta$, $\tilde{m}_\Delta^\ell \ll \tilde{m}_\Delta$ or $\tilde{m}_\Delta^\ell \sim \tilde{m}_\Delta$. Each of them exhibit different features.

III. Mixed leptogenesis models:
In these models the parameters controlling the gauge reaction densities strengths are all of the same order i.e. $\tilde{m}_1 \sim \tilde{m}_\Delta^\ell \sim \tilde{m}_\Delta$.

[5] This is to be compared with the pure triplet leptogenesis scenario [15] where the generation of the L asymmetry is entirely determined by only three parameters: $\tilde{m}_\Delta$, $\tilde{m}_\Delta^\ell$, M_Δ.

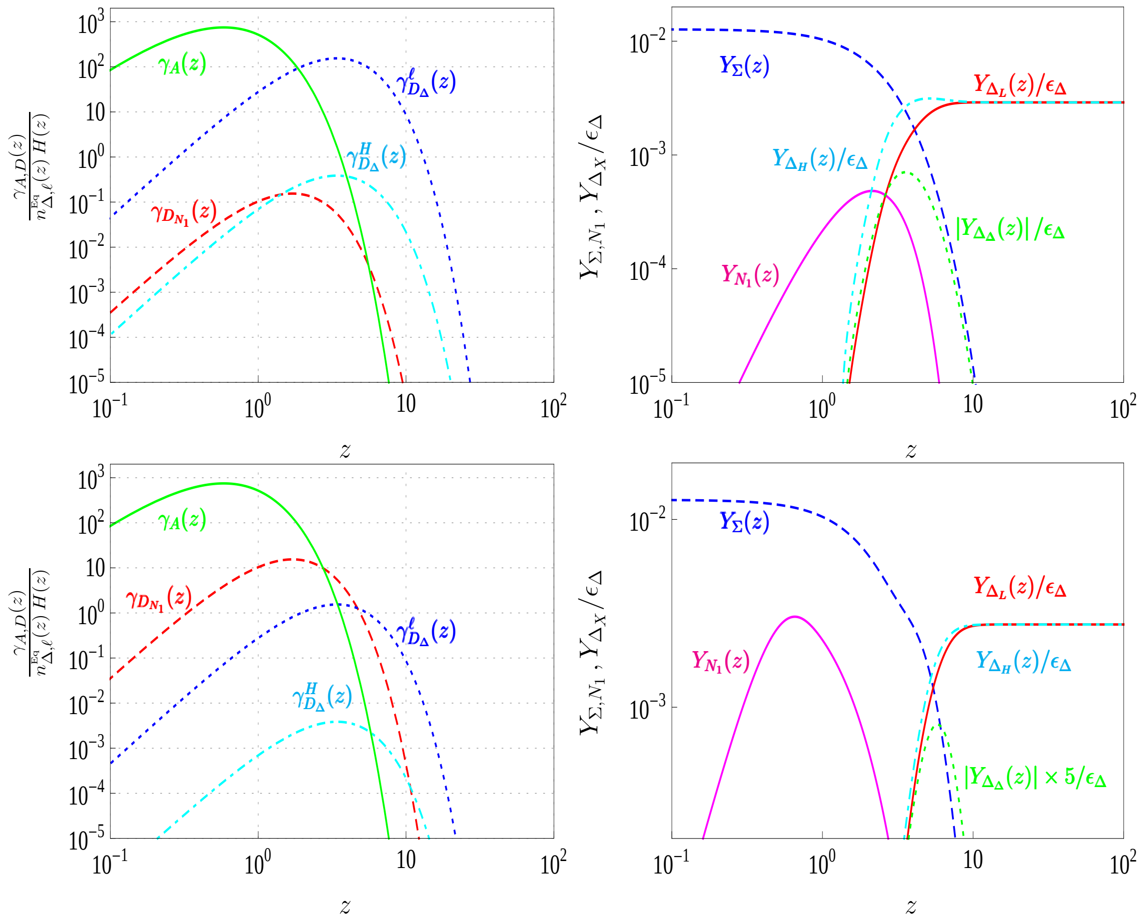

Figure 4: *Reaction densities for triplet and RH neutrino processes (left-panels) and evolution of the different densities (right-panels) entering in the kinetic equations for the scenarios of purely triplet leptogenesis (upper panels) and singlet dominated models (lower panels) [32]. See the text for more details.*

For the sake of illustration in figure 4 we show two numerical examples for scenarios I and II. They were obtained with the parameter space points P_I=($\tilde{m}_1$, $\tilde{m}_\Delta$, $\tilde{m}_\Delta^\ell$,M_Δ,r) =(10^{-4} eV, 10^{-2} eV, 10^{-1} eV, 10^{10} GeV,2) and P_{II}=($\tilde{m}_1$, $\tilde{m}_\Delta$, $\tilde{m}_\Delta^\ell$,M_Δ,r) =(10^{-2} eV, 10^{-4} eV, 10^{-3} eV, 10^{10} GeV,2) for fixed $\epsilon_\Delta = 10^{-6}$ and $\epsilon_{N_1} = 10^{-5}$ and assuming initial vanishing asymmetries.

2.3 Leptogenesis in type-III seesaw

In type-III [11] seesaw the states S_α correspond to fermion electroweak triplets (here we consider 3 for definiteness) with vanishing hypercharge. In a general basis the interactions of these states are given by the following Lagrangian

$$- \mathcal{L}^{(III)} = -\overline{\boldsymbol{T}}_\alpha \not{D} \boldsymbol{T}_\alpha + \overline{\ell}\, \boldsymbol{h}^* \boldsymbol{T} \widetilde{H} + \frac{1}{2}\overline{\boldsymbol{T}}^\dagger_\alpha C \boldsymbol{M_T} \boldsymbol{T}_\alpha + \text{h.c.}\,, \tag{28}$$

where the fermion triplets can be written as a matrix

$$\boldsymbol{T}_\alpha = \boldsymbol{\tau}\cdot\boldsymbol{T}_\alpha = \begin{pmatrix} T^0_\alpha & \sqrt{2}T^+_\alpha \\ \sqrt{2}T^-_\alpha & -T^0_\alpha \end{pmatrix}\,, \tag{29}$$

with $T^0 = T^3$, $T^\pm = (T^1 \mp \mathrm{i}T^2)/\sqrt{2}$. In this notation, the covariant derivative is defined as $D_\mu = \partial_\mu - \mathrm{i}g\tau^a W^a_\mu/2$ (a being $SU(2)$ indices). Lepton number is broken by the Majorana triplet mass terms and the effective light neutrino mass matrix has the same structure than in type-I seesaw, eq. (3), with the right-handed neutrino mass matrix replaced by that of the triplets and $\boldsymbol{m_D} = v\,\boldsymbol{h}$:

$$\boldsymbol{m_\nu^{\mathrm{eff}}} = \boldsymbol{m_\nu^{III}} = \sum_{\alpha=1,2,3} M^{-1}_{T_\alpha}\boldsymbol{m_{D_\alpha}}\otimes\boldsymbol{m_{D_\alpha}}\,, \tag{30}$$

where we are using the same conventions used in the type-I case discussion.

In what concerns leptogenesis, in several aspects, these models resemble models based on type-I seesaw. For example assuming a hierarchical triplet mass spectrum $M_{T_\alpha} < M_{T_\beta}$ ($\alpha < \beta$) the $B-L$ asymmetry is completely produced by T_1 decays. There is, however, a significant difference arising from the fact that the triplets couple to the standard model electroweak gauge bosons. Thus, at high temperatures the triplet distribution is thermalized by gauge reactions, and only when these reactions are frozen a net $B-L$ asymmetry can be built [14, 52].

