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Simplified procedure for estimating the effects of the non linearity of aerodynamic coefficients

Abstract in German : Since the early developments in wind engineering, aerodynamic coefficients have been linearized in turbulence analysis. On the other hand general and complex methods based on generalized power spectral densities can now handle this kind of non linearity. Between these two extremes, it would be desirable to give simple estimation of the effects of the non linearity of aerodynamic coefficients on the response of structures.
This paper aims at giving an overview of these general and complex methodologies (that could be applied in other contexts), and, in a second step, presenting an original procedure to give estimation of the effects of the non linearity of aerodynamic coefficients. This procedure will be illustrated on the drag coefficient of the Viaduct of Millau.

Wind analysis of a structure subjected to non-linear random dynamic loading

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1 Introduction

Thanks to new materials and more precise computation methods, today’s engineers often design very flexible structures. Even if they present an attractive aesthetic interest, they can however exhibit serious vibration problems.
In the civil engineering field, many dynamic loadings come from natural events. As a particular case, since the famous and tragic break-down of the Tacoma-Narrows Bridge, the wind loading of structures has shown more and more interest. The very first developments of Davenport [4] have been upgraded in order to account for complex phenomena like galloping, flutter, etc. It is quite amazing to see that all these complex phenomena are still nowadays represented by linear wind loading models [2]. In this paper, we will go back to the very first developments of Davenport, avoid the assumptions of linearization and see how a non linear wind loading model can bring new observations.
The first two sections are devoted to the presentation of some characteristics of the non linear wind loading. Even if this characterization of the loading is already very precious, designers of structure are often interested in determining the characteristics of the response. This will be presented in section 4.
Finally, an illustration of these methods will show how to measure easily the importance of the non linearity of an aerodynamic coefficient. In this example, the non linearity of the drag coefficient of the Viaduct of Millau (France) will be studied.

2 Quasi-steady wind model

Forces acting on a body immerged in a fluid result from the normal pressures acting on it. In civil engineering applications, three forces (drag, lift and moment) are generally considered. For example, the aerodynamic drag force F_D acting on a fixed body in a uniform flow with constant velocity V can be expressed by [11]:

$$F_D = \frac{1}{2} C_D \rho B V^2 \tag{1}$$

where ρ and B represent respectively the air density and the width of the body, i.e. the bridge deck in our further application.
Even if CFD models can simulate the flow around a bluff body, the aerodynamic coefficients are generally measured by means of wind tunnel tests. Fig. 1 shows examples of aerodynamic coefficients.
It can be seen that these coefficients are significantly dependent on the wind angle of attack i, i.e. the angle between the wind direction and the bridge deck.
Civil engineering structures are built in the atmospheric boundary layer. In this region the wind flow is known to be turbulent; it is composed of a mean velocity U and longitudinal $u(t)$ and transverse $v(t)$ fluctuations. Davenport [4] proposed to express the forces acting on a

structure immerged in such a turbulent flow by the same relation:

$$F_D(t) = \frac{1}{2} C_D\left[i(t)\right] \rho B V^2(t) \qquad (2)$$

where both the wind angle of attack $i(t)$ and the squared wind velocity $V^2(t)$ are now time-dependent.

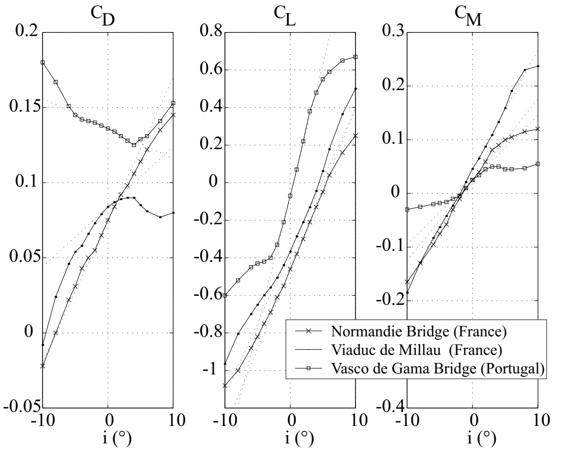

Fig. 1. Examples of aerodynamic coefficients

With notations of Fig. 2-b, these two quantities can be expressed by:

$$i(t) = ArcTan\left(\frac{v(t) - \dot{h}(t)}{U + u(t) - \dot{p}(t)}\right) - \alpha(t) \qquad (3)$$

$$V^2(t) = \left(U + u(t) - \dot{p}(t)\right)^2 + \left(v(t) - \dot{h}(t)\right)^2 \qquad (4)$$

where relative velocities (with respect to the bridge deck) have been considered in order to represent a fluid-structure interaction.

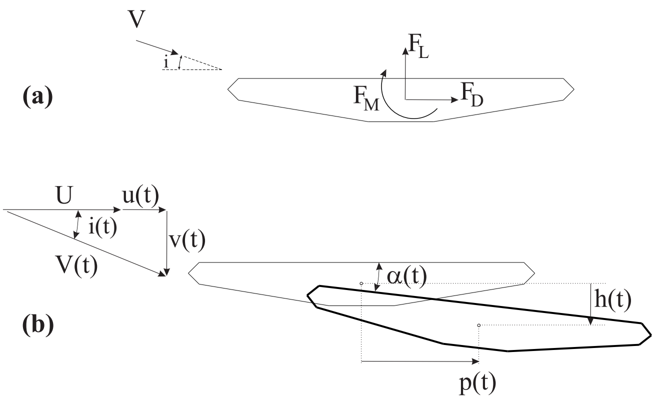

Fig. 2. (a) Aerodynamic forces in a uniform flow ; (b) Components of turbulence and displacements of the deck

It is generally assumed that the components of the turbulence, $u(t)$ and $v(t)$, can be correctly represented by Gaussian stochastic processes [11]. Equations (2) to (4) show thus that the aerodynamic forces are non linear functions of Gaussian processes. Furthermore, as can be seen on Fig. 1, the aerodynamic coefficient is also a non linear function of the wind angle of attack.

As mathematical and numerical tools weren't adapted to study this kind of loading, the wind engineering pioneers decided to linearize the expression of loading. Thanks to a Taylor series expansion of the aerodynamic coefficient:

$$C_D(i) = c_0 + i c_1 + \frac{i^2}{2!} c_2 + \frac{i^3}{3!} c_3 + \ldots \qquad (5)$$

and a Taylor series expansion of relations (3) and (4), the non linear expression of the loading in a 2-D flow could be transformed into a linear loading (see Equ. 8).

In this paper, we would like to investigate the effects of the non linearity of the loading. It can be shown ([6], [7], [8]) that this non linearity can be treated in closed forms provided the non linearity is a polynomial expression.

It is thus decided to consider, as a most general loading, the quadratic expression obtained when developing the exact expression of loading (Equ. (2)) in Taylor series:

$$\frac{F_D(t)}{\frac{1}{2}\rho B U^2} = c_0 + 2c_0 \frac{u(t)}{U} + c_1 \frac{v(t)}{U} + c_0 \frac{u^2(t)}{U^2} + c_1 \frac{u(t)v(t)}{U^2} + \left(c_0 + \frac{c_2}{2}\right)\frac{v^2(t)}{U^2} \qquad (6)$$

In order to estimate the effects of the non linearity of the aerodynamic coefficients, a second loading can be considered by adding the assumption of linear aerodynamic coefficient ($c_2 = 0$):

$$\frac{F_D(t)}{\frac{1}{2}\rho B U^2} = c_0 + 2c_0 \frac{u(t)}{U} + c_1 \frac{v(t)}{U} + c_0 \frac{u^2(t)}{U^2} + c_1 \frac{u(t)v(t)}{U^2} + c_0 \frac{v^2(t)}{U^2} \qquad (7)$$

As a second approximation of the most general loading (Equ. (6)), the linear approximation adopted by Davenport can also be derived:

$$\frac{F_D(t)}{\frac{1}{2}\rho B U^2} = c_0 + 2c_0 \frac{u(t)}{U} + c_1 \frac{v(t)}{U} \qquad (8)$$

It is important to notice that the purpose of this paper is to enlighten the effects of the non linearity of the loading. Some researchers have already worked in that field but generally in a 1-D flow ([6], [10]). In such a case, the quadratic expression of the loading (Equ. 6) reduces to:

$$\frac{F_D(t)}{\frac{1}{2}\rho B U^2} = c_0 + 2c_0 \frac{u(t)}{U} + c_0 \frac{u^2(t)}{U^2} \qquad (9)$$

which shows that the effects of the non linearity of the aerodynamic coefficient (c_2) can't be studied in a 1-D flow. In such a turbulence field, it is thus just possible to study the difference between a quadratic and a linear loading model:

$$\frac{F_D(t)}{\frac{1}{2}\rho B U^2} = c_0 + 2c_0 \frac{u(t)}{U} \qquad (10)$$

In the following, the main focus will be on Equs. (6) to (8) since it is desired to show the influence of the aerodynamic coefficient.

3 Non Gaussianity of the wind loading

In this section, it will be shown that the probabilistic shape of the loading can be severely affected by the non linear terms of the loading.

3.1 Illustration by Monte Carlo simulation

As an example, let us consider the drag coefficient of the Viaduct of Millau (Fig. 3). The linear and quadratic approximations of this coefficient are represented on Fig. 3. They have been obtained by a least square approximation with weighting functions proportional to the probability density function of the wind angle of attack. For these two approximations, the optimized values of the coefficients are respectively:

$$c_0 = 0.0840\,;\, c_1 = 0.1671 \qquad (11)$$

$$c_0 = 0.0830 \;\;;\, c_1 = 0.2020\,;\, c_2 = -3.3999 \qquad (12)$$

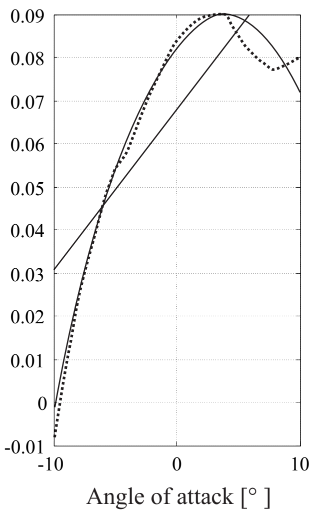

Fig. 3. Linear and quadratic approximation of the drag coefficient (Viaduct of Millau)

For this example, it will be supposed that the wind intensity is 15% for each turbulence component ($I_u = \sigma_u/U = 0.15$ and $I_v = \sigma_v/U = 0.15$).