As done in sections 2.1 and 2.2, in what follows, we will discuss the generation of the $B-L$ asymmetry in these models in the one-flavor approximation (the effects of flavor have been considered in [53]). At $\mathcal{O}(\boldsymbol{h}^2)$, the leading order in the couplings $\boldsymbol{h}$, the kinetic equations consist of T_1 decays and off-shell $\Delta L = 2$ processes. The main difference with the conventional leptogenesis scenario is the inclusion of the couplings of T_1 with gauge bosons. The Boltzmann equations in this case read

$$\begin{aligned}\dot{Y}_{T_1} &= -\left(y_{T_1}-1\right)\gamma_{D_{T_1}} - \left(y^2_{T_1}-1\right)\gamma_A\,,\\ \dot{Y}_{\Delta_{B-L}} &= -\left[\left(y_{T_1}-1\right)\epsilon_{T_1} + \frac{y_{\Delta_{B-L}}}{2}\right]\gamma_{D_{T_1}}\,.\end{aligned} \tag{31}$$

The Yukawa reaction density $\gamma_{D_{T_1}}$ is given by eq. (8), changing $\boldsymbol{\lambda}\to\boldsymbol{h}$ and $M_{N_1}\to M_{T_1}$ in the definition of $\tilde{m}_1$ (eq. (7)) whereas the gauge reaction density by (19) using, of course, the corresponding fermion triplet reduced cross section (see appendix A). The CP violating asymmetry is a factor of three smaller than in type-I seesaw due to contractions of the $SU(2)$ indices in the Yukawa interaction terms entering in the 1-loop corrections, thus

$$\epsilon_{T_1} = \sum_{i=e,\mu,\tau}\epsilon^{\ell_i}_{T_1} = -\frac{1}{16\pi v^2}\sum_\beta \frac{1}{\sqrt{\omega_\beta}}\frac{\mathbb{I}\mathrm{m}[(\boldsymbol{m_D}^\dagger\boldsymbol{m_D})^2_{\beta 1}]}{(\boldsymbol{m_D}^\dagger\boldsymbol{m_D})_{11}}\,. \tag{32}$$

From the formal integration of the $B-L$ asymmetry kinetic equation in (31) the asymmetry can be written as

$$Y_{\Delta_{B-L}}(z) = -3\epsilon_{T_1}\,Y^{\mathrm{Eq}}_{T_1}(z\to 0)\,\eta(z)\,. \tag{33}$$

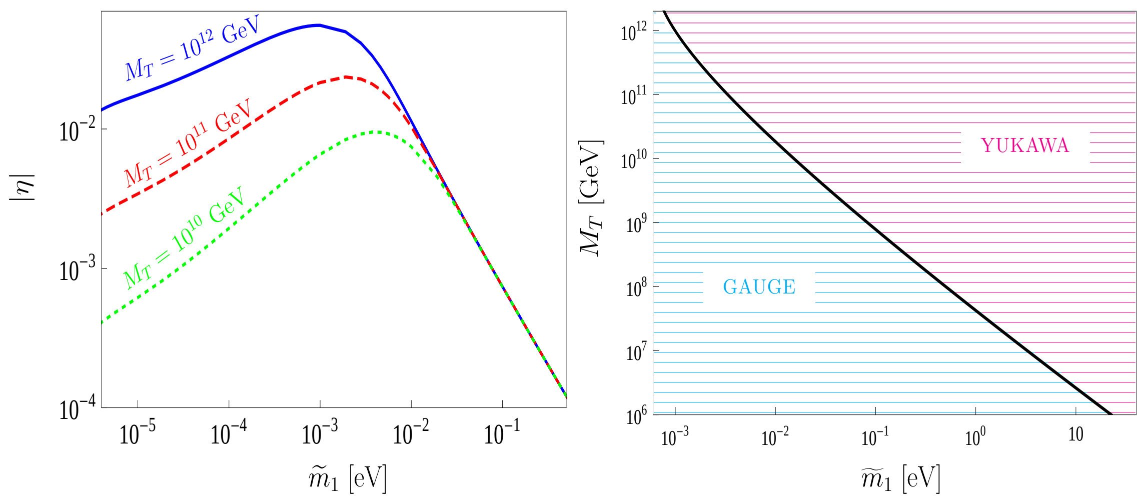

Figure 5: Efficiency factor as a function of $\widetilde{m}_1$ in the flavor aligned case (left panel) and regions for which gauge interactions freeze out after (lower region) and before (upper region) Yukawa reaction decoupling (right panel) in type-III leptogenesis.

The expression is similar to the one obtained in the type-I case but the efficiency is different, as it now includes the gauge reaction density. The factor of 3 comes from the $SU(2)$ degrees of freedom of T_1.

A precise determination of the $B-L$ asymmetry relies on numerical solutions of the kinetic equations, which in this case—even at the leading order in the couplings—requires ϵ_{T_1}, $\tilde{m}_1$ and also the triplet mass M_{T_1} to be specified. The results for the efficiency factor are shown in fig. 5 (left panel) where a strong dependence with M_{T_1} can be seen. This dependence, introduced by the gauge reactions, diminishes as $\tilde{m}_1$ increases and disappears at certain $\widetilde{m}_1^{\rm min}$. This implies that above this value T_1 leptogenesis proceeds as in type-I seesaw [6]. Thus, as highlighted in [53], in this type of models the generation of the $B-L$ asymmetry can proceed either in in a region determined by the condition $\widetilde{m}_1 < \widetilde{m}_1^{\rm min}$ ("gauge region") or conversely in a region defined by $\widetilde{m}_1 > \widetilde{m}_1^{\rm min}$ ("Yukawa region"). These regions are displayed in figure 5 (right panel).

3 Leptogenesis in the flavor symmetric phase

We now turn to the discussion of the implications of the presence of lepton flavor symmetries for leptogenesis in models based on type-I seesaw. In general in these models four energy scales can be distinguished: a cutoff scale Λ (typically a scale of heavy matter), the lepton number breaking scale M_N, the flavons scale M_ϕ—determined by the scale of the fields that trigger flavor symmetry breaking—and the scale at which the flavor symmetry is broken, denoted hereafter by v_F. The

[6]In standard leptogenesis at $\mathcal{O}(\boldsymbol{\lambda}^2)$ the efficiency does not depend on the RH neutrino mass.

scale of heavy matter is constrained to be the largest one, the remaining three scales, being free parameters, can follow any hierarchy. In principle six possible hierarchical patterns can be considered; however, since lepton number is an intrinsic feature of seesaw models these possibilities can be split in two generic scenarios:

I The flavor symmetry related scales M_ϕ and v_F are larger than the number breaking scale.

II The flavor symmetry related scales M_ϕ and v_F are smaller than the number breaking scale.

The scale at which leptogenesis takes place is intimately related with the lepton number violating scale. Accordingly in scenarios I leptogenesis proceeds once the flavor symmetry is already broken whereas in scenarios II leptogenesis takes place when the Lagrangian and the vacuum are still flavor invariant i.e in the flavor symmetric phase. The former is considered in sections 4 and 5, the latter cases are the subject of this section.

From now on we will assume the Lagrangian and the vacuum to be invariant under a flavor group G_F. The standard model leptons and RH neutrinos, thus, belong to G_F representations $R_a^{(X)} \sim (X_1, \ldots, X_m)$ (with $X = N, \ell, e$ and $a, b, c \ldots$ denoting G_F indices) in such a way that all the terms in (2) are G_F singlets. As can be seen in (13) a vanishing ϵ_{N_1} implies in turn a vanishing $B - L$ asymmetry. Two conditions have to be satisfied in order to get $\epsilon_{N_1} \neq 0$: (i) Mass splittings among the RH states, otherwise the loop integrals arising from the vertex and wave function corrections do not acquire an imaginary part; (ii) the matrix $\boldsymbol{m_D}^\dagger \boldsymbol{m_D}$ must have non-zero and imaginary off-diagonal elements. The first condition is satisfied if the RH neutrinos belong to different G_F representations (RH neutrinos belonging to the same representation have a common universal mass). But the second condition can never be achieved in the flavor symmetric phase: recovering the correct kinetic terms for the RH neutrinos and lepton doublets requires $R_a^{(N)*} R_b^{(N)} = \delta_{ab}$ and $R_a^{(\ell)} R_b^{(\ell)*} = \delta_{ab}$, for the lepton doublets transforming according to $\bar{\ell} \sim R_a^{(\ell)}$. Taking the scalar electroweak doublet as a G_F singlet, the Yukawa coupling matrix $\boldsymbol{\lambda}$ is determined by the Clebsch-Gordan coefficients arising from the contraction $R_a^{(\ell)} R_b^{(N)}$, thus implying that the matrix $\boldsymbol{\lambda}^\dagger \boldsymbol{\lambda}$ arises from the contractions $R_a^{(N)*} R_b^{(\ell)} R_c^{(\ell)*} R_d^{(N)} = \delta_{ad}\delta_{bc}$ [25].

A non-vanishing $B - L$ asymmetry is possible only if new contributions to the CP violating asymmetry exist ($\epsilon_{N_1}^{\rm New}$) i.e. if the flavons play a role, which they can do as propagating states or virtually via loop corrections. In both cases the kinematical constraint $M_\phi < M_N$ (where M_N is the mass parameter of the $R_a^{(N)}$ representation) must be guaranteed, as otherwise either RH neutrino decays to flavons are kinematical forbidden or the loop integral in which the flavons intervene can not acquire an imaginary part. The flavor models one can envisage can be described by a Lagrangian involving effective operators or models with ultraviolet completions, regardless of the approach the presence of new energy scales, different from that of lepton number violation, can have an impact in the way leptogenesis takes place.

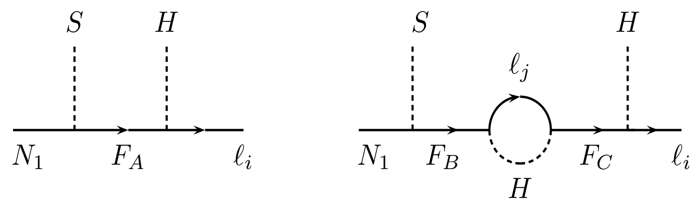

Figure 6: Tree-level and one-loop self-energy diagrams determining $\epsilon_{N_1}^{(\text{New})\,\ell_i}$.

In [25], where the conditions for leptogenesis in the flavor symmetric phase were established, an A_4 inspired model involving effective operators was analyzed in full detail. In contrast, [23, 24] discussed an ultraviolet completed flavor toy model that we now discuss with the purpose of illustrating the previous statements.

We will consider a setup inspired by $U(1)_X$ flavor models *à la* Froggatt-Nielsen [7]. Thus, in addition to the standard model fields and RH neutrinos, the setup also contains vectorlike fermion fields F and a complex scalar field S (flavon), all of them being electroweak singlets. With the horizontal charge assignment $X(\ell, F) = +1$, $X(S) = -1$ and $X(H, N) = 0$ the following Lagrangian can be written

$$-\mathcal{L} = \bar{\ell}\,\boldsymbol{h}\,F\,H + \bar{N}\,\boldsymbol{\lambda}\,F\,S + \frac{1}{2}\bar{N}^T\,C\,\boldsymbol{M_N}\,N + \bar{F}\,\boldsymbol{M_F}\,F\,. \tag{34}$$

Here the Yukawa coupling matrices $\boldsymbol{h}$ and $\boldsymbol{\lambda}$ are 3×3 matrices in flavor space. The $U(1)_X$ symmetry is spontaneously broken by the vacuum expectation value of the complex scalar field, $\langle S \rangle = v_F$. In addition to the $U(1)_X$ symmetry the terms in the Lagrangian (34) preserve a global $U(1)$ symmetry with charge assignments $L(\ell, F, N) = +1$ and $L(H, S) = 0$. This symmetry is only broken by the RH Majorana mass term and thus can be identified with lepton number. In this setup the scale Λ corresponds to M_F, and M_ϕ to M_S. Since leptogenesis in the flavor symmetric phase requires $M_N > v_F, M_S$ the following hierarchies follow $M_F > M_N > v_F, M_S$. With G_F being Abelian the standard contribution to the CP asymmetry does not vanish, but due to the absence of the tree-level coupling $\bar{\ell} N \tilde{H}$—enforced by the flavor charge assignments—ϵ_{N_1} arises at the second loop-order, rendering its value far below the one needed for successful leptogenesis ($\epsilon_{N_1} \gtrsim 10^{-6}$). Therefore, leptogenesis is viable only if new contributions to the CP violating asymmetry are present.

With the couplings in (34), and due to the kinematical constraint $M_F > M_N, M_S$, RH neutrinos have three body decay modes, $N_\alpha \to S H \ell_i$. So in this case the flavon S intervenes in the generation of the $B - L$ asymmetry as a propagating state. The interference between the tree-level decay and the one-loop self-energy correction diagrams shown in fig. 6 determine the new contribution to the flavored CP violating asymmetry, which at leading order in the mass ratio $r_A = M_{N_1}^2/M_{F_A}^2$ reads [23]

$$\epsilon_{N_1}^{(\text{New})\,\ell_i} = \frac{3}{128\pi}\frac{1}{(\tilde{\boldsymbol{\lambda}}\tilde{\boldsymbol{\lambda}})_{11}}\sum_j \mathbb{I}\text{m}\left[\left(\boldsymbol{h}\,\hat{\boldsymbol{r}}^2\boldsymbol{h}^\dagger\right)_{ji}\tilde{\lambda}_{1j}\tilde{\lambda}^*_{1i}\right]\,, \tag{35}$$

[7]Leptogenesis in models based on the Froggatt-Nielsen mechanism have been studied in [54].

where $\hat{\boldsymbol{r}} = \text{diag}(M^2_{N_1}/M^2_{F_1}, M^2_{N_1}/M^2_{F_2}, M^2_{N_1}/M^2_{F_3})$ and the effective couplings $\tilde{\boldsymbol{\lambda}}$ are defined as

$$\tilde{\boldsymbol{\lambda}} = v_F \boldsymbol{\lambda} \,\hat{\boldsymbol{M}}_{\boldsymbol{F}}^{-1}\, \boldsymbol{h}^\dagger \,. \tag{36}$$

The total CP violating asymmetry obtained from (35) by summing over the flavor indices vanishes

$$\epsilon_{N_1}^{\text{New}} = \sum_{i=e,\mu,\tau} \epsilon_{N_1}^{(\text{New})\,\ell_i} = \frac{3}{128\pi} \frac{1}{(\tilde{\boldsymbol{\lambda}}\tilde{\boldsymbol{\lambda}})_{11}} \sum_j \mathbb{Im}\left[\tilde{\boldsymbol{\lambda}}\,\boldsymbol{h}\,\hat{\boldsymbol{r}}^2 \boldsymbol{h}^\dagger\,\tilde{\boldsymbol{\lambda}}^\dagger\right]_{11} = 0\,. \tag{37}$$

Accordingly in the resulting scheme leptogenesis becomes possible only via flavor dynamics and in that sense it is a purely flavored leptogenesis realization [24, 55]. Note that since in this simple case $N_{2,3}$ are not involved in the loop corrections the RH neutrino mass splittings are not relevant. Even if they were relevant a mass splitting could always be accommodated due to the Abelian nature of G_F. When G_F is non-Abelian and the RH neutrinos are placed in multiplets, as already stressed the mass splittings can only be achieved if they belong to different multiplets.