In order to compare the wind models, a long Gaussian sample (N = 500000 points) has been generated for each component of the wind turbulence. Then the non-dimensional expressions of the five loadings (Eqs. (6) to (10)) have been used in order to establish histograms of the non-dimensional drag force (Fig. 4).

In Fig. 4, thick lines represent the linear models; in this case, the loading is proportional to the turbulence and is thus also a Gaussian process.

In a 1-D turbulence field, it is often admitted that the non linearity of the loading can bring important modifications to the statistics of the loading ([6], [10]). The histograms relating to the 2-D flow exhibit a much more evident non Gaussian shape. In this case, the non Gaussianity of the loading really has to be taken into account. As a particular case, when the whole second order expression in kept (Equ. 6), the probability density function is severely skewed to the left.

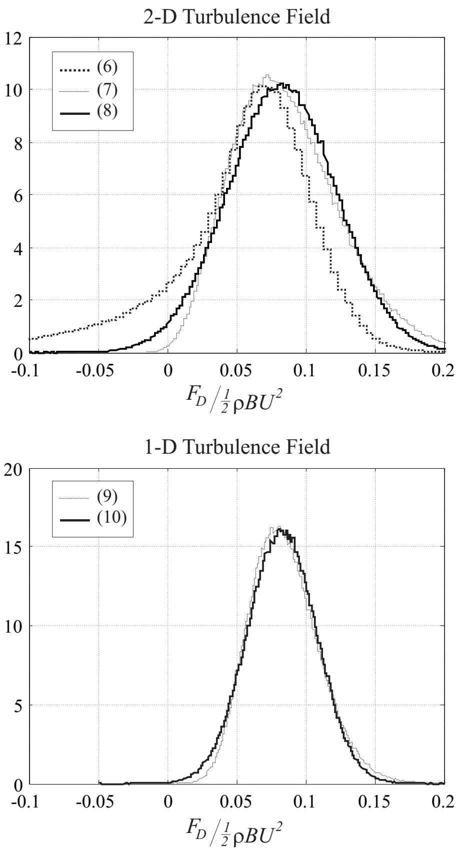

Fig. 4. Probability density function of the non dimensional drag force obtained with different wind models

The characterization of a non Gaussian random variable *f* requires the knowledge of higher statistical moments. The mean and the variance are no longer sufficient to fully describe the statistics.

At the third order, it is necessary to add the skewness coefficient:

$$\gamma_3 = \frac{E\left[\left(f-\mu_f\right)^3\right]}{\sigma_f^{\,3}} \tag{13}$$

which describes the asymmetry of the probability density function. In this relation, μ_f and σ_f represent the mean and the standard deviation of the random variable *f*. At the fourth order, the excess coefficient:

$$\gamma_e = \gamma_4 - 3 = \frac{E\left[\left(f-\mu_f\right)^4\right]}{\sigma_f^{\,4}} - 3 \tag{14}$$

represents the shape of the tail of the probability density function. The definition of these coefficients is particularly efficient since they both equal zero for Gaussian variables.

Table 1 summarises the statistical characteristics (up to the fourth order) of the generated forces. It can be seen that :

- models (8) and (10) correspond effectively to linear (gaussian) models;
- the non linearity of the aerodynamic coefficient (6) is significant since it gives large skewness and excess coefficients
- comparison between (7) and (9) shows that the importance of the non linearity of the loading (excluding the non linearity of the aerodynamic coefficient) is more significant in a 2-D field than in a 1-D one.

Table 1. Statistical characteristics of the forces computed by Monte Carlo simulation - Loadings (6) to (10).

Monte Carlo simulation				
	Mean	Std. Dev.	Skewness	Excess
(6)	0.0484	0.0649	-2.0669	7.8811
(7)	0.0867	0.0397	0.6177	0.5336
(8)	0.0829	0.0392	0.0007	0.0027
(9)	0.0848	0.0250	0.4481	0.2721
(10)	0.0830	0.0249	0.0018	0.0041

It should be noticed that the determination of the statistical properties of the loading would be very heavy if Monte Carlo simulations had to be repeated many times. The next paragraph presents a mathematical approach that gives very fast and exact results.

3.2 Probabilistic approach of the loading

Provided the joint density function of some elementary variables is known, the probability theory allows computing the statistical properties of any function of these variables.

For instance, the statistical characteristics associated to each loading model can be derived in a mathematical, and thus rigorous, way. These relations are reported to *annex I* for the quadratic loading in a 2-D turbulence field (Equ. 6). The application of these formulae can lead to exact values of statistical moments. They are represented in Table 2.

The comparison of Tables 1 and 2 shows that the Monte Carlo simulation represents accurately (but much more heavily) the non Gaussian phenomenon.

Fig. 4 lets guess that an important feature concerning non Gaussian processes is their extremum statistics. Indeed, it can be seen that the likelihood to have large negative applied forces is much larger in the non linear model. Gurley and al. [9] proposed an interesting method to account for the effects of the non Gaussianity of a stochastic process on its extreme values. For an observation duration *T*, the expected extreme values of a process *f* can be estimated by :

$$f_{MIN} = \mu_f - k_{NG}^{-} g_f \sigma_f \ ; \ f_{MAX} = \mu_f + k_{NG}^{+} g_f \sigma_f \tag{15}$$

where g_f represents the peak factor (as if the process was Gaussian) and k_{NG}^{+} and k_{NG}^{-} are modification factors accounting for the non Gaussianity of the process. These two quantities can be expressed in terms of the statistical properties that have been established before (γ_3, γ_e). They are given in References [9], [5].