4 Leptogenesis with flavor symmetries: type-I seesaw

The connection between flavor symmetry enforced Tribimaximal (TB) mixing and leptogenesis was investigated by [26], considering models based on A_4 and $Z_7 \rtimes Z_3$. The conclusion derived was that due to the specific construction those models implement, the relevant quantity $\boldsymbol{\mathcal{M}} \equiv \boldsymbol{m_D}^\dagger\, \boldsymbol{m_D}$ is proportional to the identity matrix $\mathbb{I}$ and therefore the CP asymmetry must vanish at leading order (LO) $\mathcal{O}(\eta^0)$, with $\eta \equiv V/\Lambda$ and V a generic flavon vacuum expectation value $\langle\phi\rangle = V$. Importantly, it was noted that there was a difference between having TB at low energy accidentally (which allows leptogenesis to be viable) and TB being enforced by a symmetry. Deviations from the exact mixing limit were also considered and the magnitude of the CP asymmetry was estimated as being connected to the magnitude of the next-to-leading order (NLO), $\mathcal{O}(\eta^1)$, deviations of the mixing angles. These conclusions were illustrated by considering the SUSY model $A_4 \times Z_3$ of [56]. In [27], two specific A_4 models were carefully studied (taking into account washout effects) in order to derive the correlations between the deviation from the exact mixing limit and the generation of leptonic asymmetries. The existing collection of particular cases was generalized into two model-independent results in [28, 29]. Although the conclusions of both generalizations are to some extent equivalent, they rely on different assumptions and it is worth considering both in detail. While [28] is based on group theoretical arguments, [29] is based on general arguments hinging explicitly on the absence of unnatural fine-tuning.

The group theoretical proof [28] starts by assuming invariance of the Lagrangian in (2) under a generic flavor group G_F in the limit $v_F = 0$. Under this assumption the Dirac and RH neutrino mass matrices must remain invariant under G_F

transformations of ℓ and N, namely

$$X \to \Omega_X(g)\, X \quad (\text{with } X = \ell, N)\,, \tag{38}$$

where $\Omega_X(g)$ corresponds to unitary representations of the group G_F for the generic group element g. Different conclusions can be derived depending on whether the representations are irreducible or not:

- If 3 RH neutrinos are in a 3-dimensional irreducible representation the CP asymmetry vanishes at LO. Invariance of the Lagrangian implies the following equality
$$\boldsymbol{\mathcal{M}} = \Omega_N(g)^\dagger \boldsymbol{\mathcal{M}} \Omega_N(g)\,. \tag{39}$$
Since the irreducible representation is 3-dimensional $\Omega_N(g)$ is in general a non-diagonal matrix. Therefore as a direct consequence of this $\boldsymbol{\mathcal{M}}$ is proportional to $\mathbb{I}$ so that the equality can be verified for any group element g. In general all the parameters in (2) receive NLO corrections, from higher dimensional effective operators, and so do the total and flavored CP asymmetries. At $\mathcal{O}(\eta^1)$ two cases can be identified:
  - The loop-functions $f(\omega_\alpha)$ and $g(\omega_\alpha)$ are independent of η: The flavored CP asymmetries $\epsilon_{N_\alpha}^{\ell_i}$ arise at $\mathcal{O}(\eta)$ as they have only one power of $\boldsymbol{\mathcal{M}}$, and $(m^*_{D_{i\alpha}} m_{D_{i\beta}})$ needs not depend on η (the combination has flavor indices so the transformation properties of lepton doublets can be relevant). With the sum over the lepton flavor index i taken, the total CP asymmetry ϵ_{N_α} depends on the square of $\boldsymbol{\mathcal{M}}$ and it arises only at order η^2 (this is in agreement with the results of [26]).
  - The non-Abelian symmetry produces degeneracies in the RH neutrino mass spectrum: there is an enhancement of one order in both asymmetries ($\epsilon_{N_\alpha}^{\ell_i} \sim \mathcal{O}(\eta^0)$, $\epsilon_{N_\alpha} \sim \mathcal{O}(\eta)$) due to the loop functions $f(\omega_\alpha)$ and $g(\omega_\alpha)$ having η^{-1} dependence.
- If the RH neutrinos are in a reducible representation the conclusions do not follow so straightforwardly, but if the LO matrices $\boldsymbol{M_N}$ and $\boldsymbol{\mathcal{M}}$ are simultaneously diagonalizable then the same conclusions as in the case with irreducible representations apply. As a particular case, if the symmetry is Abelian its 1-dimensional representations are in general unable to make the asymmetry vanish, with the requirement that $\boldsymbol{\mathcal{M}}$ is diagonal simultaneously with $\boldsymbol{M_N}$ typically not being fulfilled.

The authors also investigated thoroughly a particular based in the $A_4 \times Z_3 \times U(1)_{FN}$ model ([56]).

The general argument proof in [29] starts from an exact mixing scheme (in the form-diagonalizable sense [57]). The exact mixing is the outcome of a symmetry, not accidental. The proof relies fundamentally on the assumption that the resulting effective light neutrino mass matrix can be diagonalized by a special unitary matrix that does not depend on relationships between the parameters that govern the masses. For definiteness the TB mixing was considered:

$$\hat{\boldsymbol{m}}_{\boldsymbol{\nu}} = \boldsymbol{D}\,\boldsymbol{U}_{\mathbf{TB}}{}^{T}\,\boldsymbol{m}_{\boldsymbol{\nu}}^{\mathbf{eff}}\,\boldsymbol{U}_{\mathbf{TB}}\,\boldsymbol{D}\,, \tag{40}$$

$\boldsymbol{D}$, defined in section 2.1, has the low-energy Majorana phases and $\boldsymbol{U}_{\mathbf{TB}}$ is the PMNS matrix with the corresponding TB values for the mixing angles. The Dirac and RH neutrino mass matrix are diagonalized according to

$$\begin{aligned} \hat{\boldsymbol{m}}_{\boldsymbol{D}} &= \boldsymbol{U}_{\boldsymbol{L}}{}^{\dagger}\,\boldsymbol{m}_{\boldsymbol{D}}\,\boldsymbol{U}_{\boldsymbol{R}}\,, \\ \hat{\boldsymbol{M}}_{\boldsymbol{N}} &= \boldsymbol{V}_{\boldsymbol{R}}{}^{T}\,\boldsymbol{M}_{\boldsymbol{N}}\,\boldsymbol{V}_{\boldsymbol{R}}\,, \end{aligned} \tag{41}$$

with $\boldsymbol{U}_{\boldsymbol{L},\boldsymbol{R}}$ and $\boldsymbol{V}_{\boldsymbol{R}}$ unitary matrices. Then, from the seesaw formula we can write:

$$\boldsymbol{m}_{\boldsymbol{\nu}}^{\mathbf{eff}} = -\boldsymbol{U}_{\boldsymbol{L}}\,\hat{\boldsymbol{m}}_{\boldsymbol{D}}\,\left(\boldsymbol{U}_{\boldsymbol{R}}{}^{\dagger}\,\boldsymbol{V}_{\boldsymbol{R}}\right)\,\hat{\boldsymbol{M}}_{\boldsymbol{N}}{}^{-1}\,\left(\boldsymbol{V}_{\boldsymbol{R}}{}^{T}\boldsymbol{U}_{\boldsymbol{R}}{}^{*}\right)\,\hat{\boldsymbol{m}}_{\boldsymbol{D}}\,\boldsymbol{U}_{\boldsymbol{L}}{}^{T}\,. \tag{42}$$

We assume $\boldsymbol{m}_{\boldsymbol{\nu}}^{\mathbf{eff}}$ is diagonalized by the mixing scheme without special relationships between masses:

$$\hat{\boldsymbol{m}}_{\boldsymbol{\nu}} = -\boldsymbol{D}\,\left(\boldsymbol{U}_{\mathbf{TB}}{}^{T}\boldsymbol{U}_{\boldsymbol{L}}\right)\,\hat{\boldsymbol{m}}_{\boldsymbol{D}}\,\left(\boldsymbol{U}_{\boldsymbol{R}}{}^{\dagger}\,\boldsymbol{V}_{\boldsymbol{R}}\right)\,\hat{\boldsymbol{M}}_{\boldsymbol{N}}{}^{-1}\,\left(\boldsymbol{V}_{\boldsymbol{R}}{}^{T}\boldsymbol{U}_{\boldsymbol{R}}{}^{*}\right)\,\hat{\boldsymbol{m}}_{\boldsymbol{D}}\,\left(\boldsymbol{U}_{\boldsymbol{L}}{}^{T}\boldsymbol{U}_{\mathbf{TB}}\right)\,\boldsymbol{D}\,, \tag{43}$$

therefore the matrix on the left-hand side is diagonal (denoted by the hat), which then implies that the combinations of matrices appearing on the right-hand side of the equation, $\left(\boldsymbol{U}_{\mathbf{TB}}{}^{T}\boldsymbol{U}_{\boldsymbol{L}}\right)$, $\left(\boldsymbol{U}_{\boldsymbol{R}}{}^{\dagger}\,\boldsymbol{V}_{\boldsymbol{R}}\right)$ and conversely $\left(\boldsymbol{V}_{\boldsymbol{R}}{}^{T}\boldsymbol{U}_{\boldsymbol{R}}{}^{*}\right)$, $\left(\boldsymbol{U}_{\boldsymbol{L}}{}^{T}\boldsymbol{U}_{\mathbf{TB}}\right)$ should also be diagonal (up to orthogonal rotations in case of degenerate eigenvalues, but this does not alter the conclusion). Consider for simplicity a case without degeneracies, and evaluate off-diagonal elements of the expression on the right hand side: if the matrix combinations identified above were not diagonal, then the off-diagonal elements of the right-hand side will depend on combinations of the masses of RH neutrinos and Yukawa couplings $\boldsymbol{\lambda}$, which could only vanish for very specific relations between them—which explicitly violates form-diagonalizability. Therefore $\left(\boldsymbol{U}_{\mathbf{TB}}{}^{T}\boldsymbol{U}_{\boldsymbol{L}}\right)$, $\left(\boldsymbol{U}_{\boldsymbol{R}}{}^{\dagger}\,\boldsymbol{V}_{\boldsymbol{R}}\right)$ should indeed be diagonal up to orthogonal rotations of degenerate eigenvalues. Assuming no degeneracies this implies

$$\boldsymbol{U}_{\boldsymbol{L}} = \boldsymbol{U}_{\mathbf{TB}}\,\hat{\boldsymbol{P}}_{\boldsymbol{L}}\,, \qquad \boldsymbol{U}_{\boldsymbol{R}}{}^{\dagger} = \hat{\boldsymbol{P}}_{\boldsymbol{R}}\,\boldsymbol{V}_{\boldsymbol{R}}{}^{\dagger}\,, \tag{44}$$

with $\hat{\boldsymbol{P}}_{\boldsymbol{L},\boldsymbol{R}} = \mathrm{diag}(e^{\mathrm{i}\alpha_1^{L,R}}, e^{\mathrm{i}\alpha_2^{L,R}}, e^{\mathrm{i}\alpha_3^{L,R}})$. These relations allow to fix the structure of the Dirac mass matrix as

$$\boldsymbol{m}_{\boldsymbol{D}} = \boldsymbol{U}_{\mathbf{TB}}\,\hat{\boldsymbol{D}}^{*}\hat{\boldsymbol{m}}_{\boldsymbol{D}}\,, \tag{45}$$

that when compared with the Casas-Ibarra parametrization in (5) leads to $\boldsymbol{R} = \hat{\boldsymbol{m}}_{\boldsymbol{\nu}}{}^{-1/2}\,\hat{\boldsymbol{m}}_{\boldsymbol{D}}\,\hat{\boldsymbol{M}}_{\boldsymbol{N}}{}^{-1/2}$, showing that $\boldsymbol{R}$ is diagonal and real. As the total CP asymmetry can be expressed as:

$$\epsilon_{N_\alpha} = -\frac{3M_{N_\alpha}}{8\pi v^2}\frac{\mathbb{I}\mathrm{m}\left[\sum_i m_{\nu_i}^2 R_{i\alpha}^2\right]}{\sum_i m_{\nu_i}|R_{i\alpha}|^2}\,, \tag{46}$$

then the asymmetry must vanish. Alternatively one can consider the following: $(\boldsymbol{U_R}^\dagger \boldsymbol{V_R})$ is diagonal from our assumption, this means that the basis where $\boldsymbol{m_D}$ is diagonal and the basis where $\boldsymbol{M_N}$ is diagonal have a special relationship (this is often denoted as form-dominance [58] and is essentially also the requirement outlined in the group theoretical approach of [28] in the case of reducible representations). We can simply start with the diagonal basis of $\boldsymbol{m_D}$, use $\boldsymbol{U_R}$ to bring it to the general basis, $\boldsymbol{V_R}$ to bring it to the basis of diagonal $\boldsymbol{M_N}$ and see that in that basis $\boldsymbol{m_D}$ is essentially $\boldsymbol{U_{\mathrm{TB}}}\,\boldsymbol{\hat{m}_D}$—its columns are the eigenvectors of the mixing scheme. Naturally when $\boldsymbol{m_D}^\dagger \boldsymbol{m_D}$ is taken the mixing cancels out and the relevant quantity for the asymmetry is diagonal (consistently with [26, 28]). Although the example uses TB mixing for definiteness, it should be stressed that any exact mixing scheme enforced by a symmetry leads to the same conclusion. The paper also looked into several particular cases of TB mixing, dividing them into classes of models according to the structure of $\boldsymbol{m_D}$ and $\boldsymbol{M_N}$. The structure of NLO contributions was considered explicitly with expansions around the the LO values, leading to:

$$\begin{aligned}\boldsymbol{m_D}'^\dagger \boldsymbol{m_D}' = \boldsymbol{m_D}^\dagger \boldsymbol{m_D} + \boldsymbol{m_D}^\dagger \Big(& \boldsymbol{U_\ell^{(1)}}^\dagger \boldsymbol{m_D} + \boldsymbol{U_L} \boldsymbol{U_L^{(1)}} \boldsymbol{\hat{m}_D} \boldsymbol{U_R}^\dagger \boldsymbol{V_R} + \boldsymbol{U_L} \boldsymbol{\hat{m}_D}' \boldsymbol{U_R}^\dagger \boldsymbol{V_R} + \\ & + \boldsymbol{U_L} \boldsymbol{\hat{m}_D} \boldsymbol{U_R^{(1)}}^\dagger \boldsymbol{U_R}^\dagger \boldsymbol{V_R} + \boldsymbol{m_D} \boldsymbol{V_R^{(1)}} \Big) + \text{h.c.}\,. \end{aligned} \tag{47}$$

The superscript (1) refers to those quantities corrected by NLO contributions and $\boldsymbol{U_\ell}$ diagonalizes the charged lepton mass matrix (we started on the basis where it is diagonal at LO, but it becomes non-diagonal after NLO corrections are introduced). The A_4 model of [36] was used to illustrate the conclusions and to highlight how it can be possible to link low and high-energy CP violation parameters. Finally it was noted that with added degrees of freedom (such as from having type-II seesaw) it would be possible to generate an asymmetry even while remaining in the exact mixing limit.