As designers are often interested in estimating extreme values, the main reason to compute the skewness and excess coefficients is finally to use them in the estimation of the modification factors k_{NG}^{+} and k_{NG}^{-}.

Table 2. Statistical characteristics of the forces computed by the probability theory - Loadings (6) to (10).

Analytical approach				
	Mean	Std. Dev.	Skewness	Excess
(6)	0.0485	0.0649	-2.0565	7.8020
(7)	0.0867	0.0397	0.6147	0.5125
(8)	0.0830	0.0392	0	0
(9)	0.0849	0.0250	0.4458	0.2655
(10)	0.0830	0.0249	0	0

For loadings (6) to (8), the extreme values have been computed for several values of the turbulence intensities. In the rest of this document, we will suppose that both wind intensities (I_u and I_v) are equal.

The extreme values are represented at Fig. 5 where it can be seen that the non linearity of the aerodynamic coefficient (Equs. (6) and (7)) presents a serious influence on the extreme values of loading. Furthermore, this figure shows that the non linearity of the aerodynamic coefficient is the most accountable for these effects. In this particular case, the variance of the loading and its excess coefficient (obtained for loading

(6)) are so large that the expected extreme values to be considered are larger in the positive direction as well as in the negative one.

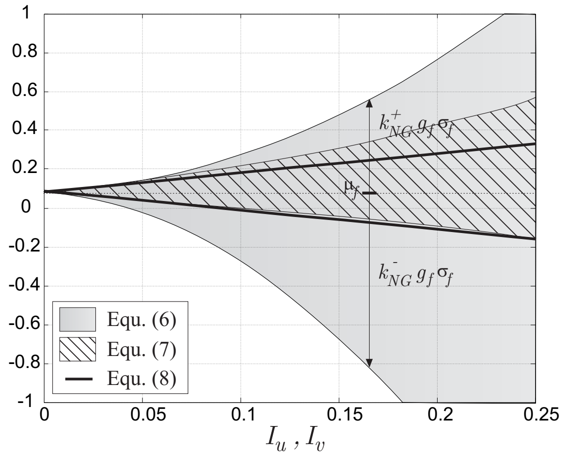

Fig. 5 Extreme values of non dimensional loading as a function of the turbulence intensities (T=600s)

4. Spectral analysis of structures subjected to random loading

Even if it is important to describe to loading as precisely as possible, designers are however mainly interested in determining the displacements of the structure, the internal forces and the stresses. In this section, we will give a schematic summary of the procedure to follow in order to establish the response of a structure subjected to a random dynamic loading.

4.1 Second order characteristics

The second order characteristics of a random loading are represented by its power spectral density (PSD). For example, the PSD of the applied force represents the distribution in the frequency domain of the variance of the process. The frequency content of the response , i.e. its PSD (see Fig. 6), can be obtained by multiplying the PSD of the force by the squared transfer function of the single degree of freedom system. The surface under the resulting function (function of the frequency) is equal to the variance of the response.

4.2 Third order characteristics

At the third order, the reasoning is identical but somewhat more complex since one dimension is added. The third order statistical characteristics of the loading are represented by its bispectrum. This function represents the distribution, in a 2-D frequency space, of the third order statistical moment of the loading. On the second line of Fig. 6, an example of typical wind force bispectrum is given. Exactly as the transfer function (2^{nd} order) represents the way a frequency content must be filtered, at the third order, the Volterra kernel represents the way the bispectrum of the force must be filtered in order to obtain the bispectrum of the response.