Not long after these two important generalizations, further results were presented by [30] and [31], clarifying some points that we summarize very briefly here. Assuming that the symmetries of the mass matrices involved in type-I are residual symmetries of the Lagrangian, [30] shows that $\boldsymbol{\mathcal{M}}$ is diagonal and therefore the asymmetry vanishes. They also consider the exact mixing schemes so characteristic of models with flavor symmetries and connect that requirement with their assumption: if the effective neutrino mass matrix has nonzero determinant, then the Lagrangian contains the maximal residual symmetry (that of the mass matrices) and so leptogenesis can not proceed at LO and in fact even when the determinant vanishes, ϵ_{N_α} is still zero at LO. The implication of form-dominance [58] on the Casas-Ibarra matrix $\boldsymbol{R}$ is considered in detail in [31]: the vanishing CP asymmetry is not particular to TB. Rather, exact mixing schemes enforced by symmetries are a particular case of form-dominance [58]. The main conclusions are that form-dominance by itself is sufficient to make the CP asymmetry vanish and that it is possible to violate form-dominance softly without perturbing the mixing. The cases considered earlier in

[34] were summarised, and they exemplify very clearly the separation between TB and form-dominance.

Before concluding this section it is important to stress that corrections to the exact mixing scheme are typically expected at NLO as is explicitly considered in the literature (see e.g. [22]). An exception which does preserve exact mixing is one of the renormalizable UV complete models in [59] (due to the lack of certain messengers). Furthermore, in [60] it is shown that RG corrections can also provide the deviations necessary to lift the vanishing CP asymmetries.

5 Leptogenesis with flavor symmetries: type-I and II seesaws

Recently, a model-independent analysis in the style of [29] considered cases with both type-I and type-II seesaw [32]. Flavor models limit themselves to type-I and/or II seesaws with few exceptions (e.g. [61]). As noted in [29], in general the CP asymmetries involving the additional degrees of freedom can be non-vanishing even in the exact mixing limit and [32] considered the framework with both seesaw types in detail. It was shown that non-vanishing CP asymmetries depend on the existence of repeated eigenvectors across the seesaw types. The main point is the following: leptogenesis can become viable through the CP asymmetries in which the triplets intervene i.e. $\epsilon_{N_\alpha}^{\Delta}$ or ϵ_Δ (see eqs. (22) and (25)), depending—of course—on whether it proceeds via RH neutrino or scalar triplet dynamics (or both as in the case treated in sec. 2.2). Both CP asymmetries depend on the imaginary parts of

$$\boldsymbol{\mathcal{Y}}\,\mu = \boldsymbol{m_D}\,\boldsymbol{Y}^*\,\boldsymbol{m_D}^T\,\mu\,. \tag{48}$$

Since the parameter μ is in general complex, and the presence of G_F does not allow a definitive statement about its phase, the CP asymmetries are non-vanishing even if the matrix $\boldsymbol{\mathcal{Y}}$ turns out to be real. Vanishing $\boldsymbol{\mathcal{Y}}$, however, implies $\epsilon_{N_\alpha}^{\Delta}, \epsilon_\Delta = 0$. In that sense the quantity to be analyzed is $\boldsymbol{\mathcal{Y}}$.

Definitive conclusions about this matrix can be made by writing the effective light neutrino mass matrix as an outer product of the eigenvectors of the assumed mixing scheme [8] and its mass eigenvalues:

$$\boldsymbol{m_\nu^{\text{eff}}} = \sum_{i=e,\mu,\tau} = m_{\nu_i}\,\boldsymbol{v_i}\otimes\boldsymbol{v_i}\,. \tag{49}$$

According to (16) the eigenvectors come from the contributions of type-I and/or type-II seesaws:

$$\boldsymbol{m_\nu^X} = \sum_{i=1,2,3} m_{\nu_i}^X \boldsymbol{v_i}\otimes\boldsymbol{v_i} \qquad (X = I, II)\,. \tag{50}$$

[8]These eigenvectors are determined by the column vectors of the PMNS matrix for a fixed mixing pattern.

This decomposition is based on the assumption that both $\boldsymbol{m_\nu^I}$ and $\boldsymbol{m_\nu^{II}}$ are diagonalized by the PMNS matrix fixed by the assumed mixing scheme. This needs not be the case but if it is not then somehow a contribution that is incompatible with the mixing scheme is present in both seesaw types in just the correct quantities to cancel each other out (which amounts to unrealistic fine-tuning given the separate physical degrees of freedom involved). With the decomposition in (50) we classify the possible models:

A) General models: The eigenvectors $\boldsymbol{v_i}$, defining the effective light neutrino mass matrix, stem from both type-I and type-II contributions. Note that in this case in addition to the pieces involving the eigenvectors $\boldsymbol{v_i}$ each (or only one) seesaw contribution may involve also the identity matrix $\mathbb{I}$.

B) Intermediate models: The eigenvectors $\boldsymbol{v_i}$ entirely arise from either type-I or type-II contributions.

C) Minimal models: Two eigenvectors $\boldsymbol{v_i}$ stem from the type-I (type-II) contributions and the third one $\boldsymbol{v_k}$ (with $\boldsymbol{v_i} \cdot \boldsymbol{v_k} = 0$) from type-II (type-I).

Note that being able to parametrize each seesaw contribution with these eigenvectors does not mean they are all explicitly present. A common scenario can be the explicit presence of only a single eigenvector in a given seesaw type in either cases B or C (with at least one more eigenvector present in the other seesaw). Another relevant observation is that the underlying symmetry may be arranging structures which can be reparametrized in terms of the eigenvectors, meaning one does not necessarily need separate physical degrees of freedom to have more than one eigenvector represented—see e.g. [62] where the $\mu - \tau$ structure

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{51}$$

arises directly from specific discrete groups—as a TB-compatible contribution it can be expressed in terms of the TB eigenvectors as explicitly seen with $b = 2a$ and $c = -3a$ in the parametrization:

$$\boldsymbol{m_\nu^{\text{eff}}} = \boldsymbol{m_\nu^I} + \boldsymbol{m_\nu^{II}} = \begin{pmatrix} 4a+b & -2a+b & -2a+b \\ \cdot & a+b+c & a+b-c \\ \cdot & \cdot & a+b+c \end{pmatrix} . \tag{52}$$

With the models classified according to the eigenvectors of their mixing scheme, we can determine the structures of $\boldsymbol{m_D}$, $\boldsymbol{Y}$ and then also $\boldsymbol{\mathcal{Y}}$ (see [32] for details). However, even without determining explicitly these structures, it can be realized from the definitions in A, B and C that vanishing $\boldsymbol{\mathcal{Y}}$ occurs only when $\boldsymbol{m_D}$ and $\boldsymbol{Y}$ are orthogonal, and in principle this happens only in models of type C: in the other cases, the presence of the scalar triplet degrees of freedom allows the generation of the baryon asymmetry via leptogenesis even in the limit of an exact mixing pattern (in agreement with what was suggested in [29]).