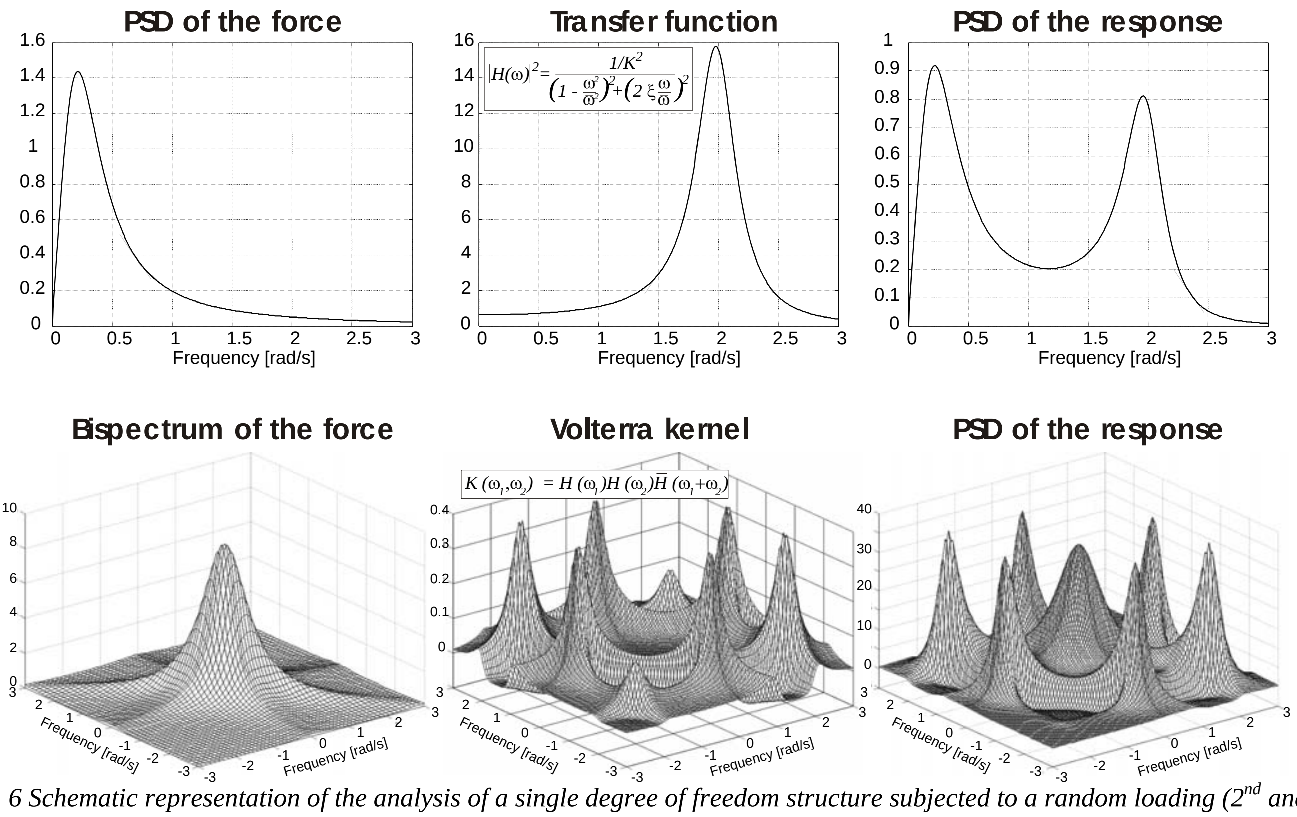

Fig. 6 Schematic representation of the analysis of a single degree of freedom structure subjected to a random loading (2^{nd} and 3^{rd} orders)

. In other words, exactly as it was done at the second order, a simple multiplication of the two lower left functions gives the bispectrum of the displacements. Then, the computation of the volume under this surface gives the third order moment, which in turn, gives an estimation of the skewness coefficient.

4.3 Fourth order characteristics

The fourth order characteristics of the response can be established following an identical procedure but working now with a supplementary dimension. This makes the graphical representation quite complicated. Anyway, it must be understood that the trispectrum (distribution of the fourth order moment in a 3-D frequency space) of the response can be obtained by multiplying the trispectrum of the force by a fourth order Volterra kernel.

5. Simple estimation of the importance of the non linearity of an aerodynamic coefficient

5.1. Method : objectives and limitations

Some finite element codes allow analyzing the non Gaussian response of large FE models, like for instance a 3-D bridge with various non linear aerodynamic coefficients, immerged in a 3-D turbulence flow. It is known that the analysis at the second order is already time-consuming when the rigorous approach presented in the previous section has to be applied. This observation is again strengthened at the third and fourth orders since the integration has to be performed on 2-D and 3-D frequency spaces.

At the very first steps of a new project, it is unthinkable to run such a complex and complete analysis. It is thus important to be able to estimate, in a very simple manner, the order of magnitude of the main quantities. As a particular case, it would be interesting to estimate the effects of the non linearity of the aerodynamic coefficients.

In this view, we propose to analyse a single degree of freedom system whose loading would be represented by Equs. 6 and 7, where C_D is the aerodynamic coefficient that has to be studied. This sort of qualification of the aerodynamic coefficient is interesting because :

- it replaces a MDOF system by a SDOF system and is hence very easy to apply;
- it can account for the real frequency content of the turbulence components *u(t)* and *v(t)* (von Kármán, EC1, Davenport, Harris, ...);
- the operation can be rapidly repeated for any eigen frequency, damping coefficient or wind intensities. This gives thus an interesting mapping that can be used in further developments of the design since, at this stage, the definitive eigen frequencies are generally not fixed yet. This kind of mapping can thus show the main tendencies: “what happens in terms of extreme values if the eigen frequency is increased? What is the influence of the damping ration?” etc.

5.2. Example

This method of qualification of aerodynamic coefficients will be applied to the drag coefficient of the Viaduct of Millau. In order to realize the developments in closed form, we will suppose that the power spectral densities of the turbulence components are given by:

$$S_u(\omega)=\frac{\alpha\sigma_u^2}{\pi\left(\alpha^2+\omega^2\right)};\quad S_v(\omega)=\frac{\alpha\sigma_v^2}{\pi\left(\alpha^2+\omega^2\right)} \tag{16}$$

where α is a shape factor (units: rad/s). It has been shown [6] that this form of PSD is close to practical turbulence PSD’s (von Kármán, Davenport, EC1...) that can be well reproduced by an appropriate choice of parameter α, generally resulting in $\alpha \in$ *[0.2 rad/s;0.5 rad/s]*.

In order to estimate the effect of the non linearity of the aerodynamic coefficient, loadings (6) and (7) will be applied to a single degree of freedom system with unit mass ($M=1$):

$$\ddot{x}(t)+2\xi\varpi\dot{x}(t)+\varpi^2x(t)=\frac{F_D(t)}{\frac{1}{2}\rho BU^2} \tag{17}$$

where ϖ and ξ represent respectively the circular eigen frequency and the damping ratio (including the aerodynamic damping).

5.2.1. First order (mean value)

The mean displacement of the system is obtained by dividing the mean applied force. The ratio between the displacements computed with and without the aerodynamic non linearity is thus equal to the same ratio on forces. At the first order, the influence of the non linearity is independent of the natural characteristics of the system (ϖ and ξ). The effects of the non linearity are thus identical for the loading and the response.

In the upper left corner of Figure 7, the mean displacement (normalized by $\frac{1/2\,\rho BU^2}{M\varpi^2}$) is represented for both considered loadings and as a function of the wind intensity.

For large intensity levels, the curvature of the aerodynamic coefficient (towards the bottom) is so important that it can even decrease the mean value of the response to zero.

5.2.2. Second order (standard deviation)

At the second order, the power spectral density of the applied force must be established. Thanks to the simple analytical expression of the PSD's of the turbulence components, this expression is rather simple. It is given in *Annex II*. The application of the procedure summarized in the previous section gives successively analytical expressions for the PSD of the displacement and finally for its standard deviation. This last expression is more complex and is not given in the paper. It is however represented at figure 7 where the influence of the non linearity of the aerodynamic coefficient can again be observed. For the simplicity of the representation, Fig. 7 represents the statistical moments of one particular system ($\varpi = 6 rad / s$; $\xi = 0.03$) but it is obvious that any other SDOF system could be used.

It could be shown that the standard deviation obtained with the quadratic aerodynamic coefficient (Equ. 6) is always larger (for any SDOF and for any aerodynamic coefficient) than the one obtained with the linear coefficient (Equ. 7). This is a consequence of the hypothesis of uncorrelated turbulence components.

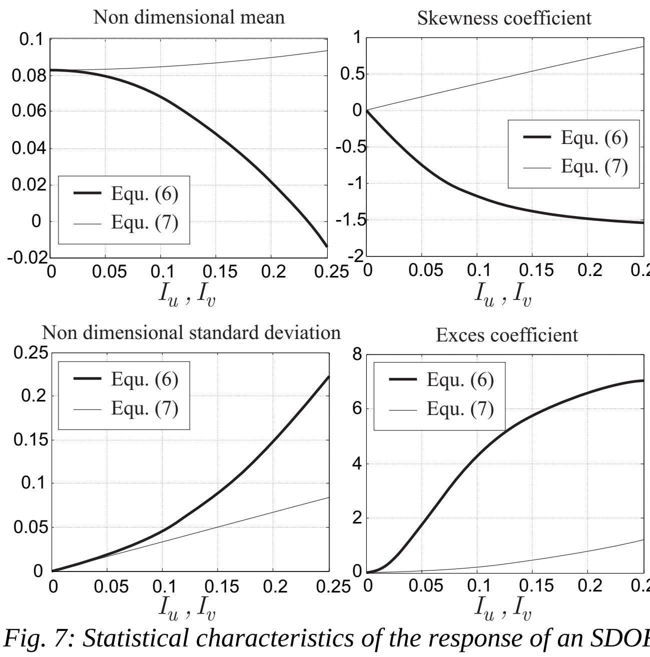

Fig. 7: Statistical characteristics of the response of an SDOF system (ϖ =6rad/s; ξ=0.03) to a non Gaussian loading including (6) or not (7) the non linearity of the drag coefficient of the Viaduct of Millau ($\alpha = 0.2 rad / s$).

5.2.3. Third order (skewness coefficient)

Computations at the third order become hard to undertake in closed forms. The expression of the bispectrum of the loading can however be established. The bispectrum of the response is obtained by multiplying this quantity by the second Volterra kernel. Finally, an analytical expression of the third order moment can be obtained.

The resulting skewness coefficient is represented in the upper right corner of Fig. 7. A comparison of this graph with the corresponding one established for the loading would show (resp. for Equs 6 and 7) that the skewness coefficient of the response is smaller than the skewness coefficient of the loading. This indicates that the structure acts like a mitigating operator and reduces thus the dissymmetry eventually existing in the loading. This observation is a general rule that can be justified by the existence, in the response, of a Gaussian dynamic behaviour and of a non Gaussian quasi-static response. The non Gaussianity of this second contribution to the response comes directly from the non Gaussianity of the loading and is mitigated by its dynamic counterpart.

This reasoning leads to the conclusion that the dissymmetry of the response is more important for structures having a mainly quasi-static behaviour.

A negatively (resp. positively) skewed loading results in a negatively (resp. positively) skewed response. In fact loadings (6) and (7) are negatively and positively skewed. This explains why the skewness coefficients of the response are completely different (Fig. 7) for loadings (6) and (7). This demonstrates again the importance of the non linearity of the aerodynamic coefficient.

5.2.4. Fourth order (excess coefficient)

The same methodology can also be applied at the fourth order. Analytical expressions can be obtained successively for the trispectrum of the loading, the trispectrum of the response, the fourth order statistical moment and finally the excess coefficient. This quantity is represented in the lower right corner of Fig 7.

The same reasoning as was led at the third order indicates that the excess coefficient of the response must be smaller than the excess coefficient of the loading.