Having identified models where leptogenesis becomes viable in the exact mixing limit, the obvious step is to study those in which the constraints enforced by G_F allow the CP asymmetry to be constrained by the low-energy data. There are in general 6 observables: 3 light neutrino masses and 3 CP phases (only 2 Majorana phases in TB mixing). Thus, models involving more than 6 parameters barely allow to make any statement about the asymmetry. The most general models in A are within that class, with 8 complex parameters. Models in which the asymmetries $\epsilon_{N_\alpha}^{\Delta}, \epsilon_\Delta$ can be constrained by the low-energy data fall within classes B or C. One can add a contribution proportional to $\mathbb{I}$ to either (or both) seesaw types, and any such contribution counts as all (and any) 3 eigenvectors, so the quantity $\mathfrak{Im}[\boldsymbol{\mathcal{Y}}]$ can be expressed in terms of the combinations of parameters defining the quantities $\boldsymbol{m_{\nu_i}^{I,II}}$. Denoting them as a_0^X and a_i^X ($X = I, II$) for $\mathbb{I}$ and the eigenvectors contributions respectively, it turns out that

$$\mathfrak{Im}\left[\boldsymbol{\mathcal{Y}}\right] = \mathfrak{Im}\left[(a_0^I a_0^{II\star}) + \sum_i \left(a_i^I a_i^{II\star}\right) + \left(\sum_i a_i^I\right) a_0^{II\star} + a_0^I \left(\sum_i a_i^{II\star}\right)\right] . \tag{53}$$

In particular for the class of models discussed in item C with only two eigenvectors $\boldsymbol{v_i}$ stemming from type-I (type-II) and $\mathbb{I}$ from type-II (type-I) we have

$$\mathfrak{Im}\left[\boldsymbol{\mathcal{Y}}\right] = \mathfrak{Im}\left[(a_0^X)^\star \sum_{\substack{i<j \\ j=2,3}} a_i^Y\right] , \tag{54}$$

with $X = I$ and $Y = II$ or vice versa. The parameters of these models are only 3 and can be well restricted by means of the solar and atmospheric squared mass differences [19] yielding tight constraints on the CP asymmetries. Figure 7 shows ϵ_Δ in models for which two eigenvectors originate from type-I and the contribution from type-II is proportional to $\mathbb{I}$, assuming a TB mixing pattern. For comparison we have also included the results for the general cases discussed in A involving contributions proportional to the $\mathbb{I}$ in type-I and II. The scatter plot was obtained by randomly scanning the parameters defining the neutrino masses and selecting those points that lead to solar and atmospheric squared mass differences within the experimental range. Figure 7 shows that in general models, even in the limit of an exact mixing pattern no statement about the CP asymmetry can be established. In contrast, in the simplified model considered, specific values of the CP asymmetry require somehow specific ranges for the triplet mass.

6 Conclusions

From a general perspective the problem of studying leptogenesis in the presence of flavor symmetries G_F depends on whether the lepton number breaking scale Λ_L is

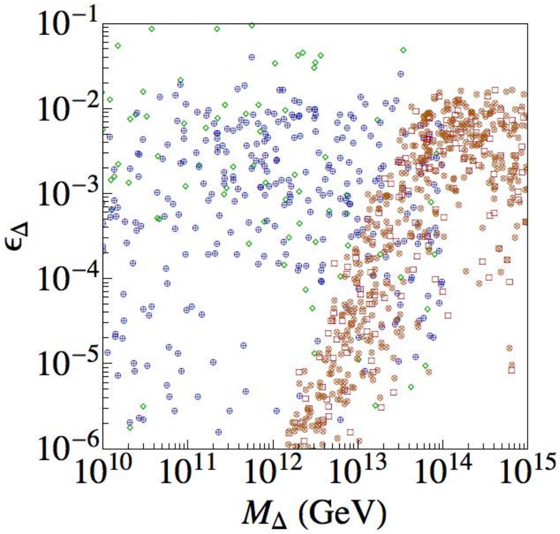

Figure 7: ϵ_Δ *as function of* M_Δ*. Red squares and orange crosses for normal and inverted hierarchy of a specific 3-parameter predictive case. Green squares and blue crosses for normal and inverted hierarchy of the general 8-parameter case [32].*

above or below the scales involved in G_F (flavor breaking and flavon scales, denoted generically v_F and M_ϕ). In the flavor symmetric phase, defined as a scenario in which $\Lambda_L > v_F, M_\phi$, as described in section 3, flavons must play a relevant role in the generation of the lepton asymmetry either as propagating or loop intermediate states. Indeed it turns out that the presence of these states apart from rendering leptogenesis viable can change the conventional picture by e.g. leading to models in which leptogenesis proceeds entirely via lepton flavor effects[9].

In the case of type-I seesaw in the flavor broken phase,defined as a scenario where $v_F, M_\phi > \Lambda_L$, the model-independent conclusion reviewed in section 4 is that CP asymmetries vanish in the exact mixing limit enforced by flavor symmetries. This is not an intrinsic feature of the exact mixing, and this result can be attributed to the property of form dominance in the neutrino mass matrices. Within the scenario of type-I seesaw with symmetry enforced mixing, interesting correlations between low energy observables (mixing angles and CP phases) and high-energy parameters (CP asymmetries) can be present when there is departure from the exact mixing limit. It is possible even in a model-independent context to identify rather generally the order of magnitudes associated with a small parameter responsible for the mixing deviations.

When other degrees of freedom that can contribute to leptogenesis are added, such as those associated with type-II seesaw, the above conclusions need not apply. Section 5 considers specifically the interplay between type-I and II, where it is possible to conclude that the associated asymmetries still vanish in special cases.

[9]The viability of these models depends on whether lepton flavor equilibrating effects can be circumvented [63].

Classifying these hybrid scenarios according to the eigenvectors of the exact mixing is helpful, and even without departure from exact mixing leptogenesis can occur whenever eigenvectors are repeated across the two seesaw types—with contributions proportional to $\mathbb{I}$ counting as any and all eigenvectors.

Finally, we note that in accordance with section 3, in the flavor symmetric phase it is possible to have non-vanishing asymmetry originating just from type-I seesaw while the type-II asymmetries vanish due to orthogonality of the eigenvectors. For this to occur there must be a specific hierarchy of scales so that the RH neutrinos decay in the flavor symmetric phase, avoiding the results described in section 4, while Δ decays in the broken phase with vanishing contributions as described in section 5.

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A Conventions and notation

In this appendix we collect the equations used in the calculations discussed in section 2. We start by specifying well known statistical and cosmological quantities.

A.1 Equilibrium distributions and Cosmological quantities

All the results presented in this short review were done using Maxwell-Boltzmann distribution functions. For type-I and type-III seesaws the equilibrium number densities read

$$n_{\ell,H}^{\rm Eq}(z) = \frac{2M^3}{\pi^2\ z^3}\,, \quad n_{N_1}^{\rm Eq}(z) = \frac{M_{N_1}^3}{\pi^2}\frac{K_2(z)}{z}\,, \quad n_X^{\rm Eq}(z) = \frac{3M_X^3}{2\pi^2}\frac{K_2(z)}{z} \quad (\text{with } \ X = \Delta, T_\alpha)\,. \tag{55}$$

Here $K_2(z)$ is the second-order modified Bessel function of the second-type and $z \equiv M/T$ where M can refer to M_{N_1,Δ,T_α} depending on the considered case (this also applies for $n_{\ell,H}^{\rm Eq}(z)$). For the type-II scenario discussed in 2.2 the N_1 equilibrium number density is given by

$$n_{N_1}^{\rm Eq}(z) = \frac{M_\Delta^3}{\pi^2} r^2 \frac{K_2(rz)}{z}\,, \tag{56}$$

with $r = M_{N_1}/M_\Delta$. The energy density $\rho(z)$ and pressure $p(z)$ become

$$\rho(z) = \frac{3M^4}{z^4\pi^2} g_* \,, \quad p(z) = \frac{M^4}{z^4\pi^2} g_* \tag{57}$$

where $g_* = \sum_{i=\text{All species}} g_i$ is the number of standard model relativistic degrees of freedom (118 for $T \gg 300$ GeV). Accordingly, the expansion rate of the Universe and entropy density can be written as

$$H(z) = \sqrt{\frac{8g_*}{\pi}} \frac{M^2}{M_{\text{Planck}}} \frac{1}{z^2} \,, \quad s(z) = \frac{4M^3}{z^3\pi^2} g_* \,. \tag{58}$$

A.2 Formal solutions of the kinetic equations

In the type-I and III seesaw cases the integration of the differential equations accounting for the evolution of the $B-L$ asymmetry leads to

$$Y_{\Delta_{B-L}}(z) = -n \times \epsilon_X \, Y_X^{\text{Eq}}(z_0) \eta(z) \,, \tag{59}$$

where $X = N_1, T_1$ and $n = 1, 3$ depending on whether the decaying state is the singlet or the triplet. Assuming a vanishing initial asymmetry ($Y_{\Delta_{B-L}}^{(\text{In})} = 0$) the efficiency function can be written as

$$\eta(z) = \frac{1}{Y_{T_\alpha}^{\text{Eq}}(z_0)} \int_{z_0}^{z} Q_X(z') \frac{dY_X(z')}{dz'} e^{-\int_{z'}^{z} dz'' P_X(z'')} \,, \tag{60}$$

with the functions $Q_X(z), P_X(z)$ given by

$$Q_{N_1}(z) = 1 \,, \quad Q_{T_1}(z) = \frac{\gamma_{D_{T_1}}}{\gamma_{D_{T_1}} + 2\gamma_A} \,, \quad P_{N_1,T_1}(z) = \frac{1}{2Y_\ell^{\text{Eq}}(z)} \frac{\gamma_{D_{N_1,T_1}}(z)}{s(z)H(z)z} \,. \tag{61}$$

Freeze-out of the asymmetry is at $z = z_f$ with $z_0 \ll z_f$. The efficiency factor is determined by $\eta = \eta(z_f)$.

The case for type-II is more involved but the kinetic equation for the L asymmetry in (18) can still be formally integrated [32]. Again, assuming an initial vanishing L asymmetry, we get

$$Y_{\Delta_L}(z) = \int_{z_i}^{z} dz' \, Q(z') \, e^{-\int_{z'}^{z} dz'' \, P(z'')} \,, \tag{62}$$

with the functions $Q(z) = Q^I(z) + Q^{II}(z)$ and $P(z)$ given by

$$Q^I(z) = \frac{1}{s(z)H(z)z} \left\{ \left[(y_{N_1}(z) - 1)\epsilon_{N_1}^{\text{tot}} - y_{\Delta_\Delta}^H(z) \right] \gamma_{D_{N_1}}(z) \right\} \,, \tag{63}$$

$$Q^{II}(z) = \frac{3}{s(z)H(z)z} \left\{ \left[(y_\Sigma(z) - 1)\epsilon_\Delta - 2K_\ell \, y_{\Delta_\Delta}(z) \right] \gamma_{D_\Delta}(z) \right\} \,, \tag{64}$$

$$P(z) = \frac{1}{s(z)H(z)z}\left[\frac{1}{Y_\ell^{\rm Eq}}\left(\gamma_{D_{N_1}}(z) + 2\,K_\ell\,\gamma_{D_\Delta}(z)\right)\right]. \tag{65}$$

Note that in $Q^{II}(z)$ we have included a factor of 3 coming from the $SU(2)$ physical degrees of freedom of the triplet. By factorizing either $\epsilon_{N_1}^{\rm tot}$ or ϵ_Δ from the functions $Q^{I,II}(z)$ and normalizing to $Y_{\rm tot}^{\rm Eq} \equiv Y_{\rm tot}^{\rm Eq}(z \to 0) = Y_{N_1}^{\rm Eq}(z) + Y_\Sigma^{\rm Eq}(z)|_{z\to 0}$ the L asymmetry in (62) can be written in terms of efficiency functions that depend on the dynamics of the scalar triplet and the fermionic singlet as done in eq. (27).

A.3 Reduced cross sections for triplet scalar and fermion

The reduced cross section for the scalar electroweak triplet involves the s-channel processes $\boldsymbol{\Delta\Delta} \to F\bar{F}, AA, HH$ (F and A stand for standard model fermions and $SU(2)\times U(1)$ gauge bosons respectively), t and u channel triplet mediated processes $\boldsymbol{\Delta\Delta} \to AA$ and the "quartic" process $\boldsymbol{\Delta\Delta} \to AA$. In powers of the kinematic factor $\omega(x) = \sqrt{1-4/x}$ (with $x = M_\Delta^2/s$) it can be split in three pieces [15]:

$$\begin{aligned}
\widehat{\sigma}_1(x) &= \frac{1}{\pi}\left[g^4\left(5+\frac{34}{x}\right)+\frac{3}{2}g'^4\left(1+\frac{4}{x}\right)\right]\omega(x)\,,\\
\widehat{\sigma}_2(x) &= \frac{1}{8\pi}\left(25g^4+\frac{41}{2}g'^4\right)\omega(x)^3\,,\\
\widehat{\sigma}_3(x) &= \frac{6}{\pi x^2}\left[4g^4(x-1)+g'^4(x-2)\right]\ln\left[\frac{1+\omega(x)}{1-\omega(x)}\right]\,,
\end{aligned} \tag{66}$$

with $\widehat{\sigma}_A(x) = \sum_{i=1}^3 \widehat{\sigma}_i(x)$ and g, g' the $SU(2)$ and $U(1)$ gauge couplings.

For the fermion $SU(2)$ triplet the reduced cross sections involves the gauge boson mediated s-channel processes $T_\alpha T_\alpha \leftrightarrow \ell\bar{\ell}$ and $T_\alpha T_\alpha \leftrightarrow q\bar{q}$ and the t and u-channel triplet mediated process $T_\alpha T_\alpha \leftrightarrow A_\mu A^\mu$. The full result where now $x = M_{T_1}^2/s$, reads [14]:

$$\widehat{\sigma}_A(x) = \frac{6g^4}{\pi}\left(1+\frac{2}{x}\right)\omega(x)+\frac{2g^4}{\pi}\left[3\left(1+\frac{4}{x}-\frac{4}{x^2}\right)\log\left(\frac{1+\omega(x)}{1-\omega(x)}\right)-\left(4+\frac{17}{x}\right)\omega(x)\right]. \tag{67}$$

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