In the chosen application, the significant difference between the excess coefficients of both responses comes unsurprisingly from the same significant difference between the excess coefficients of the loading.

5.2.5. Extreme values

In the introduction to this document, it was announced that the third and fourth order moments had to be estimated in order to give more realistic extreme values (for design) than those that would be obtained with a Gaussian model.

The previous analytical developments can thus now be pushed to the estimation of the extreme values of the displacement. For the SDOF system studied here, Fig. 8 represents, as a function of the wind intensity, in the shaded (resp. hatched) areas the envelope of the displacement obtained with the quadratic (resp. linear) drag coefficient.

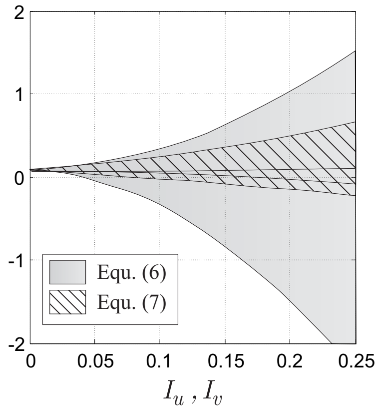

Fig. 8: Extreme values on a 600-sec duration of the response of an SDOF system (ϖ =6rad/s; ξ=0.03) to a non Gaussian loading including (6) or not (7) the non linearity of the drag coefficient of the Viaduct of Millau ($\alpha = 0.2 rad / s$).

As for the loading, if can be observed that the quadratic term of the loading exhibits a significant influence on the extreme values. Despite of the important dissymmetry of the response to the left (γ_3<0), the positive extreme values obtained with the non linear model are much larger. As for the loading, this is due to very large variance and excess coefficient.

5.2.6. Parametrical study

In the former paragraphs, the dispatching of energy between quasi-static and dynamic components was presented as an interesting indicator for the estimation of the statistical properties of the response.

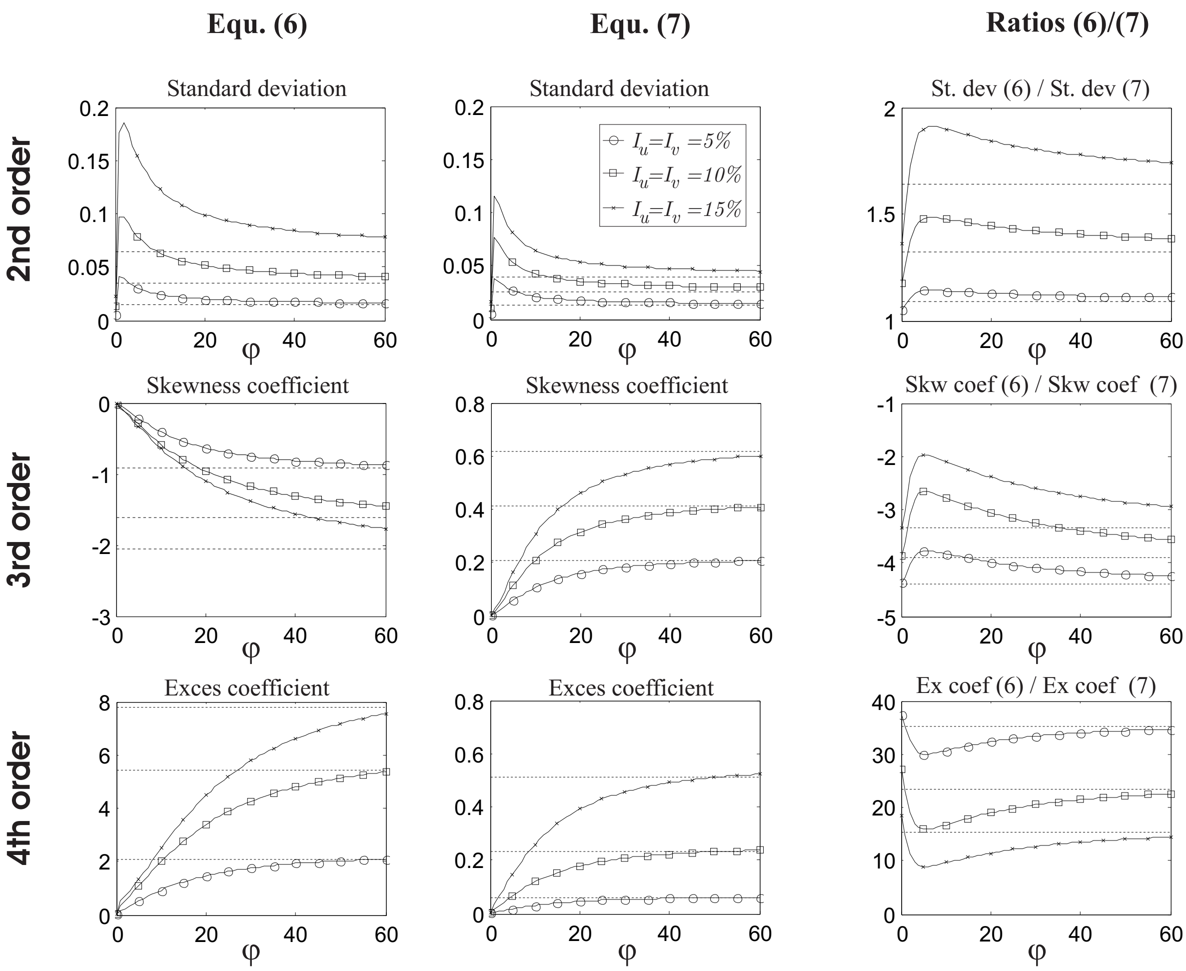

Fig. 9.Influence of the structural stiffness (φ= ϖ /α) on the statistical characteristics of the response (ξ=0.03)

Figure 9 shows the influence of the stiffness of the system (φ=ϖ/α) on the statistical properties of the response. At the second order (first row), it can be seen, as announced before, that the standard deviation obtained with the non linear model (Equ. 6). is always larger than the one obtained with the linear model (Equ. 7). The ratio (upper right corner) between results obtained with models (6) and (7) shows that the variance can be almost twice underestimated for 15% of turbulence intensity. Dotted lines represent the normalized standard deviations of the loading. Fig. 9 (upper right corner) shows that for very stiff structures (φ is large, the response is hence essentially quasi-static), the ratio between the standard deviations of the response tends towards the ratio of the standard deviations of the loading.

Estimating this last quantity is very easy. It could thus be used in case of stiff structures to estimate the effects of the non linearity of the loading on the response.

For the third and fourth orders, skewness and excess coefficients are represented. It can be seen that these coefficients tend, for stiff structures, towards the corresponding coefficients associated to the loading. On the other hand, soft structures exhibit a dynamic (resonant) behaviour and present thus a Gaussian response: skewness and excess coefficients tend towards zero for stiffness approaching zero. On the graphs of the last two rows, it can also be observed, as announced, that the skewness (excess) of the loading, represented in dotted lines, is always more important than the skewness (excess) of the response.

6. Conclusions

The analysis of a structure subjected to a non Gaussian random loading can be divided into two steps.

The first important stage is the characterization of the loading. It is important because the statistical moments of the loading can be estimated very easily and can be used to give rough estimations (mainly for stiff structures) of the statistical characteristics of the response. Some information like extreme values of the loading can already show the importance (or not) of the non linearity of the aerodynamic coefficient.

Some sophisticated methods (using bispectra and trispectra) allow establishing the statistical characteristics of the response of a structure subjected to a dynamic random loading. These methods have been presented and applied to estimate the effects of the non linearity of the drag coefficient of the Viaduct of Millau on the response of an SDOF system. Even if main tendencies can be established by looking at the characteristics of the loading, these rigorous methods allowed giving exact values of the statistical moments of the response. The studied case showed that the non linearity of this drag coefficient influences much the extreme displacement, through important increases of the variance and excess coefficient.

Some general comments have also been formulated concerning the influence of the non linearity of the loading:

- due to a mitigating effect of the dynamic component of the response, skewness and excess coefficients of the response are smaller than the corresponding coefficients of the loading;
- it is unnecessary to estimate the non Gaussian characteristics of structures having insignificant quasi-static component; they can be considered as Gaussian.

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8. References

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Annex I

Statistical properties of loading (6), up to the fourth order:

$$\mu = c_0\left(1 + I_u{}^2\right) + \left(c_0 + \frac{c_2}{2}\right) I_v{}^2$$

$$\sigma^2 = 2c_0^2 I_u{}^2\left(2 + I_u{}^2\right) + c_1^2 I_v{}^2\left(1 + I_u{}^2\right) + \frac{1}{2}\left(2c_0 + c_2\right)^2 I_v{}^4$$

$$k_3 = 8c_0^3 I_u{}^4\left(3 + I_u{}^2\right) + 6c_0 c_1^2 I_u{}^2 I_v{}^2\left(2 + I_u{}^2\right) + 3c_1^2\left(2c_0 + c_2\right) I_v{}^4\left(1 + I_u{}^2\right) + \left(2c_0 + c_2\right)^3 I_v{}^6$$

$$\gamma_3 = \frac{k_3}{\sigma^3}$$

$$k_4 = 48c_0^4 I_u{}^6\left(4 + I_u{}^2\right) + 48c_0^2 c_1^2 I_u{}^4 I_v{}^2\left(3 + I_u{}^2\right) + 6c_1^2\left(c_1^2 + 4c_0\left(2c_0 + c_2\right)\right) I_u{}^2 I_v{}^4\left(2 + I_u{}^2\right) + 12c_1^2\left(2c_0 + c_2\right)^2 I_v{}^6\left(1 + I_u{}^2\right) + 3\left(2c_0 + c_2\right)^4 I_v{}^8$$

$$\gamma_e = \frac{k_4}{\sigma^4}$$

Annex II

The power spectral density of the applied force (6) is composed of two terms:

$$S_f = \left(4c_0^2 I_u^2 + c_1^2 I_v^2\right)\frac{\alpha}{\pi\left(\alpha^2 + \omega^2\right)} + \left(2c_0^2 I_u^4 + c_1^2 I_u^2 I_v^2 + 2\left(c_0 + \frac{c_2}{2}\right)^2 I_v^4\right)\frac{2\alpha}{\pi\left(4\alpha^2 + \omega^2\right)}$$

The first term corresponds to the linear terms of loading; the second one comes from the autoconvolutions of the PSD's of the turbulence components and is typical of the non linear loading.